Cosmic-ray Modulation and the Structure of the Heliospheric Magnetic Field

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Abstract

We explore how heliomagnetic coordinates can be utilized to model the transport of cosmic rays in heliospheric magnetic field configurations where the magnetic field has an organized meridional component and the field lines have a complex geometry. The Fisk field is considered as an outstanding example. We also discuss cases when the global heliospheric field undergoes a reorganization, such as a tilted dipole model with the tilt-axis varying in time. Our numerical code, at its present stage, includes parallel diffusion only. Preliminary results are presented for illustrative purposes.

1 Introduction:

The modulation of cosmic rays in the heliosphere critically depends on the large-scale structure of the heliospheric magnetic field (HMF). The large-scale HMF determines the pattern of drift motion as well as the preferential direction of diffusion.

In a steady Archimedean spiral configuration $B_\vartheta = 0$, i.e the HMF does not have meridional component. Fisk (1996) has suggested that the HMF is not a simple Archimedean spiral, but has an organized $B_\vartheta \neq 0$ component which can establish a very complex but organized latitudinal transport between high and low heliographic latitudes. Cosmic-ray transport in a Fisk field is inherently 3-dimensional which poses a serious challenge for both analitycal and numerical models (e.g. Kóta & Jokipii, 1997). An additional technical difficulty is that field lines at the poles are in general not aligned to the radial direction thus the use of polar coordinates becomes problematic.

The Fisk-field called attention to the potential significance of a regular B_ϑ component. The problem, however, is more general. Almost any global reorganization in the HMF generates, *inevitably*, a B_ϑ component. It is the purpose of the present work to explore a particular way to handle this problem by using *heliomagnetic coordinates*, which are attached to the field lines. This technique was applied successfully by Kóta & Jokipii (1983) and Hattingh & Burger (1995) to model cosmic-ray transport in a rigidly corotating Parker field with a tilted Heliospheric Current Sheet (HCS). Here we advance this concept to address more complex HMF structures.

The heliomagnetic coordinates offer a promising avenue if diffusion is primarily field aligned. The geometry of the field lines is probably less important if perpendicular diffusion due to the random mixing of field lines is effective. The present work is a progress report, we do not aim quantitive comparison with observations at this stage. Our numerical code, in its present form, considers parallel diffusion along the magnetic field only, drift and perpendicular diffusion are not yet included. Consequently the predicted variations are irrealistically large. These will certainly be reduced when perpendicular transport is incorporated.

2 Heliomagnetic Coordinates:

The equation governing the variation of the omnidirectional cosmic-ray density, $f(x_i, p, t)$, in the position, x_i, momentum, p, and time, t, was written down by Parker (1965). In Cartesian coordinates:

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial x_i}\Big(\kappa_{ij}\frac{\partial f}{\partial x_j}\Big) + (V_j + V_{Dj})\frac{\partial f}{\partial x_j} = \frac{p}{3}\frac{\partial V_j}{\partial x_j}\frac{\partial f}{\partial p} + Q \qquad (1)$$

where V_j and V_{Dj} are the convection and drift velocities, respectively. Q accounts for sources. The anisotropic diffusion tensor, κ_{ij}, can be written as

$$\kappa_{ij} = \kappa_\perp \delta_{ij} + (\kappa_{||} - \kappa_\perp) b_i b_j \qquad (2)$$

with $b_i = B_i/B$ denoting the unit vector pointing in the direction of the field.

What we call heliomagnetic coordinates is essentially using (beside the radial distance, r) the angular variables Θ and Φ, which identify the footpoints of the respective field line at a reference time, t_0. Then, Θ and Φ remain constant on a field line. This choice of curvilinear coordinates calls for the use of co-variant and contra-variant coordinates. The form of (2) for the diffusion tensor, expressed in covariant form, becomes

$$\kappa^{ij} = \kappa_\perp g^{ij} + (\kappa_\| - \kappa_\perp) b^i b^j \tag{3}$$

while Parker's equation (1) takes the form

$$\frac{\partial f}{\partial t} - \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i}\Big(\sqrt{g}\kappa^{ij}\frac{\partial f}{\partial x^j}\Big) + (V^j + V_D^j)\frac{\partial f}{\partial x^j} = p\frac{(\nabla \mathbf{V})}{3}\frac{\partial f}{\partial p} + Q \tag{4}$$

where g is the determinant of the metric tensor, g_{ij}, and $g^{ij}g_{jk} = \delta^i_k$. The structure of the HMF and the geometry of the field lines appear in the metric tensor, g_{ij}, which will, in general, evolve in time according to the dynamics of the footpoints, and then propagate outward in radius at the solar wind speed, V.

