Stochastic Fermi acceleration in turbulent fields with non-vanishing wave helicities

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Abstract

Energetic particle acceleration at turbulent circularly polarized Alfvén waves is considered using Monte Carlo simulations. We show that the scattering forward-backward asymmetry occurring for such waves allows for the first order acceleration effects to occur in the stochastic acceleration process, enabling in favorable conditions for more effective acceleration in comparison to the linearly polarized Alfvén waves of the same amplitude. An analytic solution for a simple acceleration model is also presented. The present work is still in progress.

1 Introduction

The acceleration of charged particles to superthermal energies occurs in a number of astronomical objects harboring a turbulent magnetized plasma in the process of stochastic Fermi acceleration. Such mechanisms were considered, e.g., for providing relativistic electrons in extragalactic radiosources or for energetic ions during impulsive solar flares. Ostrowski & Siemieniec-Ozięblo (1997) demonstrated that the forward-backward asymmetry of particle scattering (as measured in the scattering center rest frame) at randomly moving scattering centers can lead to a first-order regular acceleration term, in addition to the one resulting from the momentum diffusion. A physical example of such asymmetric scatterer provides a (finite amplitude) circularly polarized Alfvén wave (Siemieniec-Ozięblo et al. 1999). In the present paper we present preliminary results of Monte Carlo modelling of the particle acceleration/diffusion process for protons interacting with finite amplitude circularly polarized Alfvén waves.

2 A simple analytic model

Let us present a simple example of the process of particle momentum diffusion in the presence of a regular acceleration/deceleration term $\dot{p}_{reg}$ =const . For simplicity we consider constant momentum diffusion coefficient D also. The evolution of the particle phase space is given by the kinetic equation for the distribution function $f(p,t)$:

$$\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\, D\, \frac{\partial f(p,t)}{\partial p}\right\} - \frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\, \dot{p}_{reg} f(p,t)\right\} \quad . \tag{2.1}$$

The initial distribution is assumed to be $f(p,0) = \frac{1}{p^2}\,\delta(p-p_0)$. We look for a time dependent solution of Eq. 2.1 in the finite interval (p_0, $p_{\max}$), and the final result is obtained in the limit of $p_{\max} \to \infty$. With

$$f(p,t) = \frac{e^{\alpha(p-p_0)}}{p}\, g(p,t) \quad , \tag{2.2}$$

where $\alpha \equiv \dot{p}_{\rm reg}/2D$, we have to solve the equation satisfied by $g(p,t)$:

$$\frac{1}{D}\frac{\partial g}{\partial t} = \frac{\partial^2 g}{\partial p^2} + g\left(-\frac{2\alpha}{p} - \alpha^2\right) \quad . \tag{2.3}$$

After applying the standard separability condition we look for the solution in the form

$$g\,(p,t) = \sum_{n=1}^{\infty} C_n\, e^{\lambda_n^2 t} \quad v_n(p) \quad . \tag{2.4}$$

The phase-space density $g(p,t)$ can be expressed as an infinite sum of functions $v_n(p)$ with expansion coefficients C_n. Each function $v_n(p)$ satisfies the equation below, corresponding to the appropriate eigenvalue

$$\frac{d^2v}{dp^2} + v \quad \left[(\lambda_n^2 - \alpha^2) - \frac{2\alpha}{p}\right] = 0 \quad . \tag{2.5}$$

The above has a form of the Schrödinger equation for the particle wave function in the Coulomb potential and its solution can be expressed in terms of the Coulomb wave function $F_0(\eta, p)$. From the two linearly independent solutions we choose that which is regular at $p = 0$:

$$g\,(p,t) = \sum_{n=1}^{\infty} C_n\, e^{-\lambda_n^2 t} \quad F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p_0\right) \quad F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p\right), \tag{2.6}$$

where the eigenvalues $\kappa_n^2 = \lambda_n^2/D - \alpha^2$ have to be calculated from boundary condition $g(p_{max}, t) = 0$. It translates for the Coulomb function as $F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p_{max}\right) = 0$. The coefficients C_n are given by an orthogonality condition for eigenfunctions F_0, i.e.

$$C_n = \int_0^{p_{max}} \left[F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p\right)\right]^2 dp \quad , \tag{2.7}$$

