Particle Drifts in a Fluctuating Magnetic Field

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Abstract

We examine the drifts of particles in a fluctuating magnetic field using direct numerical simulation of particle trajectories. We superimpose a randomly fluctuating magnetic field upon a background uniform field, as in previous papers. We show that the relation $\kappa_{ij} = \langle w_i \Delta x_j \rangle$is particularly useful, in that it allows direct computation of the antisymmetric diffusion coefficient. We focus on deviations from the standard result $\mathbf{v_d} = (pcw/3q)\ \nabla \times (\mathbf{B}/B^2)$ caused by fluctuations in the magnetic field.

1 Introduction:

Particle drifts in a magnetic field which has a mean which varies with position are a basic aspect of the motion of energetic particles or cosmic rays. In general, the motion of cosmic rays is composed of the diffusive motion caused by the scattering of the particles due to the fluctuating part of magnetic field and the drift motions resulting from large-scale gradient and curvature of the average magnetic field. The nature of the diffusive transport, and the relation of the diffusion coefficients to the turbulent structure of the magnetic field has been extensively studied over the years. In particular, the diffusion parallel to the average magnetic field seems to be fairly well understood, whereas the perpendicular diffusion $\kappa_\perp$ is less so (Fisk et al., 1998). In addition to the perpendicular and parallel diffusion, determined by the *symmetric* part of the diffusion tensor, a mean magnetic field produces in general an *antisymmetric* part to the diffusion tensor, usually termed κ_A. In general we may write the diffusion tensor as

$$\kappa_{ij} = \kappa_\perp \delta_{ij} - \left(\kappa_\perp - \kappa_{||} \right) \frac{B_i B_j}{B^2} + \kappa_A \epsilon_{ijk} \frac{B_k}{|B|}, \tag{1}$$

where ϵ_{ijk} is the totally antisymmetric tensor. The drift velocity $\mathbf{v_D}$(averaged over the nearly-isotropic distribution) may be shown to be precisely the divergence of the antisymmetric part of the diffusion tensor (Jokipii, Levy, & Hubbard, 1977). Depending on the situation, one may work in terms of either the drift velocity itself or the antisymmetric diffusion tensor. In the following we will use the term drift velocity or antisymmetric diffusion tensor interchangeably.

In this paper we examine the nature of the gradient and curvature drifts in the presence of turbulent fluctuations. The standard expression for the drift velocity of a charged particle of mass m, charge q, momentum p, and speed w in a magnetic field $\mathbf{B}$, in the limit that the scattering mean free path is much larger than the gyro-radius r_g, is $\mathbf{v_d} = (pcw/3q)\ \nabla \times (\mathbf{B}/B^2)$, where c is the speed of light. The corresponding $\kappa_A = w r_g/3$. This is the limit most-frequently used, since we expect that the mean free path is generally somewhat larger than the gyro-radius. A finite amount of scattering should reduce this somewhat.

A simple analysis based on the venerable billiard ball scattering picture suggests that scattering by fluctuating magnetic field might reduce the drifts by a noticeable amount for cosmic rays in the heliosphere (Jokipii, 1993; see also, Burger & Moraal, 1990). Similarly some analyses of the modulation of galactic cosmic rays by the solar wind suggest that the drift motions in the heliospheric magnetic field are significantly reduced from the classical value given above (e.g Potgieter, Le Roux, & Burger, 1989). In this special case the expressions for $\kappa_\perp$ and κ_A become, in terms of the ratio η of the mean free path λ to the gyroradius r_g,

$$\kappa_\perp = \frac{\kappa_{||}}{1+\eta^2} \tag{2}$$

$$\kappa_A = \frac{\kappa_{||}\eta}{1+\eta^2} \tag{3}$$

where $\kappa_{||}$ is the parallel diffusion coefficient. Again $\mathbf{v}_d$ is the divergence of the antisymmetric part of the diffusion tensor.

Here we utilize direct numerical simulations of particle motions in the turbulent magnetic field to analyze the effects of fluctuations on the drifts. As far as we know, this has not been done before.