The virtue of magnetic coordinates is that the magnetic field has only one non-zero component, both b^Θ and b^Φ vanish. The components of the drift velocity, V_{Dj}, are easy to obtain, and the HCS can be defined as a $\Theta = const.$ surface. The random transverse field component suggested by Jokipii & Kóta (1989) to impede fast polar transport can be incorporated as well. At the same time, the method has its drawbacks and limitations. The metric tensor, g_{ij}, becomes ill-conditioned at large radii where the HMF is predominantly azimuthal. For similar reasons the solar wind speed should preferably be uniform, and the motion of footpoints needs to be regular and tractable.

3 Numerical Results and Discussion:

We consider HMF models where the motion of the footpoints can be described as a rotation but the axis of rotation is allowed to vary in time. For instance, such is the Fisk field which is generated by the combination of two rotations, the momentary axis changes in time. The solar wind speed is taken uniformly 500 km/s, the outer boundary is placed at 60 AU. At this stage, the code can handle parallel diffusion only, thus drift and diffusion are not included. In the simulations we assume $\kappa_\| = \kappa_0(r/r_0)(P/P_0)\beta$, so that $\kappa = 1.510^{23}\beta cm^2/s$ at $P = 1$ GV rigidity, and $r = 1$ AU (β is the particle velocity expressed in units of the speed of light).

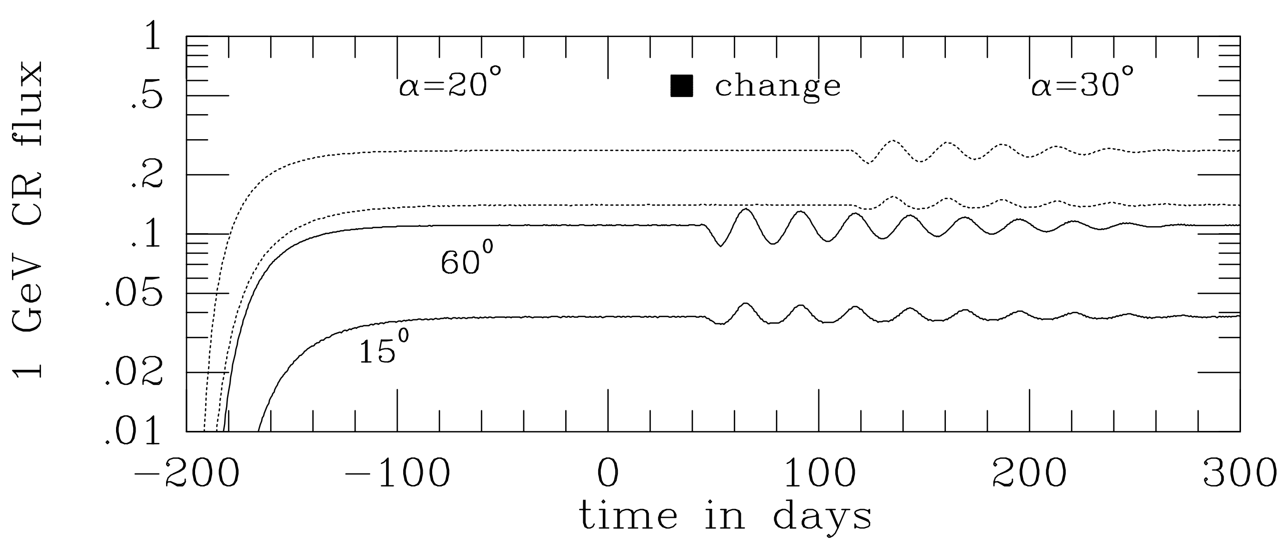

Figure 1: Variation of simulated cosmic-ray flux in an Archimedean spiral field model. Observers are at latitudes 15^o and 60^o at 4 AU (solid lines) and 25 AU (dotted lines) from the sun. The tilt angle of the HCS is changed from 20^o to 30^o at the mark.

