Ion injection and acceleration at modified shocks

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Abstract

The theory of diffusive particle acceleration explains the spectral properties of the cosmic rays below energies of $\simeq 10^{15}$ eV as produced at strong shocks in supernova remnants (SNR's). To supply the observed flux of cosmic rays, a significant fraction of the energy released by a supernova has to be transfered to cosmic rays. The key to the question of the efficiency of SNR's in producing cosmic rays is the injection process from thermal energies. A self-consistent model has to take into account the interaction of the accelerated particles with magneto-hydrodynamic waves, which generate the particle diffusion, a requisite for the acceleration process. Such a nonlinear model of the turbulent background plasma has been developed recently (Malkov 1998). We use this model for the first numerical treatment of the gas dynamics and the diffusion-convection equation at a quasi-parallel strong shock, which incorporates a plasma-physical injection model to investigate the cosmic ray production.

1 Introduction

The problem of the efficiency of particle acceleration at shocks by the first order Fermi acceleration process, and the strength of the back-reaction of these particles on the plasma flow, is intimately related to the injection process. This describes the rate at which particles are not only part of the thermal plasma, which is compressed and heated when it passes the shock, but become subject to energy gain due to the Fermi process, described by the diffusion-convection equation (e.g. Skilling 1975). We will follow closely a picture of this injection process which has been developed by Malkov & Völk (1995) and Malkov (1998).

The spatial diffusion of particles, which is an essential part of their acceleration process, is produced by magneto-hydrodynamic waves, which are in turn generated by particles streaming along the magnetic field $\boldsymbol{B}_0$. We will assume the field $\boldsymbol{B}_0$ to be parallel to the shock normal (x-direction). The magnetic field, which corresponds to a circularly polarised wave can be written as $\boldsymbol{B} = B_0\boldsymbol{e}_x + B_\perp(\boldsymbol{e}_y \cos k_0 x - \boldsymbol{e}_z \sin k_0 x)$. The amplitude $B_\perp$ will be amplified downstream by a factor $\rho/\rho^* = r$, where ρ and ρ^* are the plasma densities downstream and upstream respectively and r is the compression ratio. The downstream field can be described by a parameter ϵ, for which we assume $\epsilon := B_0/B_\perp \ll 1$, in the case of strong shocks considered here. Note that the perpendicular component of the magnetic field leads effectively to a quasi-alternating field downstream of the shock for particles moving along the shock normal. Only particles with a gyro radius $r_{\mathrm{g}\perp} = p\,c\,\sin\alpha/(eB_\perp)$ for which the condition $k_0 r_{\mathrm{g}\perp} > 1$ is fulfilled, would be able to have a net velocity with respect to the wave frame, i.e. the downstream plasma would be transparent. The fraction of these particles, which are also in the appropriate part of the phase space (depending on the shock speed) would be able to cross the shock from downstream to upstream. For the protons of the plasma, the resonance condition for the generation of the plasma waves gives $k_0 v_{\mathrm{p}} \approx \omega_0 = \omega_\perp B_0/B_\perp$, where the cyclotron frequency of protons is given by $\omega_\perp = eB_\perp/(m_{\mathrm{p}}c)$, and v_{p} is the mean downstream thermal velocity of the protons. We now have for the thermal protons $k_0 r_{\mathrm{g}\perp} \approx \epsilon \ll 1$, which means, that most of the downstream thermal protons would be confined by the wave, and only particles with higher velocity (the tail of the Maxwell distribution) are able to leak through the shock. Ions with mass to charge ratio higher than protons, have a proportional larger gyro radius, so that the injection efficiency of protons, would yield a lower limit for the ions. On the other hand, for thermal electrons a plasma with such proton generated waves would not be transparent. However, reflection off the shock could become efficient. In the following we will focus on the wave generating protons.

To find the part of the thermal distribution for which the magnetised plasma is transparent, and therefore forms the injection pool, Malkov (1998) solves analytically the equations of motion for protons in such self generated waves. This is a nonlinear problem, because of the feedback of the leaking particles on the transparency of the downstream plasma, mediated by the waves generated in the upstream region, which are swept to downstream with the plasma flow. He finds a transparency function $\tau_{\rm esc}$ which depends mainly on the particle velocity v, the velocity of the shock in the downstream plasma frame u_2 and the parameter ϵ. This function expresses the fraction of particles which are able to leak through the magnetic waves, divided by the part of the phase space for which particles would be able to cross from downstream to upstream when no waves are present. Furthermore, as a result of the above described feedback, he is able to constrain the parameter ϵ, leaving essentially no free parameter.

