A Two-Dimensional Self-Consistent Model of Galactic Cosmic Rays in the Heliosphere

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Abstract

We present initial results from our new 2-D, self-consistent model of galactic cosmic rays in the heliosphere. This work can be viewed as an extension of a number of self-consistent 1-D models reported by several authors during the past 3-4 years. Even in 1-D models, cosmic rays have a significant impact in the outer heliosphere, slowing down the wind and modifying the termination shock; the opposite is also true, e.g., particles are accelerated at the the shock. Our study focuses on the latitudinal variations in the solar wind flow caused by the energetic particles. The solar wind portion of the model is based on a shock-capturing scheme with magnetic field as a tracer; the cosmic-ray part is described by the Parker's equation and includes all relevant drift motions.

We report a new effect that is not seen in 1-D models. Our preliminary results indicate that the wind is deflected towards the equatorial plane and becomes substantially faster downstream of the termination shock near the current sheet (for the case of $A > 0$). We attribute this effect to the lateral gradients in the cosmic ray pressure that squeeze the flow towards the equatorial plane.

1 Introduction:

The past decade has shown a number of advances in one-dimensional self-consistent models of the heliosphere that included cosmic rays. The latter have been found to have a complex effect on the heliospheric termination shock. First, the supersonic upstream flow is decelerated by the radial cosmic-ray pressure gradient creating a shock precursor in addition to the gas-pressure-mediated subshock (Drury and Völk, 1981). Second, the shock moves inward closer to the Sun as a result of decreased dynamic pressure of the wind upstream (see Ko, Jokipii and Webb, 1988). Third, the flow downstream of the shock is modified as well, e.g., gas pressure becomes higher approaching the shock as a result of momentum conservation (e.g., Donohue and Zank, 1993).

While, originally, momentum-integrated quantities were used for the particles, recently a more consistent approach has been adopted which favors solving the full cosmic-ray transport equation of Parker (1965). le Roux and Ptuskin (1995) and le Roux and Fichtner (1997) came up with a series of sophisticated 1-D models that included both galactic (GCRs) and anomalous cosmic rays.

On the other hand, 2-dimensional simulations of the cosmic rays as *test particles* have been available for decades (see, for example, Jokipii and Kopriva (1979), Jokipii and Davila (1981), Potgieter and Moraal (1985), Haasbroek and Potgieter (1995) and Jokipii, Kota and Merényi (1993)). It has been found that drifts in the interplanetary magnetic field play an important role, often more so than diffusion, in transporting the galactic cosmic rays (GCRs) inside the heliosphere. In particular, GCRs have enhanced access to the interior regions near the pole while at low latitudes their propagation is inhibited by the positive (for $A > 0$ solar period) drift within the current sheet. Since it's impossible to treat the drifts correctly in 1 dimension, we believe an important part of the physical picture is being left out of such models.

This paper presents our efforts to develop a 2-dimensional model of the GCR-modified heliosphere. While our results are still preliminary, we believe that a new physical phenomena does exist in the postshock region, as will be described below.

2 2-D Model Description:

Our model uses a spherical coordinate system with symmetry assumed about the polar axis; another symmetry is about the equatorial plane (all quantities in the lower hemisphere are their mirror reflections except

B_r and B_ϕ which change sign). The solar wind is described by the usual set of hydrodynamic equations (not written here due to space limitations), modified by the cosmic ray pressure term. In our current version the magnetic force is not included since the magnetic pressure is not very significant compared with the solar wind dynamic pressure (upstream) and the gas and GCR pressures downstream. However some of our simulations exhibited regions where this pressure could become large. Later revisions of our model will include the dynamic effects of the field on the solar wind. We compute the azimuthal component B_ϕ of the magnetic field from the Faraday's law

$$\frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial\left(ru_r B_\phi\right)}{\partial r} - \frac{\left(u_\theta B_\phi\right)}{\partial\theta}. \tag{1}$$

The other two components of the field are usually much smaller, except at small radii, but they still are computed for consistency using the steady-state version of the Faraday's law $u_r B_\theta = u_\theta B_r$. These can be combined with the Maxwell's equation $\nabla\cdot\mathbf{B} = 0$ to solve for the radial field component. Because extremely-large diffusion and drifts over the polar regions create considerable problems with numerical stability we adopted a modified field model from Jokipii and Kota (1989), also used in the GCR modulation context by Haasbroek and Potgeiter (1995). This reduced the polar transport by several orders of magnitude, due to the fact that the modified field falls off as r^{-1} rather than r^{-2} for the radial field. It can be shown that if the spatial and temporal scales of the modified field fluctuations are much smaller than the grid size and the time step, respectively, it's possible to write the following equation for the modified component of the field:

$$\frac{\partial B_\theta^m}{\partial t} = -\frac{1}{r}\frac{\partial\left(ru_r B_\theta^m\right)}{\partial r}. \tag{2}$$