First, we consider a tilted dipole model with a random transverse component (Jokipii & Kóta, 1989) to impede the easy diffusive access in the polar regions. The tilt angle, α is changed from the initial 20^o to 30^o between day 30 and day 40. The results of this model calculation are shown in Figure 1. In the absence of drift the solution settles to a steady state after an initial period. Modulation is essentially determined by the length of the field lines to the outer boundary. Solid curves refer to observers at 4 AU, dotted curves refer to observers at 25 AU. The shifts of levels are due to radial and latitudinal gradients. Latitudinal gradients arise since the field lines are less tightly wound at higher latitudes. As the tilt of the magnetic axis is changed from 20^o to 30^o, 26-day waves appear simultaneously at both latitudes, and these variations last for several solar rotations. Clearly, cosmic rays sense the variations in the field between the observer and the outer boundary. The delay in the onset time, in this simple model, corresponds to a propagation speed equal to the solar wind speed. Conceiveably, the inclusion of drift will modify this simple picture.

Though the model is overly simplified, it demonstartes that a global reorganization of the the current sheet and the HMF may produce enhanced simultaneous 26-day variations at varous latitudes. Similar simultaneous 26-day waves were observed at Ulysses and IMP in early 1996 (McKibben, 1998).

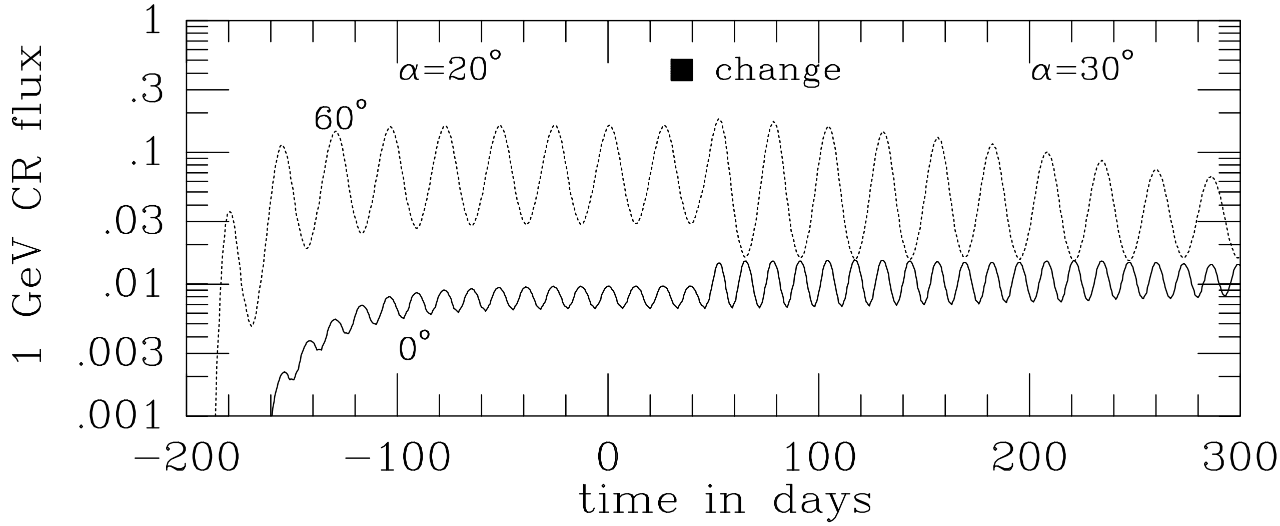

Figure 2: Variation of the cosmic-ray flux at the equator (solid line) and 60^o latitude (dotted line) at 4 AU from the sun obtained in a model calculation (see text) for a Fisk field configuration. The tilt of the offset axis is changed from 20^o to 30^o at the mark.