Now, the final time dependent distribution reads as:

$$f\,(p,t) \propto \frac{e^{\alpha(p-p_0)}}{p} \quad \sum_{n=1}^{\infty} \left[C_n\, e^{-\lambda_n^2 t} \quad F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p_0\right) \quad F_0\left(\frac{\alpha}{\kappa_n}, \kappa_n p\right)\right] \quad . \tag{2.8}$$

3 Numerical modelling

Let us consider an infinite region of tenuous plasma with a uniform mean magnetic field along z-axis, perturbed with propagating Alfvén waves. Test particles are injected at random positions into this medium and their trajectories are followed by integration of the particle equations of motion. Due to the presence of waves, particles move diffusively in space and momentum. In a chosen sequence of time instants, $t_{\rm i}$, particle spatial positions and momentum values are recorded. Based on the information collected for numerous particles one derives the particle momentum distribution at any given time. In the present simulations we consider propagation of relativistic protons with the same initial velocity $v_{\rm in} = 0.99c$.

3.1 The wave field model In the case of high amplitude waves, there are no analytic models available reproducing the turbulent field structure. Because of that, we consider a model of MHD turbulence represented as a superposition of sinusoidal Alfvén waves. In the simulations, for any individual particle a separate set of wave field parameters is selected. As a result all averages taken over particles include also averaging over multiple magnetic field realizations. In the model we take a superposition of plane Alfvén waves propagating along the z-axis, in the positive (forward) and the negative (backward) direction. Two planar polarizations, along the x and y axes, are considered. In the computations a discrete number of 144 sinusoidal waves, of each polarization, is used. Related to the wave 'i' the magnetic field fluctuation vector $\delta\mathbf{B}^{(i)}$ is given in the form:

$$\delta\mathbf{B}^i = \left\{ \begin{array}{c} \delta B_o^i \sin(k^i z - \omega^i t - \Phi^i) \\ 0 \\ 0 \end{array} \right\} \quad \text{or} \quad \left\{ \begin{array}{c} 0 \\ \delta B_o^i \sin(k^i z - \omega^i t - \Phi^i) \\ 0 \end{array} \right\} \tag{3.1}$$

for linearly polarized Alfvén waves and

$$\delta\mathbf{B}^i = \left\{ \begin{array}{c} \delta B_o^i \sin(k^i z - \omega^i t + \Phi^i) \\ \mp\delta B_o^i \cos(k^i z - \omega^i t + \Phi^i) \\ 0 \end{array} \right\} \tag{3.2}$$