For the GCRs the Parker's transport equation was used:

$$\frac{\partial f}{\partial t} + u_i\frac{\partial f}{\partial x_i} + v_{d,i}\frac{\partial f}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\kappa_{ij}\frac{\partial f}{\partial x_j}\right) = \frac{1}{3}\frac{\partial u_i}{\partial x_i}\frac{\partial f}{\partial\ln p}, \tag{3}$$

where $\mathbf{v_d}$ is the particle drift velocity described by (assuming all particles are protons):

$$v_{d,r} = \frac{pcv}{3e}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{B_\phi}{B^2}\right), \qquad v_{d,\theta} = -\frac{pcv}{3e}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{B_\phi}{B^2}\right), \tag{4}$$

while the diffusion coefficients we used are from Jokipii and Davila (1981). If B_0 is the magnetic field strength at Earth and P, particle's rigidity, then the diffusion tensor component parallel to the average magnetic field is

$$\kappa_{||} = \kappa_0 P^{1/2}\frac{v}{c}\frac{B_0}{|B|}, \tag{5}$$

based on the assumption that the spatial variation of the mean free path scales with the particle's gyroradius. The perpendicular component is taken to be a fixed fraction of this as long as $\kappa_\perp \ll \kappa_{||}$.

Since the magnetic field changes polarity across the equatorial current sheet, the drift velocity contains a delta-function. We employ two different approaches in dealing with the current sheet. In the first we treat the drift velocity as a boundary conditions as shown in Jokipii and Kopriva (1979). Because the angular resolution in our model is relatively low, we suspected that results obtained with this type of condition could be strongly affected by numerical errors. Therefore we also carried out calculations for the wide current sheet model of Potgieter and Moraal (1985). Fortunately, the results from the wider current sheet were basically consistent with the boundary condition approach.

Solar wind equations were solved with the second order Godunov-type numerical scheme of Colella (1990) and the HLLE Riemann solver of Einfeldt et al. (1991). For the cosmic ray part we used a second order ADI split scheme based on the scheme of McKee and Mitchell (1970). While our wind properties were uniform in latitude this was done to isolate the important physical aspect of this problem; this is not a limitation of the model which can handle non-spherical termination shocks as well. All results presented in the next section are for the $A > 0$ epoch, i.e., $B_r > 0$, $B_\phi < 0$ in the northern hemisphere.

3 Preliminary Results:

We have performed a number of simulations with different resolutions and equatorial boundary conditions. For results presented here spatial resolution was 1 AU in the radial direction by 0.5^0 in latitude. This relatively low resolution was imposed by computer time requirement. The results presented here are not steady state. Because the system evolves on extremely long time scales, longer even than the 11-year solar cycle, the magnetic field reversal would change the sense of particle transport and following the system for a longer time would make little sense. The wind parameters are similar to those used in le Roux and Fichtner (1997) while the diffusion $\kappa_0 = 1.5 \cdot 10^{22} \mathrm{cm}^2 \mathrm{s}^{-1}$ and $\kappa_\perp = 0.05\kappa_{||}$. At the outer boundary, the gas pressure was set to 1.1 eV cm^{-3} and the GCR pressure 0.4 eV cm^{-3}. The CR-absorbing lower boundary was placed at 5 AU to avoid possible numerical errors due to insufficient resolution.

Figure 1 shows radial-velocity contours for the 8 degree-wide current sheet. While the shock has moved inwards, due to pressure of the GCRs, its shape remains spherical. We can see a significant change in the flow pattern downstream of the shock. There is a substantial increase in wind velocity downstream of the shock in the equatorial region. Apparently, the wind is being deflected away from the pole and towards the equator by the lateral GCR gradients.

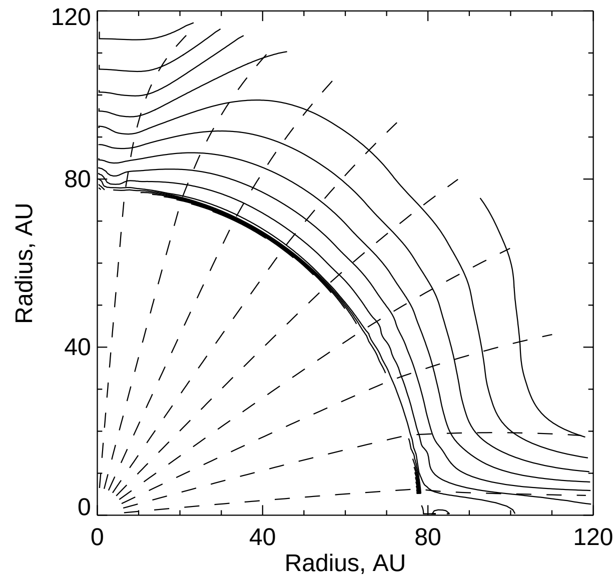

Figure 1: Radial wind velocity contours (solid) and streamlines (dashed).

