Interacting and Interplanetary Proton Spectra in Parallel Shock Waves

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Abstract

Energy spectra of interacting and interplanetary protons are studied in a model, where the acceleration takes place at a coronal shock wave propagating parallel to the magnetic field. Time integrated spectra are calculated analytically, and shown to be determined by (i) the scattering center compression ratio at the shock and (ii) the number of diffusion lengths in the upstream region. It is shown how the differential energy spectrum of interplanetary protons and ratio of interplanetary to interacting protons at high energies can be used as a probe for these parameters.

1 Introduction

Protons accelerated on/near the Sun can be directly measured in the interplanetary medium, or alternatively their properties may be deduced from measurements of secondary emissions, neutrons and γ-rays, produced during nuclear interactions at the Sun (see Ramaty et al. 1993 for a review). The ratio (Γ) of the interplanetary proton number to the number of protons interacting at the Sun is an important result of recent solar observations. This ratio varies in a very wide range, from about ~ 0.01 to $\gtrsim 1$, being typically in order of unity if a post-impulsive-phase acceleration is present. Here, we study the spectra of interacting and interplanetary protons in a model, where they are accelerated diffusively (Axford, Leer, & Skadron 1977; Bell 1978; Blandford & Ostriker 1978; Krymsky 1977) in a coronal shock wave traveling parallel to some open magnetic field lines outwards from the Sun applying and developing the ideas of Ellison & Ramaty (1985) and Lee & Fisk (1982).

It is commonly accepted that the majority of ions in large solar energetic particle (SEP) events are due to shock acceleration in outward propagating coronal/interplanetary shocks related to, e.g., coronal mass ejections (Reames 1993). It is therefore important to study whether this acceleration mechanism can really account for the SEP spectrum, but equally important is to study the spectrum of protons precipitating in sub-coronal regions behind the shock wave. Under which conditions (if any) can we produce the observed ratios $\Gamma \sim 1$ and power-law energy spectra of the two proton populations? Since shock acceleration involves solar-frame bulk motion of the plasma, it may be questioned whether particles leaving the shock in the downstream region ever reach the sub-coronal regions where they should interact. Also, since there has to be turbulence in front of the shock to hinder particle escape if any acceleration of particles is to occur, we need to study how the spectrum of the escaping particles is related to the spectrum at the shock.

2 The Model

We assume that the acceleration of energetic particles is due to first-order Fermi acceleration at a parallel shock. The shock is propagating with a constant speed $V_{\rm s}$ into a medium at rest with an exponentially decreasing magnetic field, $B(\zeta) = B_0 e^{-\zeta/L_B}$, where ζ is the coordinate measured along the magnetic field lines and $L_B = -B/(\partial B/\partial\zeta)$ is the (constant) scale length of the magnetic field. We take the particles to be scattered by Alfvén waves propagating along the magnetic field lines on both sides of the shock in a region that has spatial extent of $L_{1[2]}$ upstream [downstream] of the shock. In the upstream region, the waves are taken to propagate outwards from the Sun. In the downstream region, the turbulence then always consists of waves propagating in both directions, but the outward-propagating wave component is dominating in intensity so that the neglect of stochastic acceleration in the downstream region is justified (Vainio & Schlickeiser 1999). The upstream Alfvén speed and the compression ratio of the shock are taken to be constants.

Since there is no resonant cross-field diffusion in our model, the particles remain on their initial magnetic field lines forever. A bundle of neighboring field lines, thus, defines a flux tube. We may describe the linear particle density $f(\zeta, p, t) = d^2N/d\zeta\, dp$ in the region $V_\mathrm{s}t - L_2 < \zeta < V_\mathrm{s}t + L_1$ with a Fokker–Planck equation

