

# Equilibrium Charge of Energetic Ions from the Balance of Electron Capture and Loss Cross-Sections

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## Abstract

The well known semiempirical expressions for *Effective Charge* ( $q^*$ ) of energetic ions during their passage through matter do not allow to draw inferences about the involved essential processes themselves. Here, we propose a theoretical approach to estimate  $q^*$  in terms of electron capture and loss cross-sections with explicit dependence on the medium temperature (T), either for atomic matter or plasmas, whatever the energy change process undergone by particles, acceleration or deceleration.

## 1 Introduction:

Energetic ions undergo charge interchange (electron capture and loss) during their passage through matter when the amount of traversed matter is high enough for the establishment of those atomic interactions (relatively high density and/or long flight time in a given volume). The behavior of the charge of energetic ions as a function of their velocity during such interactions with the traversed medium is known as *Effective Charge*  $q^*$ : such evolution of  $q^*$  as ions change their velocity (due to any acceleration or deceleration process) is basically defined by the balance between the cross-sections of both processes - electron capture and loss - that energetic ions undergo with the medium. The knowledge of  $q^*$  is limited to experiments of stopping power in atomic matter for some kind of projectil ions and target atoms (e.g. Betz, 1972), and described by semiempirical equations of the form  $q_{semp}^* = Z[1 - \xi \exp(-a\beta/Z^{2/3})]$ , where  $\beta = V/c$  is the particle velocity in terms of the light velocity  $c$ ,  $Z$  is the atomic number; in atomic solids and gases  $\xi = 1$  and  $a = 125 - 130$ . However, these expressions do not give us information about the processes themselves that cause the evolution of the ion charge as its velocity is being degrading, neither can be applied in finite temperature (T) matter, nor in the case in which ions instead of stopping are undergoing acceleration. Extrapolation of those semiempirical expressions to particle acceleration studies in astrophysical sources is not justified, because acceleration is not just the opposite process of stopping, the former is a process of electromagnetic nature and stopping is an atomic process.

We propose here that an adequate approach to the study of *Effective Charge* must be based on the balance between the cross-sections of electron capture and loss, and for the goal of generality must also apply in T-dependent matter, whatever the ionization degree of the medium and whatever the process that causes the evolution of ion velocity - acceleration or deceleration. Unfortunately, present day cross-sections do not consider the temperature of the target medium: a procedure to build a global picture of cross-sections from thermal energies up to cosmic ray energies was described by Pérez-Peraza et al (1985), on basis to the classical works of Nikolaev (1965) for atomic matter, and Bethe and Salpeter (1957) for ionized matter; T-dependence was basically introduced through the relative velocity  $V_r$  between the projectil-ion velocity  $V$  and the thermal velocity  $V_t$  of targets (electrons or atoms, depending on whether we are dealing with plasmas or atomic matter) and by normalizing to the *local thermal charge*  $Q_L(Z, T)$  of the ion at the temperature of the background medium from which some ions are accelerated, or, in which energetic ions are decelerated toward thermalization with the medium at  $q^* = Q_L$ .  $Q_L(Z, T)$  values were taken from the ionization equilibrium tables of Arnaud and Ruthenflug (1985).

