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The Process Leading to the Formation of a 'Knee' in the Proton Spectrum at Energies of about 1 TeV

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Abstract

It is shown, that the spectrum of protons, formed as a result of acceleration at the initial stage of the supernova shell expansion, inevitably has a 'knee' in the range of energies of about 1 TeV. The formation of a knee is caused by a specific dependence of the effective cross-section of inelastic interaction and therefore, should depend weakly on the type of the supernova and various mechanisms of particle acceleration.

The equation of nucleon transport (1) was considered in many papers, especially 30-35 years ago during the period of intensive study of the hardron high energy component in the Earth's atmosphere.

$$\frac{\partial J(E,x)}{\partial x} = -\frac{J(E,x)}{\lambda(E)} + \int_E^{\infty} \frac{J(E^1,x) f(E,E^1) dE^1}{\lambda(E^1)} \qquad (1)$$

Here $J(E,x)$ - is the intensity of the flux of nucleons with energy E at atmospheric depth x g/cm^2, $\lambda(E)$ is the range for inelastic interactions of nucleons with energy E, . $f(E,E^1)$ is the probability for a particle to lose the energy $(E-E^1)$.

It was already mentioned then, that for a constant cross-section of inelastic interaction the power-law spectrum $J(E,x=0) = AE^{-\beta_0}$ remains power-law with a constant spectral index β_0 at all atmospheric depth x g/cm^{-2}.

The integral term in (1) somewhat increases the range for absorption of the nucleon flux. Without this term $J(E,x) = AE^{-\beta} e^{-x/\lambda}$, and taking into account the incomplete loss of energy (the integral term in (1)) $J(E,x) = AE^{-\beta_0} e^{-x/L}$, where $L/\lambda \approx 1.2 \div 1.3$. Therefore, below we will consider the solution of (1) without the integral term, since it does not change the situation significantly.

Let us consider (1) without the integral term but with energy dependent $\lambda(E)$:

$$\frac{\partial J(E,x)}{\partial x} = -\frac{J(E,x)}{\lambda(E)} \qquad (2)$$

For boundary conditions $J(x=0,E) = AE^{-\beta_0}$, the solution for (2) acquires the form:

$$J(E,x) = AE^{-\beta_0} e^{-\frac{x}{\lambda(E)}} \qquad (3)$$

In an atmosphere consisting of medium nuclei the effective cross-section for inelastic interaction may be approximated with good accuracy in the form (Ranft, 1997):

$$\sigma^{in} = \sigma_0 = \text{const at } E < E_0$$

$$\sigma^{in} = \sigma_0 = (1 + b \ln E) \text{ at } E > E_0$$

where E_0~500÷1000 GeV.

We will rewrite $\sigma^{in}(E) = \sigma_0 (1 + be^{-(\frac{E_0}{E})^2} \ln E)$, which reduces to σ =const at $E < E_0$ and $\sigma = \sigma_0 (1 + b \ln E)$ at $E > E_0$.

Since $\lambda(E) \sim \frac{1}{\sigma(E)}$, the solution of (3) acquires the form:

$$J(E,x) = AE^{-\beta_0} e^{-\frac{x}{\lambda_0}\left(1+be^{-(E_0/E)^2} \ln E\right)} \qquad (4)$$

But $e^{-\frac{x\left(b\ln E e^{-\left(\frac{E_0}{E}\right)^2}\right)}{\lambda_0}} \approx E^{-\delta}$, where $\delta = \frac{x}{\lambda_0} b e^{-\left(\frac{E_0}{E}\right)^2}$. Hence, at $E > E_0$ the spectral index of nucleons $\beta = \beta_0 + \frac{x}{\lambda_0} b$ grows with increasing x g/cm^{-2} .

This phenomenon was discovered in hardron spectra in the Earth's atmosphere back in 1965 and is explained by the logarithmic dependence of σ^{in} (Grigorov, Rapoport, Savenko, 1965).

Account for the integral term in (1) yields $\delta(x) = \frac{x}{L} b$ instead of $\frac{x}{\lambda} b$ (Akimov, Kozlov, 1975; Grigorov, 1977).

Returning to the solution of (4): $\beta(x) = \beta_0 + b\frac{x}{L} e^{-\left(\frac{E_0}{E}\right)^2}$

As it follows from this solution, at $E < E_0$ for all depths $\beta(x) - \beta_0 \cong 0$, i.e. the spectral index does not change and remains equal to the index of the primary spectrum.

