The Structure of a Cosmic-Ray–Plasma System

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Abstract

Hydrodynamic approach is a fairly good approximation for studying the structure and evolution of a cosmic-ray–plasma. In this contribution we consider a four-fluid model which comprises the thermal plasma, cosmic rays and two oppositely propagating Alfvén waves. In general there are three energy exchange mechanisms, namely, work done by plasma flow, cosmic ray streaming instability and stochastic acceleration. We present several steady state profiles of the system which demonstrate the interplay between the aforementioned mechanisms.

1 Introduction:

Cosmic rays interact with thermal plasma via hydromagnetic irregularities or waves in the plasma. Scattered by the irregularities (say by gyro-resonant scattering), cosmic rays advect and diffuse through the plasma. Cosmic rays acquire energy from the plasma if the plasma flow is systematically converging. This is called the first order Fermi process. As they advect with the plasma, cosmic rays excite hydromagnetic waves in the plasma via streaming instability. When waves of different phase velocities are present, cosmic rays diffuse in the momentum space also. This is called the second order Fermi process or stochastic acceleration. The system is self-consistent and is called a cosmic-ray–plasma system.

As one can imagine to solve the system in distribution function approach is very difficult (see Malkov 1997a, b). We adopt the hydrodynamic approach, which is a fairly good approximation for studying the dynamics and structure of cosmic-ray–plasma systems (see e.g., Drury and Völk 1981; Axford, Leer & McKenzie 1982; McKenzie and Völk 1982; Ko 1992). In this approach every component is considered as a fluid. For instance, cosmic rays and waves (magnetic field also if one likes) are treated as massless fluids but with significant energy density or pressure.

In section 2 we discuss the four-fluid model. In section 3 we provide some steady state results and a brief discussion.

2 Four-Fluid Model:

Ko (1992) proposed a fairly comprehensive version of the hydrodynamic approach. That is a four-fluid model which comprises thermal plasma, cosmic rays and two oppositely propagating Alfvén waves. The governing equations are the total mass and momentum equations, and energy equations of various components (i.e., kinetic energy and thermal energy of plasma, cosmic ray energy and wave energies).

In one dimension with magnetic field parallel to the plasma flow, we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho U) = 0\,, \tag{1}$$

$$\rho\frac{\partial U}{\partial t} + \rho U\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}(P_{\rm th} + P_{\rm c} + P_{\rm w}^{+} + P_{\rm w}^{-})\,, \tag{2}$$

$$\frac{\partial E_{\rm k}}{\partial t} + \frac{\partial F_{\rm k}}{\partial x} = -U\frac{\partial}{\partial x}(P_{\rm th} + P_{\rm c} + P_{\rm w}^{+} + P_{\rm w}^{-})\,, \tag{3}$$

$$\frac{\partial E_{\rm th}}{\partial t} + \frac{\partial F_{\rm th}}{\partial x} = U\frac{\partial P_{\rm th}}{\partial x}\,, \tag{4}$$

$$\frac{\partial E_{\rm c}}{\partial t} + \frac{\partial F_{\rm c}}{\partial x} = [U + (e_+ - e_-)V_{\rm A}]\frac{\partial P_{\rm c}}{\partial x} + \frac{P_{\rm c}}{\tau}\,, \tag{5}$$

$$\frac{\partial E_{\rm w}^\pm}{\partial t} + \frac{\partial F_{\rm w}^\pm}{\partial x} = U\frac{\partial P_{\rm w}^\pm}{\partial x} \mp e_\pm V_{\rm A}\frac{\partial P_{\rm c}}{\partial x} - \frac{P_{\rm c}}{2\tau}\,, \tag{6}$$

where ρ and U are density and velocity of the plasma; $_{\rm k}$, $_{\rm th}$, $_{\rm c}$ and $_{\rm w}$ denote the kinetic part of the plasma, the thermal part of the plasma, cosmic ray and wave, respectively; and $\pm$ denote forward and backward propagating waves. The energy densities and energy fluxes are given by $E_{\rm k} = \frac{1}{2}P_{\rm k} = \frac{1}{2}\rho U^2$, $E_{\rm th} = P_{\rm th}/(\gamma_{\rm g}-1)$, $E_{\rm c} = P_{\rm c}/(\gamma_{\rm c}-1)$, $E_{\rm w}^\pm = 2P_{\rm w}^\pm$, $F_{\rm k} = E_{\rm k}U$, $F_{\rm th} = (E_{\rm th}+P_{\rm th})U$, $F_{\rm c} = (E_{\rm c}+P_{\rm c})[U+(e_+-e_-)V_{\rm A}] - \kappa\partial E_{\rm c}/\partial x$ and $F_{\rm w}^\pm = E_{\rm w}^\pm(U\pm V_{\rm A}) + P_{\rm w}^\pm U$. The Alfvén speed is given by $V_{\rm A} = B/\sqrt{\mu_\circ\rho}$, and B is constant in one dimensional problem.

