

# The Role of Different Perpendicular Diffusion Models in Anomalous and Galactic Cosmic Ray Modulation

J.A. le Roux<sup>1</sup>, G.P. Zank<sup>1</sup>, V.S.Ptuskin<sup>2</sup>, and H. Moraal<sup>3</sup>

<sup>1</sup> *Bartol Research Institute, University of Delaware, Newark, DE 19716, USA*

<sup>2</sup> *IZMIRAN, Troitsk, Moscow District, Russia*

<sup>3</sup> *Space Research Unit, School of Physics, Potchefstroom University, Potchefstroom, South Africa*

## Abstract

We evaluate 3 different theories for perpendicular diffusion dominated by field line random walk by calculating anomalous and galactic cosmic ray spectra on the basis of standard cosmic ray transport theory and comparing them with certain features in observed 1996 spectra.

## 1 Introduction:

Currently, the perpendicular diffusion ( $\kappa_{\perp}$ ) of cosmic rays (CRs) is not well understood which hampers our understanding of CR modulation. In response, we presented calculations of the cosmic ray (CR) radial diffusion coefficient ( $\kappa_{rr}$ ) in **another contribution to this conference** (required reading!) using 3 different plausible theoretical models for  $\kappa_{\perp}$  dominated by field line random-walking effects (le Roux et al., 1999a, b). The dependence of  $\kappa_{rr}$  on radial distance ( $r$ ) was determined theoretically on the basis of a MHD model for the transport of predominantly 2D turbulence in the solar wind (Zank et al., 1996, le Roux et al., 1999a, b). To evaluate the  $\kappa_{\perp}$  models, we use a CR modulation model limited to negligible CR transport in the polar direction. We compare our simulated CR spectra with certain key features in observed anomalous CR (ACR) and galactic CR (GCR) He obtained during a time of relatively small latitudinal gradients (1996).

## 2 The Modulation Model:

We solve the standard CR transport equation assuming spherical symmetry and apply it in the ecliptic plane in the upwind direction. In spherical coordinates, the equation for the isotropic part  $f(r, p, t)$  of the nearly isotropic CR distribution function at radial distance  $r$  from the Sun for particle momentum  $p$  at time  $t$  is

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa_{rr} \frac{\partial f}{\partial r} \right) - U \frac{\partial f}{\partial r} + \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 U) \frac{\partial f}{\partial \ln p} + Q(r, p) \quad (1)$$

where  $U$  is the solar wind flow speed for a radially expanding solar wind ( $U = Ue_r$ ), and  $Q$  is a source function for ACRs. The model can be used for GCR or ACR transport. The model includes a solar wind termination shock at  $r_{sh} = 85$  AU where  $U$  drops by a factor of 3.1 from  $400 \text{ km s}^{-1}$ , consistent with a termination shock weakened by pickup ions. At the shock, CRs are assumed to undergo diffusive shock acceleration. Downstream of the shock  $U \propto 1/r^2$  implying the assumption of incompressible radial flow. The modulation boundary is assumed to be at 120 AU where a local interstellar GCR He<sup>++</sup> spectrum is specified as  $j_T = 0.38E_k^{0.52}/(E_k + 0.25E_0)^{2.6}$  particles  $\text{m}^{-2} \text{sr}^{-1} (\text{MeV/nuc})^{-1}$  where  $E_k$  is kinetic energy in GeV  $\text{nuc}^{-1}$ , and  $E_0$  is the rest mass energy of a proton. In our study, we focus on ACR and GCR He because the modulated peaks of these 2 species are better separated than those of hydrogen. The source term  $Q$  is given by  $Q = Q_0 \delta(r-r_{sh}) \delta(R-R_{inj})$  where  $Q_0$  is a free parameter,  $r_{sh}$  is the shock distance from the Sun,  $R$  is rigidity and  $R_{inj} \approx 0.1$  GV is the injection rigidity for ACRs. The injection rigidity is chosen at an appropriately high value for which diffusive shock acceleration theory should be applicable at the quasi-perpendicular termination shock.