Figure 2 plots the cosmic-ray pressure contours for both the test-particle and the self-consistent case. It can be seen that the cosmic ray distribution is virtually unchanged. A slight difference is due to the changed position of the shock. We have found an increase in magnetic field strength near the equator (due to general compression in the flow). At the pole, however, it's the modified filed component, B_θ^m that becomes large, due to the wind slowing down in the radial direction. Still, the particle distribution is insensitive to relatively small (less than a factor of 2) changes in transport coefficient (see Jokipii and Davila, 1981, that paper was also used for reference to verify the accuracy of our cosmic-ray code).

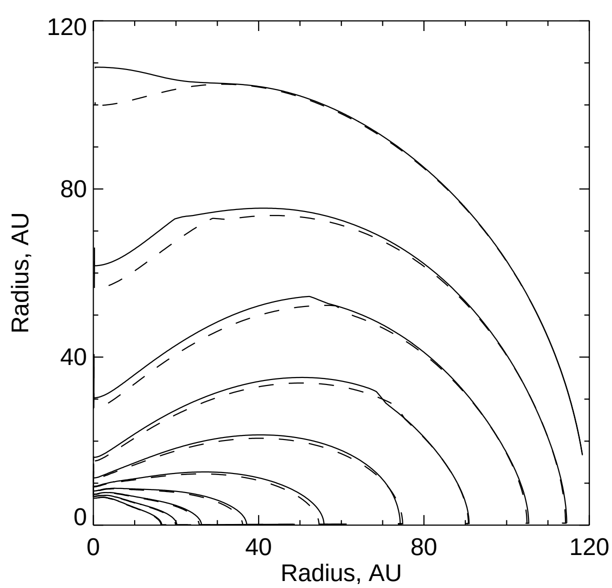

Figure 2: GCR pressure contours. Solid lines are self-consistent results, dashed lines are for the test-particle case. All contours are evenly spaced at 5% decrements

In Figure 3, we plot the radial wind velocity, gas and GCR pressure at the pole and the equator. As expected, cosmic rays have larger radial gradients near the current sheet than at the pole. The termination shock has moved inwards a distance of about 3 AU. The flow is slowed down in the polar region as it is being deflected away from the polar axis. At the equator, the wind is considerably faster and more dense up to the far boundary. A wide cosmic-ray shock precursor is visible in the supersonic wind. Note a gas pressure (P_g) build up behind the shock near the equator. There is a corresponding increase in u_θ immediately behind the shock that we believe balances the increase in P_g.

4 Conclusion:

We have verified by running a range of tests that the effects we observe are due mostly to radial drifts. Lateral drifts only smooth out the particle distribution over latitude. Faster GCR radial transport over the poles create significant lateral gradients in the particle distribution which drive the wind towards the equator through their pressure gradient. The cosmic rays are affected as well by the changing diffusion coefficient and drift velocity due to compression and radial speed variation, although this effect is very small.

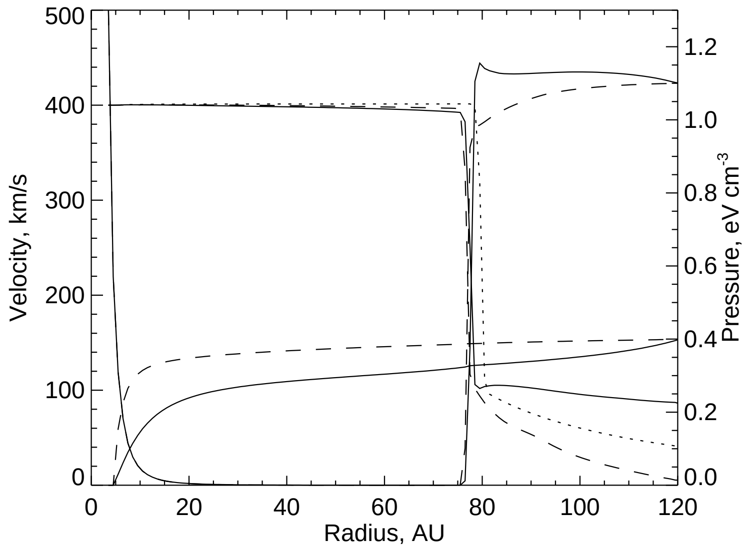

Figure 3: Radial wind velocity (curves starting at 400 km/s on the left) and pressure, and GCR pressure at 0^0 latitude (solid lines) and at 90^0 (dashed lines). GCR pressure curves have no jump at the shock. Dotted line is the wind speed without the cosmic rays.

At this point we are limited by the low resolution to GCRs only. The parameter κ/u describing shock acceleration of anomalous cosmic rays, is of the order of 40 AU for 1 GeV protons but only 0.3 AU for 1 MeV particles, assuming Equation 5 is still valid. Although there is some indication that κ is somewhat larger at lower energies, higher resolution is still required in order to separate the gas subshock from the precursor and allow the particles to be accelerated at the shock.

Another interesting phenomena which we have not studied is the possibility of negative radial flow velocity in the outer heliosphere near polar regions. It is not clear whether this effect has a physical significance, as such inflow would require to specify the density at the outer boundary which is not known. In reality, this could mean that the boundary itself (e.g., the heliopause) moves inward over the pole.

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