Ko (1992) gave a simple model of the coupling between plasma, cosmic rays and waves. The diffusion coefficient κ, stochastic acceleration rate $1/\tau$, and $e_\pm$ (which are related to streaming instability) are given by,

$$\kappa = \frac{c^2}{3\alpha(P_{\rm w}^+ + P_{\rm w}^-)}\,, \quad \frac{1}{\tau} = 16\alpha\frac{V_{\rm A}^2}{c^2}\frac{P_{\rm w}^+P_{\rm w}^-}{(P_{\rm w}^+ + P_{\rm w}^-)}\,, \quad e_\pm = \frac{P_{\rm w}^\pm}{(P_{\rm w}^+ + P_{\rm w}^-)}\,, \tag{7}$$

where c is the speed of light, and α indicates the strength of coupling.

In steady state, there are six integration constants. They are magnetic flux $\Phi = B$, mass flux $\mathcal{J} = \rho U$, entropy constant $\mathcal{A} = P_{\rm th}/\rho_{\rm g}^\gamma$, total energy flux $\mathcal{F} = F_{\rm k} + F_{\rm th} + F_{\rm c} + F_{\rm w}^+ + F_{\rm w}^-$, total momentum $\mathcal{G} = P_{\rm k} + P_{\rm th} + P_{\rm c} + P_{\rm w}^+ + P_{\rm w}^-$ and wave-action

$$\mathcal{W}_{\rm A} = \left[F_{\rm c} + \frac{(U+V_{\rm A})^2}{V_{\rm A}}E_{\rm w}^+ - \frac{(U-V_{\rm A})^2}{V_{\rm A}}E_{\rm w}^-\right]. \tag{8}$$

3 Results and Discussion:

We seek steady state structures of the aforementioned cosmic-ray–plasma system. In systems without waves or systems where the thermal plasma is dominant (the so called non-linear test particle picture) physical solutions can be classified completely (Drury & Völk 1982; Axford, Leer & McKenzie 1982; Jiang, Chan & Ko 1996; Ko, Chan & Webb 1997; Ko 1998). Unfortunately the mathematics of the full system is too complicated for us to sort out every physical solutions. We work out several typical solutions numerically. We, however, cannot claim that we exhaust all generic structures.

A solution or structure is deemed physical if its pressures are non-negative, and it approaches uniform states both far upstream ($x \to -\infty$) and far downstream ($x \to \infty$). Moreover, due to stochastic acceleration, at least one of the three pressures $P_{\rm c}$, $P_{\rm w}^\pm$ must be zero as $x \to \pm\infty$ (see Eqs. (5) & (6)).

Recall that in cosmic-ray–plasma systems without waves there are two generic steady state structures. The flow profile is monotonically decreasing and it is either continous or with one discontinuity (i.e., a subshock) (Drury & Völk 1981; Axford, Leer & McKenzie 1982; Ko, Chan & Webb 1997). For systems with a uni-directional wave we can only consider continuous flow, because a subshock generates both waves downstream. In this case the flow profile is also monotonically decreasing (McKenzie & Völk 1982). We should point out that uniform states are physically allowable solutions in the simplified systems mentioned above but not in the full system.

In this work we concentrate only on the continuous flow profile of the full system (i.e., with both foward and backward waves). Furthermore, we consider super-Alfvénic flows only (i.e., $U/V_{\rm A} > 1$ everywhere). In all our calculations we normalise the magnetic field, velocity, density, pressures and length to $B_\circ$, $U_\circ$, $\rho_\circ$, $P_\circ$ and $L_\circ$, where $B_\circ^2/\mu_\circ = \rho_\circ U_\circ^2 = P_\circ$ and $L_\circ = c^2/\alpha P_\circ U_\circ$. To integrate the set of equations, besides assigning values to $\gamma_{\rm g}$ and $\gamma_{\rm c}$, eight constants are required, e.g., three integration constants Φ, $\mathcal{J}$, $\mathcal{F}$, and five initial values of U, $P_{\rm th}$, $P_{\rm c}$, $P_{\rm w}^+$, $P_{\rm w}^-$ at $x = 0.0$.