### 3 The Results:

We use the following features of CR He spectra observed during 1996 (e.g., McDonald, 1998) in our  $\kappa_{\perp}$  evaluation test: (1) Modulated GCR He<sup>++</sup> peaks at  $\sim 210$  MeV nuc<sup>-1</sup> at a distance of  $\sim 65$  AU where the differential intensity is 0.45 particles m<sup>-2</sup> s<sup>-1</sup> sr<sup>-1</sup> (MeV nuc)<sup>-1</sup>. The increase in the modulated peak intensities between 1 and 65 AU is a factor of  $\sim 1.5$ . (2) The ACR He<sup>+</sup> spectrum peaks at  $\sim 7$  MeV nuc<sup>-1</sup> at 65 AU and the increase in the modulated peak intensities between 1 and 65 AU is  $\sim 24$ .

In Figure 1 we present calculated ACR and GCR spectra where we used the modified quasi-linear theory



**Figure 1:** (a) Simulated differential intensity of ACR He<sup>+</sup> in particles m<sup>-2</sup> s<sup>-1</sup> sr<sup>-1</sup> (MeV nuc<sup>-1</sup>)<sup>-1</sup> in the ecliptic plane for the MQLT model of  $\kappa_{\perp}$ . The spectra are shown for radial distances 1, 23, 42, 65, and  $r_{sh}+2$  AU from the Sun where  $r_{sh} = 85$  AU is the distance to the termination shock. (b) The simulated GCR He<sup>++</sup> spectra are shown for the same distances as in (a) and for 100, and 120 AU. The curves from 1- $(r_{sh}+2)$  AU overlap too much to be distinguishable from each other. The curve labeled 120 AU denotes the local interstellar spectrum at the CR modulation boundary.

(MQLT) model of  $\kappa_{\perp}$  (see le Roux et al., 1999a). The modulated ACR He<sup>+</sup> spectra at 65 AU in Figure 1(a) peaks at  $\sim 40$  MeV nuc<sup>-1</sup> which is much higher than the observed  $\sim 7$  MeV nuc<sup>-1</sup>. This is caused by the rollover portion of the source spectrum at the shock that is located at too high energies. Since the characteristic rollover energy at the shock is determined by the condition  $\kappa_{rr}/U > r_{sh}$ , the problem is caused by a too small  $\kappa_{rr}$  in those regions. In addition, the increase in the peak density between 1 and 65 AU is just  $\sim 5.9$  instead of the observed  $\sim 24$ . The small upstream  $\kappa_{rr}$  ensures that adiabatic cooling and convection is more important than diffusion in equation (1). In this limit, radial CR intensity gradients tend to zero and the intensity  $j_T \propto E_k$ .

The calculated GCR He<sup>++</sup> spectra in Figure 1(b) are very strongly modulated between the assumed CR modulation boundary at 120 AU due to the large drop in  $\kappa_{rr}$  across the shock (le Roux et al., 1999a, b) with most of the modulation occurring between 100 and 120 AU. Upstream, the radial CR intensity gradient is zero and  $j_T \propto E_k$  for  $E_k < 0.1$  GeV nuc<sup>-1</sup> which is a typical result where adiabatic cooling and convection dominates the relatively small  $\kappa_{rr}$ . Consequently, the energy at which the modulated spectrum at 65 AU peaks ( $\sim 2$  GeV nuc<sup>-1</sup>) is clearly too high by an order of magnitude and the intensity at this peak (0.03 particles m<sup>-2</sup> s<sup>-1</sup> sr<sup>-1</sup> (MeV nuc)<sup>-1</sup>) is too low by a similar margin.