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \zeta}\left(V + \frac{\kappa}{L_B}\right) f - \frac{\partial}{\partial p}\left(\frac{V}{L_B} + \frac{\partial V}{\partial \zeta}\right)\frac{p}{3} f = \frac{\partial}{\partial \zeta}\kappa\frac{\partial f}{\partial \zeta} + s, \tag{1}$$

where p is the particle momentum, $s(\zeta, p, t) = Q(t)\delta(\zeta - V_\mathrm{s}t)\delta(p - p_\mathrm{inj})$ is the source function, p_inj is the injection momentum, $Q(t)$ gives the number of injected particles to the considered flux tube at the shock per unit time, $V(\zeta, t)$ is the phase speed of the waves scattering the particles, and $\kappa(\zeta, p, t)$ is the spatial diffusion coefficient of the particles related to the particle mean free path λ by $\kappa = \lambda v/3$. In the regions $\zeta > V_\mathrm{s}t + L_1$ and $\zeta < V_\mathrm{s}t - L_2$ the particles are taken to stream adiabatically without scattering and those particles reaching $\zeta = 0$ are taken to precipitate.

3 Results

If the positions of the boundaries relative to the shock, L_1 and L_2, are time-independent, it is most convenient to transform to the coordinate system moving with the shock with spatial variable $x = V_\mathrm{s}t - \zeta$ and wave speed $U = V_\mathrm{s} - V$. For of non-diverging magnetic field lines ($L_B \to \infty$), equation (1) can be solved analytically. In this case the adiabatic motion of the particles outside the shock is free streaming, and the particles escape at both boundaries: $f(-L_1, p) = f(+L_2, p) = 0$. Then, the time-integrated solution, $F(x, p) = \int_0^\infty f\, dt$, is (see, e.g., Ostrowski & Schlickeiser 1996)

$$F(x,p) = \begin{cases} F_0(p)\ [\exp(\beta_1 x) - \exp(-\beta_1 L_1)]/[1 - \exp(-\beta_1 L_1)], & 0 > x > -L_1; \\ F_0(p)\ [\exp(\beta_2 L_2) - \exp(\beta_2 x)]/[\exp(\beta_2 L_2) - 1], & 0 < x < L_2 \end{cases} \tag{2}$$

where $\beta_{1[2]}^{-1} = \kappa_{1[2]}/U_{1[2]}$ is the upstream [downstream] diffusion length taken here to be a function of momentum, only,

$$F_0(p) \approx \frac{3\gamma N_0 H(p - p_\mathrm{inj})}{U_1 p_\mathrm{inj}}\ \left(\frac{p_\mathrm{inj}}{p}\right)^{\gamma-2} \exp\left\{-\gamma \int_{p_\mathrm{inj}}^{p} \frac{1}{e^{\eta_1(p')} - 1}\frac{dp'}{p'}\right\}, \tag{3}$$

N_0 is the total number of injected particles, $\gamma = 3\,U_1/(U_1 - U_2)$, and $H(p)$ is the Heaviside function. The approximative equality in equation (3) means that it applies for $\beta_2 L_2 \gg \min\{\beta_1 L_1, 1\}$, only. Thus, if the number of upstream diffusion lengths, $\eta_1 \equiv \beta_1 L_1$, is large, the shock spectrum approaches the canonical power law, where the spectral index γ is fully determined by the scattering-center compression ratio at the shock, $\rho_\mathrm{c} = U_1/U_2$.

The differential energy spectrum of particles escaping at the boundaries is

$$\frac{dN_{1[2]}}{dE} = +[-]\frac{dp}{dE}\ \left(\kappa\frac{\partial F}{\partial x}\right)\bigg|_{x=-L_1[+L_2]}. \tag{4}$$

Using equations (2–4), the spectrum of interplanetary particles is obtained as

$$\frac{dN_1}{dE} = \frac{\gamma N_0}{v\, p_\mathrm{inj}\left(e^{\eta_1(p)} - 1\right)}\left(\frac{p_\mathrm{inj}}{p}\right)^{\gamma-2} \exp\left\{-\gamma \int_{p_\mathrm{inj}}^{p} \frac{1}{e^{\eta_1(p')} - 1}\frac{dp'}{p'}\right\} \tag{5}$$

and the ratio of interplanetary to interacting protons as

$$\Gamma(E) = \frac{dN_1/dE}{dN_2/dE} = \frac{\rho_\mathrm{c}}{e^{\eta_1(p)} - 1}, \tag{6}$$

where v is particle speed and we have assumed that all particles leaving the system at the downstream boundary precipitate at the solar surface.