## 2 Effective Charge from the Balance of Electron Capture and Loss:

According to Pérez-Peraza and Alvarez (1990) the mean equilibrium charge carried on by an ion depends on the number of capture and loss interactions that the ion undergoes along its path, given as  $C = NV_r\sigma_c t$  and  $P = NV_r\sigma_l t$  respectively, where  $N$  is the number density of the medium,  $V_r$  is the relative velocity between projectiles and targets,  $\sigma_c$  and  $\sigma_l$  are the electron capture and loss cross-sections respectively and  $t$  is either an acceleration time  $t_a$  (for ions gain energy from  $E$  to  $E + \Delta E$ ) or a stopping time  $t_d$  (for deceleration of ions from  $E$  to  $E - \Delta E$ ). For estimations of  $t_a$  we used the acceleration efficiency from stochastic acceleration by the fast MHD mode (Gallegos and Pérez-Peraza 1994), whereas for  $t_d$  we used the stopping power time in plasmas from Buttler and Buckingham (1962), and in atomic gases from Lindhard and Scharff (1961) and Lindhard et al (1963) for nuclear and electronic stopping and from Ginzburg and Syrovatskii (1964) for electronic stopping at high energies (ionization losses). Though  $t_a$  is  $q^*$ -independent,  $t_d$  has a From the analysis of the graphycal behavior of  $q_{semp}^*$  it can be seen that  $(dq/dV) \propto V^{-n}$ , that is, the slope  $(dq/dV)$  decreases with  $V$  from a certain initial value at  $V_o$  (in  $Q_L$ ) up to the value 0 at a  $V_{max}$  defined as the velocity where  $q$  reaches  $Z$ . Since curves are close to a paraboloid we assume that  $n = 2$ . Similarly, since  $(dq)$  increases as  $q \Rightarrow Z$ , hence we assume  $(dq/dV) \propto q$ . On the other hand, the balance between capture and loss cross-sections is such that  $C = P$  implies a low level of charge interchange, that is  $(dq)$  is small, but as soon as  $P \gg C$  or  $C \gg P$  charge interchange increases. Let us assume that such balance is described by  $F[C(q, V), P(q, V)]$ ; for the goal of simplicity we approximate such a balance function to  $F(C; P; V)$ , so, we can write the following evolution equation

$$\frac{dq}{dV} = \frac{KqF(C; P; V)}{V^2} \quad (1)$$

where  $K$  is the proportionality constant. The balance function  $F$  contains not only the relative importance of electron capture and loss, but also the relative distribution (spectrum) of projectil-ions of charge  $q$  in the velocity interval  $dV$ :  $F(C; P; V) = (V_o + f'V^2/K)\exp[f + K(1 + V_o/V)]$ , where  $f'$  is obtained from  $f = CP/(C + P)^2$ . Solution of eq. (1) leads to

$$q^* = Q_L \exp\left[\frac{CP}{(P + C)^2}\right] \exp\left[\frac{K(1 - V_t/V_r)}{1 + V_o/V_r}\right] \quad (2)$$

where  $K = [\ln(Z/Q_L) - f]/[1 - V_t/V_{max}]$  where  $V_o$  and  $V$  have been substituted by  $V_t$  and  $V_r$  respectively. It can be appreciated from eq. (2) that at  $V_r = V_t$  we obtain  $q^* = Q_L(Z, T)$  and when  $V_r = V_{max}$ , we obtain  $q^* = Z$ , where  $V_{max}$  is defined as the velocity where ions reach the nuclear charge  $Z$ . Since cross-sections are functions of  $q^*$  and  $q^* = q * (\sigma_c, \sigma_l)$  the procedure to evaluate  $q^*$ ,  $\sigma_c$  and  $\sigma_l$  is based on the *self-regulating interdependence* between  $q^*$  and the cross-section, that is, at each velocity step a *"retro-feeding"* is performed. Physically, this means that ion charge is evolving during acceleration or deceleration. Such a *"retro-feeding"* is also carried out in the stopping rates for evaluations of  $t_d$ . Results are displayed through Figures 1-4. Figs. (1-2) correspond to ion acceleration and deceleration in plasmas respectively, whereas Figs. (3-4) refer to acceleration and deceleration in atomic matter. Acceleration assumes that particles begin to be accelerated from thermal energies with the local thermal charge states reaching  $Z$  at  $\sim 50 MeV$ . For deceleration we inject hydrogenic ions in a medium of temperature  $T$  with an energy of 100 MeV reaching  $Q_L$  at its thermal energy.  $q^*$  in Figs. (a) is plotted together with the normalized  $q_{semp}^*$ , but obviously, this comparison has only sense in the case of ion deceleration.  $P$  and  $C$  are plotted in Figs. (b) giving us a picture of cross-section behavior, though in the case of deceleration the proportionality between  $C$  and  $P$  and the cross-sections is slighted modulated by the  $q^*$ -dependence of  $t_d$ .