In conclusion, the dominance of  $\kappa_{\parallel}$  in  $\kappa_{rr}$ , because  $\kappa_{\perp}$  is too small to make a contribution (see also Zank et al., 1998), is a major drawback for attempts to simulate realistic modulated CR spectra. This points to need of a more efficient  $\kappa_{\perp}$  which the MQLT model fails to provide. One reason for the failure of the theory is that the theory does not allow for a description of a perpendicular correlation length associated with 2D turbulence. This length might be much larger than the parallel correlation length associated with slab turbulence (Zank et al., 1998).

In Figure 2 we present calculated ACR and GCR He spectra where we used the modified anomalous diffusion model (MAD) model for  $\kappa_{\perp}$  (see le Roux, 1999a, b). In the simulated ACR He<sup>+</sup> spectra in Figure

2(a) the intensity peaks at  $\sim 8 \text{ MeV nuc}^{-1}$  at 65 AU, which is remarkably close to the observed value of  $\sim 7 \text{ MeV nuc}^{-1}$ . Unfortunately, the enhancement in the peak intensity from 1 to 65 AU is  $\sim 177$ , an order of magnitude too large according to our observational test. The large radial intensity gradient implies that we are in a strong diffusive-convective limit where adiabatic cooling is unimportant relative to diffusion, combined with the mentioned large positive radial dependence of  $\kappa_r$  in the outer heliosphere. That large radial gradients in the CR intensity are inevitable follows from the diffusion-convection expression that the radial gradient  $G_r \propto 1/\kappa_r$ .

In Figure 2(b) we show calculated GCR  $\text{He}^{++}$  spectra. The peak intensity in the modulated spectrum at 65 AU is at  $\sim 110 \text{ MeV nuc}^{-1}$  instead of  $\sim 210 \text{ MeV nuc}^{-1}$ . The intensity level at the peak ( $\sim 1.5 \text{ particles m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} (\text{MeV nuc}^{-1})^{-1}$ ) is also too large by a factor of 3.3. This indicates an absence of modulation due to a very large  $\kappa_{\perp}$  at large  $r$ . However, the factor of  $\sim 2.1$  enhancement in the peak intensity between 1 and 65 AU is larger than observed, just as for the ACR component and for the same reasons. As expected, there is no sign of the strong downstream CR modulation barrier found when using the MQLT model for  $\kappa_{\perp}$ .

To conclude, the MAD model for  $\kappa_{\perp}$  indicates that the simulated spectra better reproduce the observations when  $\kappa_{\perp}$  contributes significantly to  $\kappa_r$ . The drawback of the model is that the radial gradients



**Figure 2:** As Figure 1, except that the simulations were done for the MAD model of  $\kappa_{\perp}$ .

in the simulated CR intensity are too large. This is caused by the strong  $r$ -dependence of  $\kappa_{\perp}$  at large upstream  $r$  due to the resonant interaction of CRs with the energy range of the solar wind turbulence.

In Figure 3 we display calculated ACR and GCR He spectra on the basis of the nonperturbative (NP) model for  $\kappa_{\perp}$  (Zank et al., 1998; le Roux et al., 1999a, b). In Figure 3(a) the calculated ACR  $\text{He}^{+}$  spectra show a peak intensity at an energy of  $\sim 18 \text{ MeV nuc}^{-1}$  at 65 AU instead of the observed  $\sim 7 \text{ MeV nuc}^{-1}$ . The factor in the peak intensity variation between 1 and 65 AU is  $\sim 20$ , which is close to our estimated observed value of  $\sim 24$ . Overall, the ACR spectra produced with the NP model fare better in our observational test than those spectra calculated with the other 2 models of  $\kappa_{\perp}$ .

The simulated GCR  $\text{He}^{++}$  spectra are displayed in Figure 3(b). The intensity at 65 AU peaks at  $210 \text{ MeV nuc}^{-1}$  which is the same as our estimate from the observed spectra. The intensity at 65 AU peaks at a level of  $0.68 \text{ particles m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} (\text{MeV nuc}^{-1})^{-1}$  instead of the observed value of  $\sim 0.45 \text{ particles m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} (\text{MeV nuc}^{-1})^{-1}$  which is in much closer accord with the observations compared to the other 2 models of  $\kappa_{\perp}$ . The same can be said for the factor of change in the peak intensity between 1 and 65 AU which is  $\sim 2.1$  compared to the  $\sim 1.5$  observed.