The plasma flow structure, of course, also depends on the cosmic rays. These provide an energy sink for the plasma and smooth the shock structure due to a gradual deceleration of the plasma flow. We use a numerical method of solving the gas dynamics equations together with the cosmic ray transport equation which has been developed by Kang & Jones (1995), and which is outlined in more detail in Sect. 3. We show in the next section how we incorporate the above described model in this numerical method.

2 Injection model

The key part of the solution to the problem of proton (ion) injection is the above described transparency function of the plasma. Kang & Jones (1995) used a numerical injection model with two essentially free parameters which describe boundaries in momentum at which particles can be accelerated and from which these contribute to the cosmic ray pressure. The transparency function provides now a more physical definition of exactly this intermediate region. For the adiabatic wave particle interaction it is given by Malkov (1998) Eq. (33), with $\tau_{\rm esc} = 2\,\nu_{\rm esc}/(1-u_2/v)$, where the wave escape function $\nu_{\rm esc}$ is divided by the fraction of the available phase space. We use here the following approximation:

$$\tau_{\rm esc}(v,u_2) = H\left[\tilde{v}-(1+\epsilon)\right]\left(1-\frac{u_2}{v}\right)^{-1}\left(1-\frac{1}{\tilde{v}}\right)\,\exp\left\{-\left[\tilde{v}-(1+\epsilon)\right]^{-2}\right\}\,, \tag{1}$$

where the particle velocity is normalised to $\tilde{v} = vk_0/\omega_\perp$ and H is the Heaviside step function. We argued above, that $\omega_\perp/k_0 \simeq u_2/\epsilon$ (see Malkov 1998, Eq. 42). The transparency function now solely depends on the shock velocity u_2, the particle velocity v, and the relative amplitude of the wave ϵ. An important result of Malkov (1998) is the constraint on the parameter ϵ. He found $0.3 \lesssim \epsilon \lesssim 0.4$, as a result of the feedback described above. Comparison with hybrid simulations suggest $0.25 \lesssim \epsilon \lesssim 0.35$ (Malkov & Völk 1998). The function (1) is plotted in Fig. 1 for $\epsilon = 0.35$ and protons vs. their kinetic energy for three different times during the evolution of a modified shock (see below). The time dependence arises from the modification of the downstream plasma velocity by the cosmic rays. We use this function to correct the result of the cosmic ray transport equation for the upstream phase space density after each numerical time step. That means, the Maxwell distribution is restored (according to the appropriate plasma temperature) where $\tau_{\rm esc} = 0$, because here the cosmic ray acceleration has no effect. For higher velocities we multiply the difference between the new and old phase space distribution by the transparency function. Where $\tau_{\rm esc} = 1$, the solution of the transport equation is unchanged, because for these particles the plasma is completely transparent, and all of them are subject to first order Fermi acceleration. The transition between these regions is then described by the shape of the transparency function (1). The phase space distribution then changes gradually (in energy) from a Maxwell

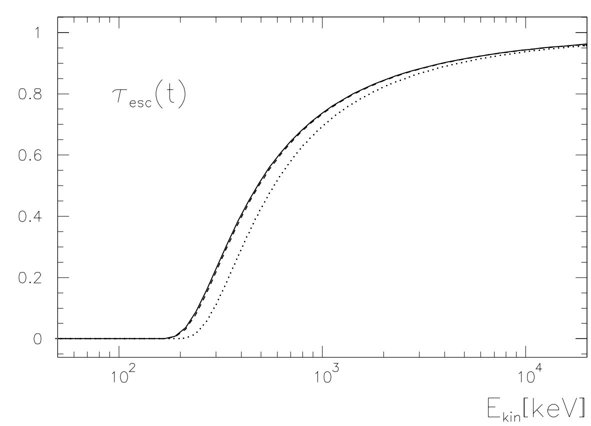

Figure 1: *Escape function Eq. (1), for protons at times $t/t_0 = 0$ (dotted), 5 (dashed) and 10 (solid line).*

distribution to a power law distribution at high energies, and it is the difference between this distribution and the Maxwell distribution, which we use to calculate the cosmic ray pressure P_c.