2 Particle Drifts:

Before proceeding to the results of the numerical simulations, we first present an analytical result which we believe has not been previously published and which enables us to simplify and make more precise the numerical analysis. For simplicity in notation, we assume without loss of generality that the average magnetic field, at least locally, is in the z direction, so that the perpendicular directions are x and y. In determining the transport coefficients from numerical simulations it is usual to work in terms of the Fokker-Planck transition moments $\langle \Delta x^2 \rangle / \Delta t$, etc (e.g., Giacalone & Jokipii, 1999). In this case the drift term appears in one or more of the first-order coefficients, for example $\langle \Delta x \rangle / \Delta t$. However, this will only be non-zero when the magnetic field has spatial variation, and this is more complicated to compute numerically. Hence it is usually more convenient to work with the antisymmetric diffusion coefficient, which is non-zero even if there are no gradients, and whose divergence is the drift velocity. But the obvious Fokker-Planck Coefficient $\langle \Delta x \Delta y \rangle / \Delta t$ is obviously symmetric. The reason is that the divergence of the antisymmetric tensor is zero if the field does not vary, and in this case the antisymmetric coefficient does not appear in the diffusion equation. But it does appear in the equation for the streaming flux, or anisotropy. We must proceed differently.

It may be shown that, in general, the equation for the streaming flux in a simple system with no convection, may be written

$$F_i = -\kappa_{ij} \frac{\partial f}{\partial x_j} \tag{4}$$

where the diffusion tensor κ_{ij} can be written $\kappa_{ij} = \langle w_i \Delta x_j \rangle$. It is easily seen that this also gives the antisymmetric part of κ_{ij}. Furthermore, this form is much simpler to compute in a simulation.

The above result can be demonstrated as follows. At some time t, we may express the value of the distribution function $f(x_i, p_i, t)$ in terms of the values of the $p_i = p_i'$ and $x_i = x_i'$ corresponding to the x_i, p_i at some other time t' (following the actual particle trajectories) by the *exact* relation

$$f(x_i, p_i, t) = f(x_i', p_i', t'), \tag{5}$$

which is simply a restatement of Liouville's theorem. Now consider the situation where the time $\Delta t = (t' - t)$ is many scattering times, but where the corresponding $\Delta x_i = (x_i' - x_i)$ is much smaller than the scale of spatial variation of f (this is equivalent to the usual diffusion approximation). Then, since f is nearly isotropic and the momentum magnitude of a particle is constant, the spatial gradient $\partial f / \partial x_i$ is approximately the same for all directions (all particles at a given p) and we may write $f(x_i', p_i', t') \approx f(x_i, p_i, t) + \Delta x_i \partial f(x_i, p_i, t) / \partial x_i$. Therefore, since the p_i' at x_i' are scrambled relative to the p_i, so that $\int w f(x_i', p_i', t') d\Omega = 0$, the diffusive flux at x_i, t is

$$\begin{aligned} F_i(p, x_i, t) &= -\int w_i f d\Omega = -\int w_i \Delta x_j d\Omega \ \frac{\partial f}{\partial x_j} \\ &= -\langle w_i \Delta x_j \rangle \frac{\partial f}{\partial x_j} \\ &= -\kappa_{ij} \frac{\partial f}{\partial x_j} \end{aligned} \tag{6}$$

which is the desired result (4). Here we make use of the fact that the diffusion coefficient κ_{ij} depends on the magnitude of p as well as x_i and t, so the integral over Ω sums over all the particles at a given p and the $\partial f / dx_i$ properly weights the sum over particles. Below we compute κ_A from the relationship $\kappa_{ij} = \langle w_i \Delta x_j \rangle$.

3 Numerical Simulations

We integrate the trajectories of particles moving under the influence of a time-independent magnetic field of the form $\mathbf{B}(\mathbf{r}) = B_0\hat{z} + \delta\mathbf{B}(\mathbf{r})$. The fluctuating component, $\delta\mathbf{B}(\mathbf{r})$, is determined in a manner similar to that which we have described previously (c.f. Giacalone & Jokipii, 1994, 1996, 1999). They are characterized by a discrete sum of individual stationary plane waves with random wave vectors, phases, and polarizations. The amplitudes are given by a Kolmogorov-like power spectum which is described mathematically in terms of three parameters: the total integrated power, σ^2, the correlation length, L_c, and the spectral index γ (for all simulations considered here we set $\gamma = 5/3$). Here we consider fluctuations which are approximately spatially homogeneous and isotropic.

Particles are injected at a given energy (which remains constant since the field is time stationary) chosen in such a way that the particle gyroradius is $0.1L_c$. For the interplanetary magnetic field with a typical correlation length of 0.01 AU and mean field strength at 1AU of 5nT, this would correspond to proton with an energy of 31.6 MeV. The particles are released isotropically in velocity space at a point in space which we arbitrarily take to be the origin. They are followed for 1000 gyroperiods, which is larger than the scattering time for all runs that we report here. The numerical scheme is described in detail in our previous articles (Giacalone & Jokipii, 1996, 1999).