The main conclusion is that the NP model of  $\kappa_{\perp}$  does the best in reproducing roughly the basic characteristics of both GCR and ACR spectra observed in 1996. This suggests that the NP model is useful for modulation studies, provided CR transport effects in the heliospheric polar direction are of lesser importance. The 3 rigidity interval dependence of  $\kappa_r$  in the outer heliosphere plays a key role in this success. The  $R^2$  dependence at low  $R$  ensures the simulation of realistic ACR spectra as were found first by Cummings et al. (1994) in their modeling efforts. The smaller  $R$ -dependence at intermediate energies

ensures that galactic CR suffer sufficient modulation at those rigidities, and the  $R^2$  dependence at high rigidities enables the fulfillment of observations that galactic CRs are not modulated at the highest rigidities. The sufficiently weak  $r$ -dependence of  $\kappa_{rr}$  in the outer heliosphere at intermediate  $R$  is an important element in the successful simulation of realistic CR intensity gradients. The need for a similar 3



**Figure 3:** As Figure 1, except that the simulations were done for the NP model of  $\kappa_{\perp}$ .

interval  $R$ -dependence for  $\kappa_{rr}$ , with the weakest  $R$ -dependence in the center interval, was first recognized by Moraal et al. (1999) as necessary for the simulation of both observed GCR and ACR spectra. Moraal et al., who followed an empirical approach in specifying the CR diffusion tensor and assumed suppressed CR transport in the polar direction in their CR modulation model, also pointed out the need for a  $\kappa_{rr}$  with a weaker  $r$ -dependence as an additional requirement for simulating observed CR spectra. The NP model for  $\kappa_{\perp}$  gives tentatively a theoretical basis for the findings of Moraal et al.

#### 4 Summary and Conclusions:

We simulated ACR and GCR modulation with a spherically symmetric CR modulation model in the limitation of negligible CR transport in the heliospheric polar direction. In the process,  $\kappa_{rr}$  was theoretically calculated using 3 different plausible models for  $\kappa_{\perp}$  (see le Roux et al., 1999a, b) and a MHD model for solar wind turbulence (Zank et al., 1996; le Roux et al., 1999a, b). The 3 models were evaluated against CR spectra observed during 1996, a period characterized by small latitudinal CR intensity gradients. The NP model for  $\kappa_{\perp}$  gave overall the best reproduction of the observed spectra. Nonetheless a word of caution is necessary, because the perpendicular correlation length in this model is a poorly constrained parameter at this stage, and particle simulations so far failed to confirm the theory for  $\kappa_{\perp}$  at low rigidities (Giacalone, 1998; Mace et al., 1999). Evidently, this theory needs further study at low rigidities.

### References

- Cummings, A.C., Stone, E.C., and Webber, W.R. 1994, JGR, 99, 11547  
 Giacalone J. 1998, Space Sci. Rev., 83, 351  
 le Roux, J.A., Zank, G.P., Ptuskin, V.S., & Moraal, H. 1999a, Proc. 26<sup>th</sup> ICRC (Salt Lake City, 1999)  
 le Roux, J.A., Zank, G.P., Ptuskin, V.S., & Moraal, H. 1999b, JGR, submitted  
 Mace, R., Matthaeus, W.R., & Bieber, J.W. 1999, ApJ, submitted  
 McDonald, F.B. 1998, Space Sci. Rev., 83, 33  
 Moraal, H., Steenberg, C.D., & Zank, G.P. 1999, Adv. Space Res., in press  
 Zank, G.P., Matthaeus, W.H., Bieber, J.W., & Moraal, H. 1998, JGR, 103, 2085  
 Zank, G.P., Matthaeus, W.H., & Smith, C.W. 1996, JGR, 101, 17093