Figure 2 shows a similar model calculation for a Fisk field. The offset axis is changed from 20^o to 30^o. The observers are 4 AU from the sun, at the equator (solid line) and at 60^o latitude (dotted line). In the Fisk field configuration an observer would, even in the absence of drifts, see recurrent variations due to the variation in the length of the field line that passes the observer (Zurbuchen, 1999). The magnitude of the 26-variation is expected to increases following the change in the offset axis. As a rule of thumb, cosmic-ray intensity close to the equator is expected to increase while intensities at high latitudes are expected to decrease as the tilt of the offset axis increases. A larger tilt of the offset axis, as could be anticipated, results in more effective latitudinal transport and thus reduces the latitudinal variation. We note that the magnitude of variations resulting from this model are irrealistic, these extreme variations are expected to reduce considerably when transverse transport is incorporated.

Figure 3 shows the results of an exercise when the offset axis of the Fisk field was changed gradually from 20^o to 160^o, executing a complete flip, between day 0 and 600. An observer near the heliographic equator would be expected to see an increase then a decrease in the level of 1 GeV cosmic rays, and the whole period of transition would be characterized by intense 26-day variations

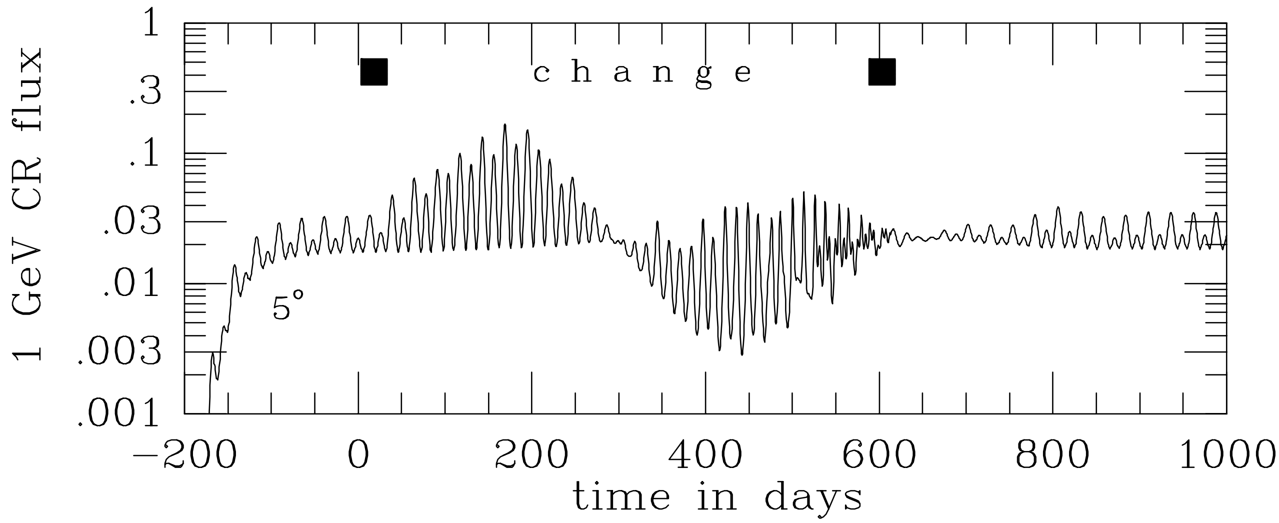

Figure 3: Simulated variation of cosmic ray flux at 5^o latitude, 4 AU from the sun in a Fisk field configuration. The tilt of the offset axis changes gradually from 20^o to 160^o, executing a complete flip, between the two marks.

4 Summary:

We have presented model calculations to demonstrate how heliomagnetic coordinates can be used in situations when the field lines follow a complex geometry. We call attention that almost any global reorganization of the HMF, for instance a global change in the HCS, may and probably will lead to a regular meridional component in the the HMF, the structure of which may be quite complex (see Kóta & Jokipii, 1997). This can affect cosmic rays and might produce simultaneous enhanced 26-variations, lasting for several solar rotations, at different latitudes.

So far our numerical code includes parallel diffusion only. Results may give insight into some phenomena but must not be taken quantitively. Conceiveably, the inclusion of drift and perpendicular diffusion will significantly reduce the magnitude of the variations that were obtained without transverse transport. The goal of our effort is to explore and quantify the effect of transverse transport on these variations.

Finally we note that the use of heliomagnetic coordinates may prove fruitful in analytical approximations.

5 Acknowledgements

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