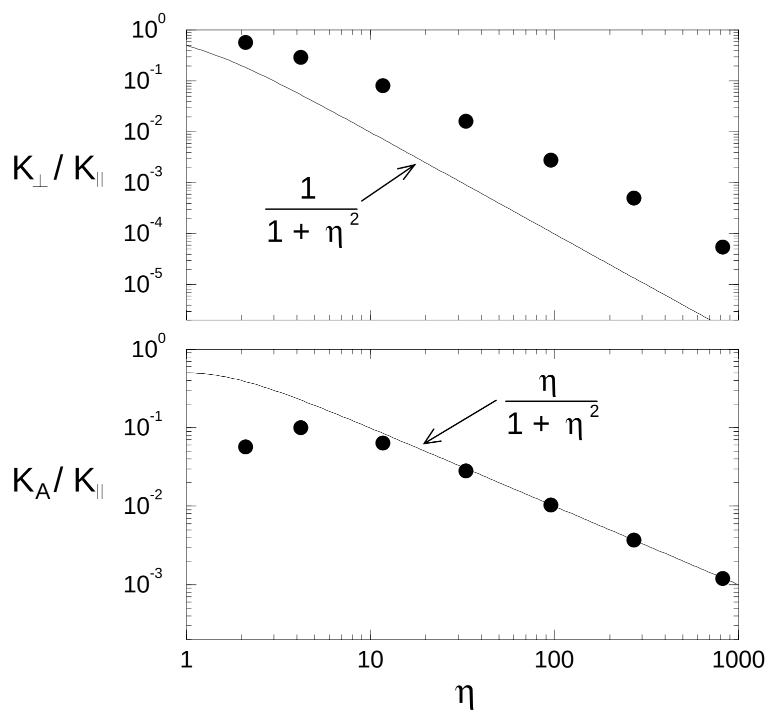

Figure 1. Comparison of numerical simulations (sold circles), and analytic theory based on classical scattering (curves).

We compute the diffusion coefficients in the following manner: the cross-field and parallel diffusion coefficients are determined by computing the averages over all particles of $\langle \Delta x^2 \rangle/(2\Delta t)$ and $\langle \Delta z^2 \rangle/(2\Delta t)$, respectively. The antisymmetric diffusion coefficients are determined from (6) as the average over all particles of $K_{xy} = \langle \Delta x w_y \rangle$, and $K_{yx} = \langle \Delta y w_x \rangle$, respectively.

In order to compare our numerical results with Equations (2) and (3), we must vary the particle mean free path. To accomplish this, we vary the power in the random fluctuations, σ^2. According to the standard quasilinear theory (e.g. Jokipii, 1966) the mean free path varies as the inverse of σ^2. We emphasize, however, that here we compute the mean free path directly from the simulations from the relationship $\lambda_{\|} = 3\kappa_{\|}/w$ (which we divide by the particle gyroradius to get η).

Shown in Figure 1 are the ratios $\kappa_\perp/\kappa_{\|}$ and $\kappa_A/\kappa_{\|}$ as a function of η. The corresponding values of the turbulence variance range from $0.03 < \sigma^2/B_0^2 < 30$. The curves on this plot are Equations (2) and (3).

Figure 1 shows that the cross-field diffusion coefficient is considerably larger than the classical scattering result of Equation (2). This is due to the fact the $\kappa_\perp$ is enhanced by the field-line random walk. This result is consistent with our earlier findings (e.g. Giacalone & Jokipii, 1999). On the other hand, the simulated values of κ_A agree nicely with the classical scattering result of Equation (3) for large values of η. In order to obtain the smaller values of η, we had to set the power in the random fluctuations considerably larger than the power in the mean field. Consequently, the field becomes almost completely random with no preferential direction. There should be no drifts under such a situation. This is the reason why the simulation results deviate noticeably from the curve. We point out however that the statistics were very poor in determining these points and that additional simulations are needed to verify these findings.

4 Summary and Conclusions

We have performed numerical simulations of charged-particles moving in turbulent magnetic fields and compared these with analytic theory. We have concentrated primarily on the drifts associated with these motions and have derived expressions for determining the antisymmetric diffusion coefficients.

We have found that the computed antisymmetric diffusion coefficient agrees well with the classical theory when mean-free path largely exceeds the particle gyroradius. On the other hand, κ_A is significantly smaller than the predicted value when the mean-free path is less than several particle gyroradius, which occurs when the power in the random fluctuations exceeds that in the mean field.

These conclusions regarding κ_A are restricted to a small range of parameters. Future work will extend this to a more comprehensive range of parameters. The small value of κ_A at $\eta < 5$ is potentially of significance for models of cosmic-ray transport in the heliosphere, where drifts play an important role.

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