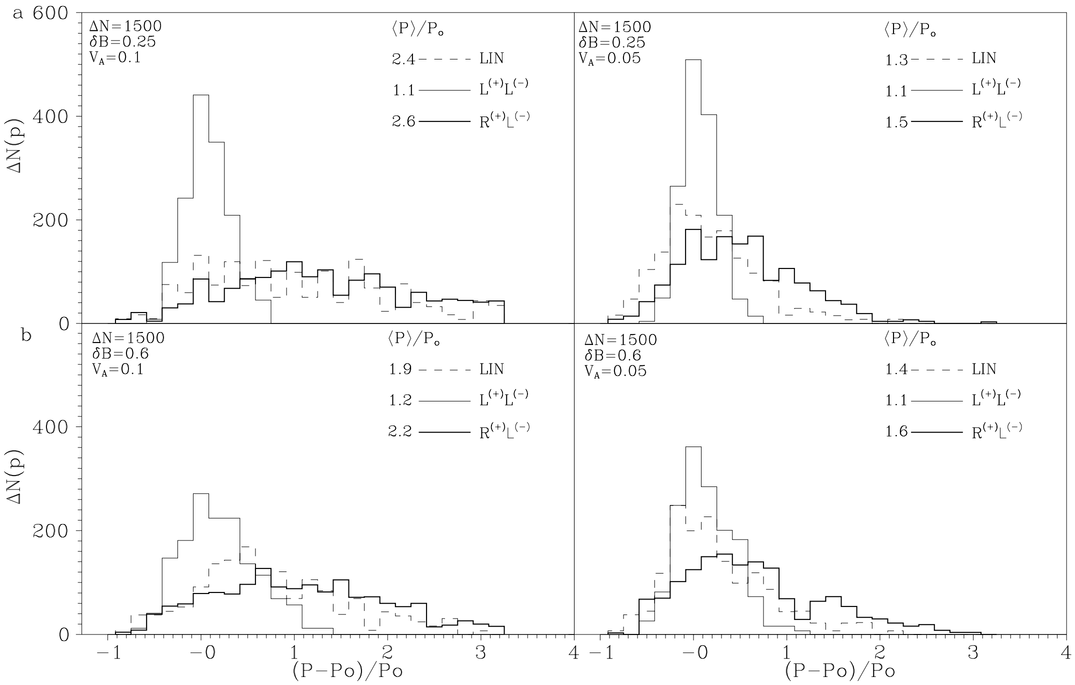

Figure 1: Histograms of generated particle momentum distributions for three considered turbulence models – $L^{(+)}L^{(-)}$ - a thin solid line, $R^{(+)}L^{(-)}$ - a thick solid line, and LIN - a dashed line. The results for $V_A = 0.1$ are presented in left panels, for $V_A = 0.05$ in right panels, and for $\delta B = 0.25$ in upper panels, for $\delta B = 0.6$ in bottom panels. In the upper right corners of each panel the respective values of $< p > /p_0$ are provided.

for left- or right-handed polarized waves, depending on the sign $+$ or $-$, respectively. The wave parameters – the wave vector k, the wave amplitude δB_o and the phase Φ – are drawn in a random manner from the flat power-law spectrum, $F(k) \propto k^{-1}$ for $k \in (k_{min} = 0.08, k_{max} = 8.0)$; a wave vector is expressed in units of the 'resonance' wave vector for an injected particle with momentum $p = p_o$, $k_{res} \equiv 2\pi/r_g(< B >, p_0)$ in the mean magnetic field $< B > \equiv < \sqrt{B_o^2 + \delta B^2} >$. The wave amplitudes are drawn in a random manner so as to keep constant the model parameter δB:

$$\left[\sum_{i=1}^{288} (\delta \mathbf{B}_0^{(i)})^2\right]^{1/2} \equiv \delta B \qquad . \tag{3.3}$$

The dispersion relation for Alfvén waves, $\omega^2 = V_A^2 k^2$, provides the respective ω parameter for any given wave. The sign of ω is defined by selecting the wave velocity V at, randomly, $\pm V_A$, but it is subject to a constraint that a number of waves moving in any direction is the same.

Simulations were performed for a set of Alfvén wave turbulence models:

(i) Linearly polarized waves in the positive and the negative direction (LIN).

(ii) Right-handed polarized waves propagating in the positive and left-handed polarized waves propagating in the negative direction ($R^{(+)}L^{(-)}$).

(iii) Left-handed polarized waves propagating in the positive and the negative direction ($L^{(+)}L^{(-)}$).

In all cases the intensities of waves propagating in the positive and the negative direction were the same. All these models were considered for $\delta B = 0.25$ or 0.6 and $V_A = 0.05$ or 0.1 . The large values for the Alfvén velocity were considered to speed up the acceleration process (and computations). The flat wave spectrum allowed to keep scattering approximately the same within the considered particle energy range. Typically 1500 particles were used in an individual run.

4 Results and final remarks

In Fig. 1 particle momentum distributions for three magnetic turbulence models – $L^{(+)}L^{(-)}$, $R^{(+)}L^{(-)}$ and LIN – are presented. In the upper right corner of each panel the respective values of $< p > /p_0$, a ratio of the average particles momentum, $< p >$, to the initial momentum are provided. In the figure we observe that the acceleration rate depends on a turbulence model considered. The most effective acceleration, when in addition to the diffusive acceleration a positive regular acceleration term arises (the particle distribution most shifted to the right), is present for the turbulence model $R^{(+)}L^{(-)}$ Then the circular polarization is opposite for opposite wave propagation directions and the scattering anisotropy leads to more effective head-on collisions. The change of one polarization in the model $L^{(+)}L^{(-)}$ into the opposite ones results in negative regular acceleration term decreasing particle energy gains. Results obtained for these two situations are compared in Fig. 1 to the ones at LIN model, where the regular term vanishes.

Possibility of acting regular acceleration processes in turbulent MHD media is of interest in all situations, where the second order Fermi acceleration is considered to energize cosmic rays. With the small Alfvén velocity such processes are usually inefficient and even small first order acceleration effects could substantially modify the resulting particle spectra. In the present paper we consider a particular situation with a non-symmetric MHD turbulence modifying the ordinary momentum diffusion acceleration. The importance of such processes in real astrophysical situations is a matter of debate. However, one may note that in several astrophysical processes, there may occur processes generating non-symmetric circular polarized waves. A good example provide waves generated by the streaming instability processes in a cosmic ray precursor region upstream of the shock wave.

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References

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