3 Numerical method and results

In order to find the time evolution of the cosmic ray energy spectrum, we solve the time dependent cosmic ray transport equation (using an implicit Crank-Nicholson scheme), together with the general equations of gas dynamics (using a TVD code, see Harten 1983), including the cosmic ray pressure P_c and the energy flux S which couples these equations. We refer to Kang & Jones (1995) for a more detailed description, and emphasize here only the main differences with that work. Very important for the injection process is the energy transfer between plasma and cosmic rays. The injection energy-density loss term can be written as $(\mathrm{d}e/\mathrm{d}t)_\mathrm{c} = -S$ where S is given by ($p \rightarrow p/m_\mathrm{p}$):

$$S = -\frac{2}{3}\pi\, m_\mathrm{p} c^2\, \frac{\partial u}{\partial x} \int_0^\infty \frac{\partial \tau_\mathrm{esc}(p)}{\partial p} p^5 f(p)\, \mathrm{d}p\,. \qquad (2)$$

Here we have used $\tau_\mathrm{esc}(0) = 0$, $\tau_\mathrm{esc}(\infty) = 1$, and $\partial\tau_\mathrm{esc}/\partial p \equiv 0$ for momentum $p \gg 1$, which is true, of course, for the representation given in Eq. (1). Given a step function $\tau_\mathrm{esc}(p) = H(p - p_0)$ the injection energy loss term used by (e.g.) Kang & Jones (1995) is revealed. The escape function τ_esc depends on the downstream plasma velocity, which is averaged over the diffusion length of the particles with momentum at the injection threshold. This dependence is quite important for the injection efficiency, and leads to a regulation mechanism similar to the above beam wave interaction. If the initial injection is so strong that a significant amount of energy is transferred from the gas to high energy particles, the downstream plasma cools, and becomes decelerated. Because the injection momentum is in the high energy cut-off of the Maxwell distribution of the plasma, the cooling decreases significantly the injection rate. However, the deceleration allows for a modest increase of the phase space of particles which can be injected. This is expressed by the u_2 dependence of Eq. (1), and can be seen also in Fig. 1 where the dotted line shows the escape function for the setup distribution, and the solid line at $t = 10\, t_0$. This velocity dependence balances partly the reduction of injection due to the cooling of the plasma. These two effects lead to a very *weak* dependence of the injection efficiency on ϵ in the vicinity of $\epsilon \approx 0.35$.

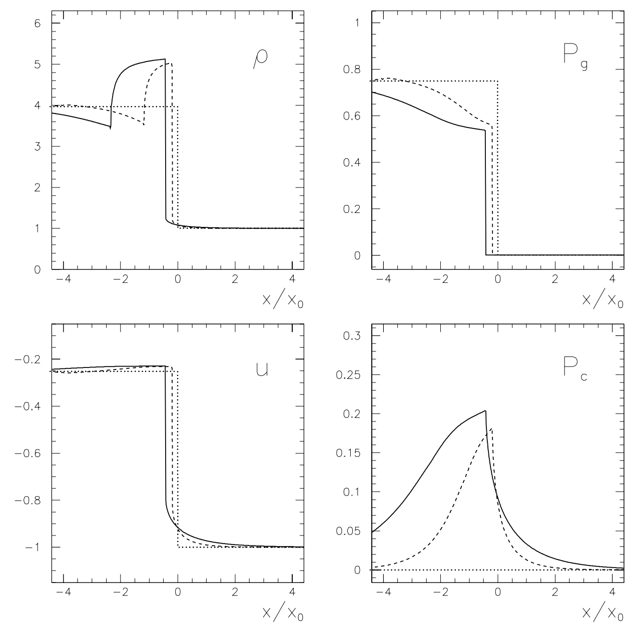

Figure 2: *Gas density ρ, pressure P_g, velocity u, and cosmic ray pressure P_c, at times $t = 0$ (dotted), $t = 5\, t_0$ (dashed) and $t = 10\, t_0$ (solid line).*

We consider here diffusion proportional to Bohm diffusion, with $\kappa = 10 \cdot \kappa_\mathrm{B}$, where $\kappa_\mathrm{B} = 3 \cdot 10^{22} p^2/(1 + p^2)^{1/2}\,\mathrm{cm^2\,s^{-1}}$, for a magnetic field $B = 1\,\mu$G. Figure 2 shows the gas density ρ, gas pressure P_g, plasma velocity u and the cosmic ray pressure P_c over the spatial length x, for different times. The scales are as follows: $t_0 = 1.2 \cdot 10^5$ s, $x_0 = 6.0 \cdot 10^{13}$ cm, $u_0 = 5000\,\mathrm{km\,s^{-1}}$, $\rho_0/m_\mathrm{p} = 1\,\mathrm{cm^{-3}}$, $P_\mathrm{g0} = 4.175 \cdot 10^{-7}\,\mathrm{erg\,cm^{-3}}$. The initial cosmic ray adiabatic index is equal to the gas adiabatic index $\gamma_\mathrm{c} = \gamma_\mathrm{g} = 5/3$, and the compression ratio is $r = 3.97$. We have used 20480 grid zones for $x/x_0 = [-18, 14]$, with the shock initially at $x = 0$, and 128 grid zones in $\log(p/m) = [-3.0, 0.3]$.