It can be shown (Vainio, Kocharov, & Laitinen 1999) by comparing the time scales of the particle transport, adiabatic deceleration and shock acceleration as described by equation (1), and the time scale of particle precipitation in the diverging magnetic field behind the shock that the solution given for the case $L_B \to \infty$ is valid as long as the following conditions are met: (i) the upstream mean free path is small enough, $\lambda_1/L_B \ll 3U_1/v$, at all particle speeds of interest; (ii) the downstream mean free path is even smaller, so that the number of diffusion lengths in the downstream region is $\eta_2 \gg 1$, when at the same time $L_2 \ll L_B$; and (iii) the injection of low-energy particles is concentrated in a region within a couple of L_B's above the solar surface. In the following, we shall assume that these conditions are met.

There is an interesting possibility to analyze the observed accelerated particle spectra. If the interplanetary proton spectrum can be approximated with a power law, $v\, dN_1/dE \propto p^{-\gamma_1}$ above the momentum $p = p_1$, equation (5) can be re-arranged as

$$e^{\eta_1(p)} - 1 = \frac{z_1 \left(p^{\gamma-2}\, v\, dN_1/dE\right)_{p=p_1} + \gamma \int_p^{p_1} p'^{\gamma-3}\, [v\, dN_1/dE](p')\, dp'}{p^{\gamma-2}\, v\, dN_1/dE}, \tag{7}$$

where $z_1 = \exp\{\eta_1(p_1))\} - 1$. In this case the parameters γ and z_1 can be given as functions of two observables, γ_1 and the ratio of interplanetary to interacting protons at $p > p_1$, Γ_∞, through the use of $\gamma = (\gamma_1 + 2 + 3\Gamma_\infty)/(1 + \Gamma_\infty)$ and $z_1 = (\gamma_1 + 2 + 3\Gamma_\infty)/[(\gamma_1 - 1)\Gamma_\infty]$.

Torsti et al. (1996) analyzed the interplanetary protons of the solar cosmic ray event on 24 May 1990. They concluded that the injection of protons into the interplanetary medium consisted of two components, first of which was released 10–40 minutes after the X-ray flare. The time scale of the this particle release is consistent with acceleration-site length scales of the order of solar radius. This flare was also a source of gamma rays and neutrons, which have been analyzed as well (Kocharov et al. 1994, 1996), and there were indications of a prolonged emission of neutrons that could be originating from shock-accelerated particles. These observational facts make the event acceptable for application of our model. The energy spectrum of the prompt-component interplanetary protons could be represented by $dN_1/dE = N_0\,(E/160\,\mathrm{MeV})^{-1.6}\,[1 + (E/360\ \mathrm{MeV})^3]^{-1}$ with $N_0 = 1.4 \cdot 10^{30}$ protons/MeV in the range $E = $ 30–1000 MeV assuming a solid angle of 2 sr for the flux tube. The number of high-energy neutrons was found to be equal to or several times less than the number of high-energy interplanetary protons.

As we apply our model to these observations we take the proton momentum $p_1 = \sqrt{120}\, m_\mathrm{p} c$ corresponding to the kinetic energy of $10\, m_\mathrm{p} c^2$, where we have $\gamma_1 = 5.0$. We then adopt $\Gamma_\infty = 1$, which gives a value of $\gamma = 5.0$. This yields $\rho_\mathrm{c} = 2.5$, a relatively small value when compared to a theoretical value of about 5.5 (Vainio & Schlickeiser 1999), which can be obtained for a shock with low Alfvénic Mach number propagating into a medium with low plasma beta (estimated here as $M_\mathrm{A} = 4$ and $\beta = 0.3$, respectively). As another case, we note that the theoretical compression ratio yields $\gamma = 3.67$ and $\Gamma_\infty = 5$, so it may marginally fit the observed data as well. For these two cases, we then have $z_1 = 2.5$ and 1.10, respectively. The integral in equation (7) performed numerically, we present the number of upstream diffusion lengths in Figure 1 and the inferred spectra of interacting protons in Figure 2 resulting for the two considered cases. Note, that the actual values should lie between the curves of these figures, although at low energies the curves for dN_2/dE should be taken as upper limits.