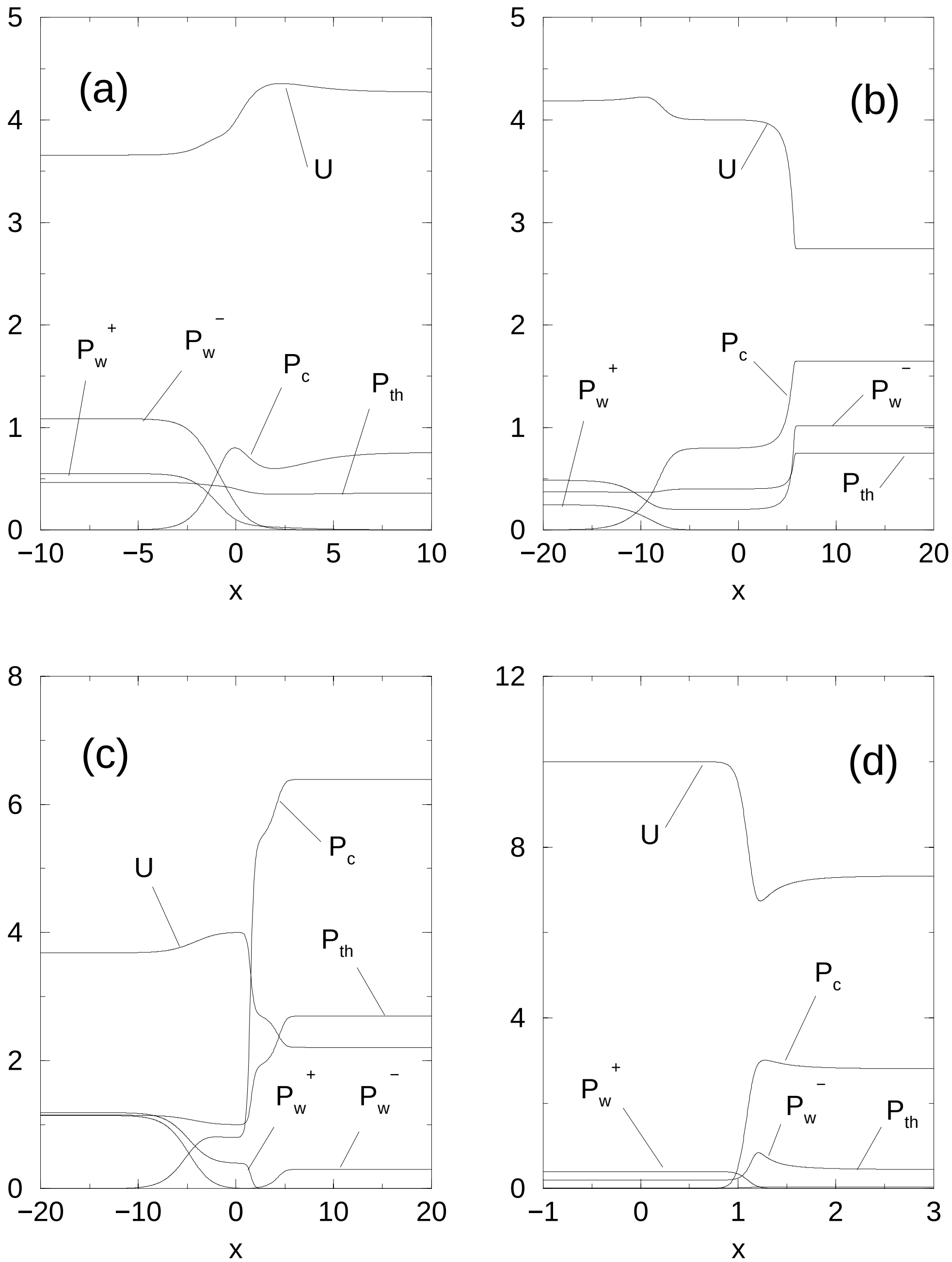

Figure 1: Profiles of cosmic-ray–plasma systems in hydrodynamic approach. The parameters of the four examples are given in the text.