## 3 Conclusions:

The interest of this preliminary work is based on the need to draw inferences about the behavior of cross-sections in finite-T-matter that up to now is not openly disposable. To do so we propose

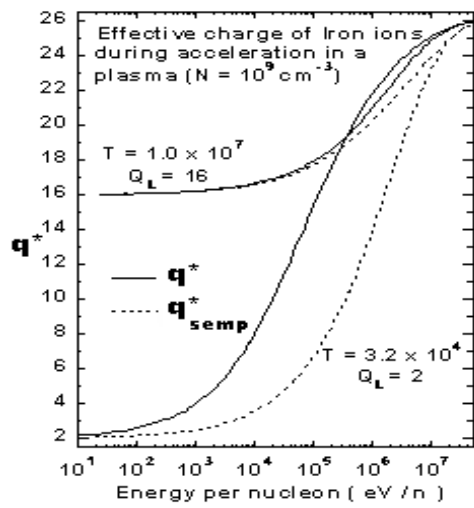


Figure 1(a)

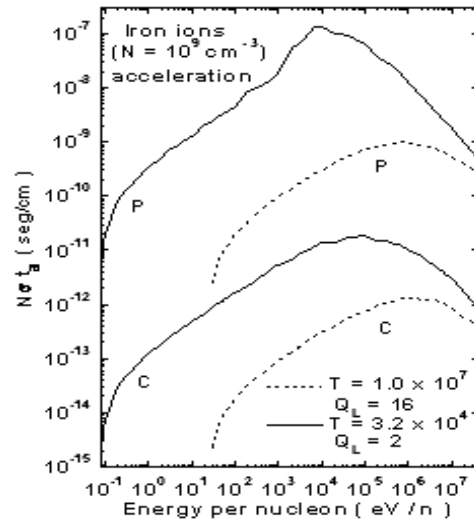


Figure 1(b)

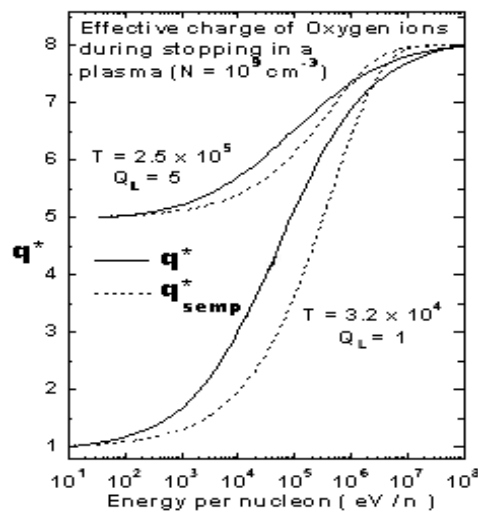


Figure 2(a)

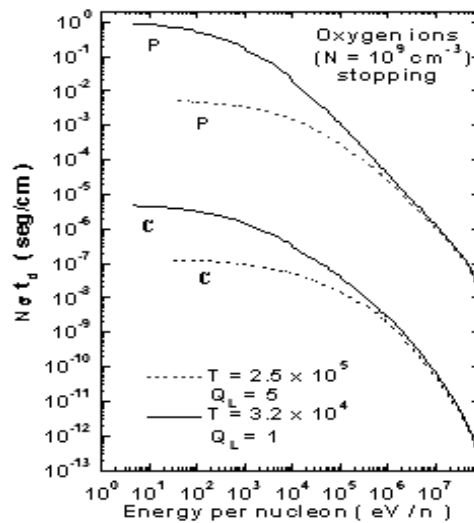


Figure 2(b)

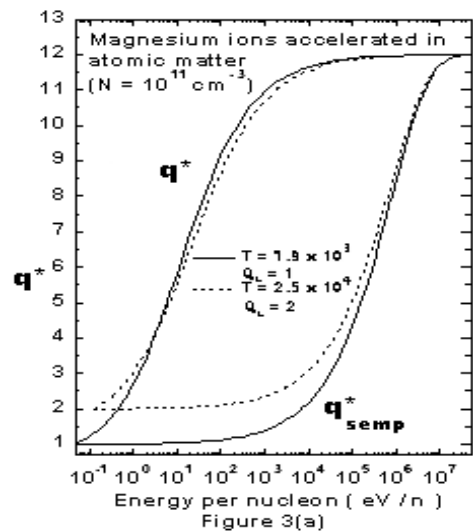


Figure 3(a)