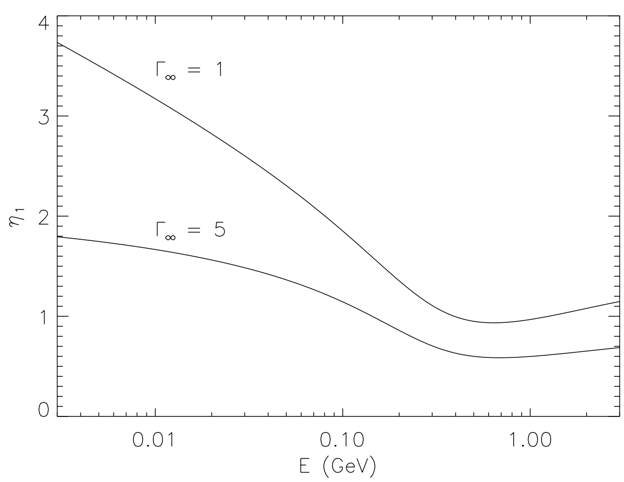

Figure 1: The number of diffusion lengths in the upstream region as a function of particle energy for two limiting cases of Γ_∞ for the 24 May 1990 SEP event.

4 Discussion

In our model, the ratio of interplanetary to interacting particles is very sensitive to the value of η_1. Very large values of this parameter can not be proposed without obtaining extremely small values of Γ never observed in gradual flares. Interplanetary mean free paths at 1 AU for $E = 10$–100 MeV protons are often observed to be of the order of 0.1 AU. WKB theory predicts that the mean free path near the Sun should be a decreasing function of heliocentric distance for typical wave spectra, if Sun is the only source for turbulence (Jokipii 1973); in contrast, we need values that are orders of magnitude smaller than this to overcome losses associated with the diverging mean magnetic field. These facts suggest that the upstream waves are self-generated by the accelerated particles through the streaming instability: even if an external source could have produced the waves necessary for the intense scattering, it would be impossible for the accelerated to ever escape upstream to be detected near the Earth. To answer, whether it is possible for the particles to generate the waves self-consistently, we need a model that is time dependent and can address also the basic question of injection of particles to the acceleration process. This is beyond any diffusion theory and needs numerical kinetic modeling.

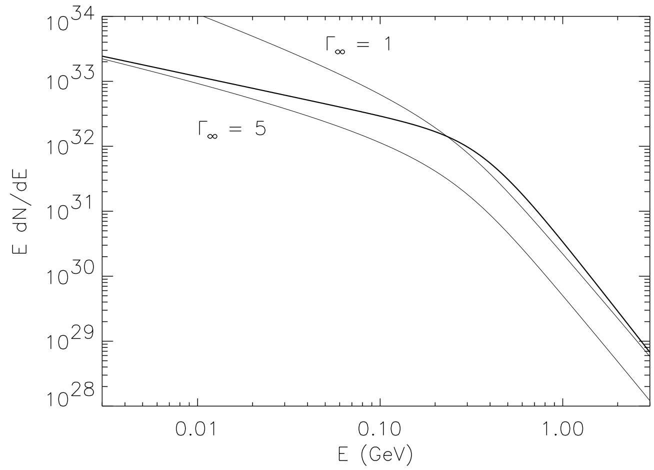

Figure 2: The measured spectrum of interplanetary protons for the 24 May 1990 SEP event (unlabeled curve) and the spectra of precipitating protons (labeled curves) deduced for two limiting cases of Γ_∞.

The sensitivity of the model to the value of η_1 gives also a good possibility to test it experimentally: we have offered our model as a natural way to explain the broken power-law spectra of the interplanetary ions observed in some SEP events (e.g., Torsti et al. 1996). In this case, the change in the spectral slope should also be seen as a change in the interplanetary to interacting proton ratio at the same energies so it should be, at least in principle, detectable even with the currently available gamma-ray and particle detectors.

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