In Fig. 1 we take $\gamma_{\rm g} = \frac{5}{3}$ and $\gamma_{\rm c} = \frac{4}{3}$; and

Fig. 1(a): $\Phi = 1.0$, $\mathcal{J} = 1.6$, $\mathcal{F} = 31.26$, and $U = 4.0$, $P_{\rm th} = 0.4$, $P_{\rm c} = 0.8$, $P_{\rm w}^{+} = 0.1$, $P_{\rm w}^{-} = 0.25$, at $x = 0.0$; moreover $\mathcal{A} = 1.842$, $\mathcal{G} = 7.95$, $\mathcal{W}_{\rm A} = 12.82$.

Fig. 1(b): $\Phi = 1.0$, $\mathcal{J} = 1.6$, $\mathcal{F} = 26.30$, and $U = 4.0$, $P_{\rm th} = 0.4$, $P_{\rm c} = 0.8$, $P_{\rm w}^{+} = 10^{-6}$, $P_{\rm w}^{-} = 0.2$, at $x = 0.0$; moreover $\mathcal{A} = 1.842$, $\mathcal{G} = 7.800$, $\mathcal{W}_{\rm A} = 6.250$.

Fig. 1(c): $\Phi = 1.0$, $\mathcal{J} = 4.0$, $\mathcal{F} = 63.53$, and $U = 4.0$, $P_{\rm th} = 1.0$, $P_{\rm c} = 0.8$, $P_{\rm w}^{+} = 0.4$, $P_{\rm w}^{-} = 0.01$, at $x = 0.0$; moreover $\mathcal{A} = 1.0$, $\mathcal{G} = 18.21$, $\mathcal{W}_{\rm A} = 35.65$.

Fig. 1(d): $\Phi = 1.5$, $\mathcal{J} = 1.0$, $\mathcal{F} = 70.40$, and $U = 10.0$, $P_{\rm th} = 0.02$, $P_{\rm c} = 10^{-8}$, $P_{\rm w}^{+} = 0.4$, $P_{\rm w}^{-} = 0.2$, at $x = 0.0$; moreover $\mathcal{A} = 0.9283$, $\mathcal{G} = 10.62$, $\mathcal{W}_{\rm A} = 34.33$.

The most significant feature in Fig. 1 is the flow and pressure profiles can be non-monotonic, which is in sharp contrast to systems without waves or systems with a uni-directional wave. Fig. 1(a) is a reminiscence of the non-linear test particle picture of Jiang, Chan & Ko (1996), where the cosmic ray pressure can be increasing non-monotonically. In Fig. 1(b) the downstream state closely resembles a system without forward wave ($P_{\rm w}^{+} = 0$), but the upstream state is totally different. Fig. 1(c) shows a prominent peak in velocity and a valley in backward wave pressure, while Fig. 1(d) shows the opposite. In these examples the cosmic ray pressure far downstream is always larger than the cosmic ray pressure far upstream, i.e., cosmic ray is always accelerated.

The rich morphology of structures are the result of the interplay between the three basic energy transfer mechanisms (see Eqs. (3)–(6)): (i) work done by plasma flow, (ii) cosmic ray streaming instability, and (iii) stochastic acceleration. (i) and (ii) are facilitated by pressure gradients, and (ii) and (iii) involve energy exchange between cosmic rays and waves. Note that (i) and (ii) can accelerate or decelerate cosmic rays, while (iii) can only accelerate. As shown in the non-linear test particle picture, work done by plasma flow is, in general, the major accelerating mechanism for cosmic rays. The relative contributions of the three mechanisms along x produce the fine details of the profile.

We haven't touched upon the other class of flow profiles, namely, profiles with a subshock. The mathematics is rather complicated, but one thing is clear. The structure ought to be qualitatively different from the structure of systems without waves. Besides non-monotonicity, the downstream state will not be uniform. (Recall that uniform state is the only physically allowable downstream state available to systems without waves). As far as the upstream state has a wave, both forward and backward waves are generated downstream by the shock. When cosmic rays and both waves are present, no uniform state is possible because of the stochastic acceleration.

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