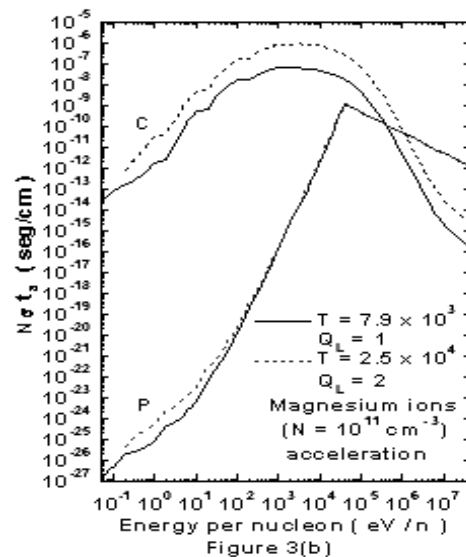
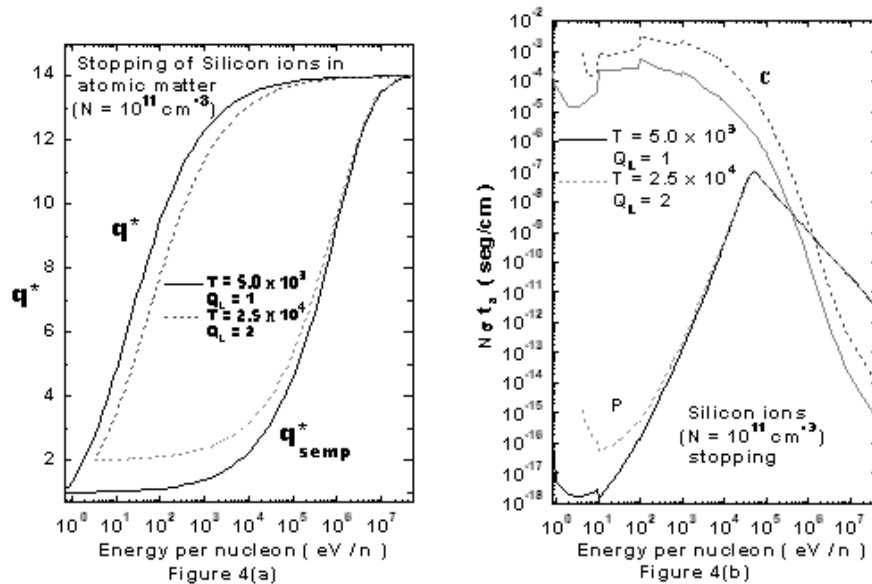


Figure 3(b)



as a second stage of this research to proceed with a *calibration* of our predictions for  $q^*$  with data on charge evolution in finite-T-matter; however, since such a data is also not disposable at present, it may eventually be created in experiments of atomic interactions in plasmas, or even from the comparison with measurements of charge states of solar energetic ions at many different velocities in a single event (provided that the amount of traversed matter in the source has been high enough for the establishment of charge transfer during that event). A "first level" calibration may be done by the confrontation of our  $q^*$  evaluated at  $T \equiv 0$  with the semiempirical one in order to derive a V-dependent function that sets  $(q^*/q_{semp}^*) \sim 1$ . Calibration of our T-dependent  $q^*$  curves with such a function should lead us to obtain a better description of T-dependent cross-sections. Such *calibrations* may provide a kind of *thermometer* about how far (or close) are our constructed cross-sections from the unknown real ones, and so, to proceed with further refinements in the estimation of them. Other implications of this work are within the frame of astrophysics as well as plasma confinement research, mainly in association with diagnostic methods based on photon-emissions following electron capture, as well as in studies of particle composition based on mass/charge selectivity effects, evolution of charge spectra, and so on.

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