In the range $E > E_0$ $\beta(x) - \beta_0 \approx b\frac{x}{L}$, i.e. the spectrum becomes 'softer' with increasing amount of matter traveled. This dependence $J(E,x)$ on x is shown in Fig.1. As it can be seen from the figure, the purely power-law spectrum of nucleons, passing through matter, acquires a 'knee' and the location of this knee $E \sim E_0$ is defined by the dependence of σ^{in} on E and weakly depends on the medium.

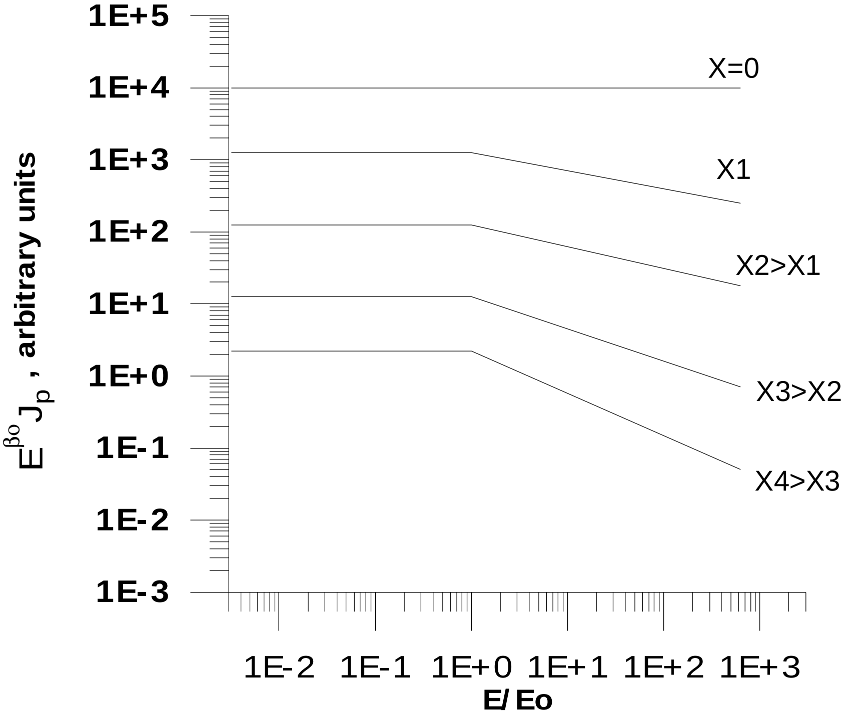

Figure 1. A schematic drawing of the change in the proton spectrum with increasing amount of matter X g/cm^{-2} from the region of acceleration .

We will now assume, that acceleration of cosmic ray particles occurs at the initial stage of the supernova expansion. Such a possibility was discussed in (Colgate, Johnson, 1960). At different distances from its edge proton fluxes are formed, which have to penetrate through different amounts of matter x g/cm^2, before exiting into the galactic space.

The exiting fluxes of protons will have spectra with an undistorted index β_0 up to $E < E_0$, and a 'knee' at $E < E_0$. The resulting spectrum will be their sum. It is obvious, that the total spectrum will have in the energy range $E > E_0$ a power index, which is greater, that at $E < E_0$, and the region of the 'knee' $E \approx E_0$ will mainly be defined by the energy dependence of $\sigma^{in}(E)$, i.e. the properties of accelerated particles and will be practically independent of the matter in which the acceleration occurs.

It is also obvious that this phenomenon cannot occur with nuclei, since for a relatively dense shell they will be disintegrated into nucleons after several inelastic collisions.

Hence, a natural assumption, that the bulk of the cosmic ray proton flux is formed at an early stage of the supernova shell expansion will inevitably lead to a 'knee' in the observed spectrum at energies of $E_0 \sim 0.5 \div 1$ TeV.

A more accurate definition of the 'knee' location, the value of $\Delta\beta = \beta(x) - \beta_0$, the form of the spectrum in the transition region from $E < E_0$ to $E > E_0$ require specific calculations, based on the choice of the acceleration model.

It cannot be ruled out, that detailed experimental study of these properties of the proton spectrum will lead to the choice of a single acceleration model and significant detailization of the acceleration process at early stages of the blast.

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