For the parameters introduced above, Fig. 3 shows the energy spectrum in form of the (at $t = 0$) normalised omni-directional flux $F(E)\mathrm{d}E \propto v\, p^2 f(p)\mathrm{d}p$ vs. proton kinetic energy downstream of the shock. At energies above the thermal particles we expect, for the strong shock ($r \simeq 4$) simulated here, the result $F(E) \propto E^{-\sigma}$, with $\sigma = \{(r + 2)/(r - 1)\}/2 = 1$, which is reproduced with high accuracy. At the thermal part of the distribution the cooling of the plasma due to the energy flux into high energy particles is responsible for the shift of the distribution towards lower energies. The initial injection rate decreases thereby to a quite stable value, as described above. Due to the steep dependence of both the Maxwell distribution, and the transparency function (Fig. 1) on particle energy, the injection energy is quite well defined, leading to a power law due to Fermi acceleration, starting shortly above thermal energies. In using the standard cosmic ray transport equation, we have, of course, made use of the diffusion approximation, which may introduce an error especially for $v \simeq u_2$. Multiplication of the initial thermal distribution with $\tau_{\rm esc}$ suggest an effective initial injection velocity of about $7 \cdot 10^6\,\mathrm{m\,s^{-1}}$ (in the shock frame). Using an eigenfunction method, Kirk & Schneider (1989) have explicitly calculated the angular distribution of accelerated particles and accounted for effects of a strong anisotropy especially at low particle velocities. They were able to calculate the injection efficiency without recourse to the diffusion approximation, and found always lower efficiencies. For the above given injection velocity, $r = 4$ and $u_0 = 5 \cdot 10^6\,\mathrm{m\,s^{-1}}$, they estimate a reduction effect of $\approx 7\%$, leaving the diffusion approximation as quite reasonable even in this regime.

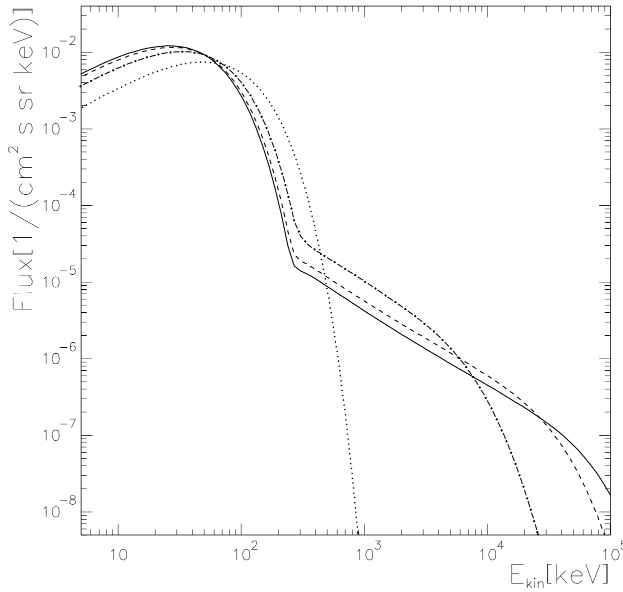

Figure 3: *Omni-directional flux vs. proton kinetic energy, for* $t = 0$ *(dotted),* $t = t_0$ *(dot-dashed),* $t = 5\,t_0$ *(dashed), and* $t = 10\,t_0$ *(solid line)*

4 Conclusions

We have presented results from a solution of the time dependent gas dynamics equation together with the cosmic ray transport equation. We have incorporated in these calculations an analytical solution of an injection model for a quasi-parallel shock, based on particle interaction with self-generated waves. We were therefore able to investigate the interaction of high energy particles, accelerated by the Fermi process, with the shocked plasma flow *without* a free parameter for the efficiency of the injection. We found the energy-flux $E \cdot F(E)$ of (non-relativistic) particles in the power law region to be about two orders of magnitude less than at the peak of the thermal distribution. This result turns out to be quite stable, due to the self-regulating mechanisms between particle injection and wave generation *and* gas modification described above.

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