Strongly nonlinear synchrotron dominated shocks in a pure pair plasma

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Abstract

We present initial numerical results of strong shocks in a pure electron positron pair plasma. Synchrotron losses provide the dominant loss mechanism and the kinetic energy of the incoming flow is efficiently converted to radiation.

1 Introduction

In a companion paper in these proceedings (OG.3.3.13) we determined criteria, in the context of the box model, for the conditions under which "pile-ups" can occur in shock accelerated electron spectra subject to synchrotron losses. Nonlinear effects were included by introducing an effective momentum-dependent velocity $U_1(p)$ for the upstream precursor. In this case it was shown that the logarithmic slope of the energetic electron spectrum becomes (Drury et al. 1999)

$$\frac{\partial \ln f}{\partial \ln p} = -3 \frac{U_1 - 4\alpha_s pL + \frac{p}{3}\frac{dU_1}{dp} - \alpha_s p^2 \frac{dL_1}{dp}}{U_1 - U_2 - 3\alpha_s pL} \tag{1}$$

where $U_1(p)$ and U_2 are the upstream and downstream flow velocities, $L(p) = L_1(p) + L_2(p)$ is the momentum dependent size of the acceleration region and the synchrotron loss rate is $\dot{p} = -\alpha_s p^2$. The pile-up criterion at $p^* = (U_1 - U_2)/3\alpha_s L$ is then

$$U_1(p) - 4U_2 - p\frac{dU_1}{dp} + 3\alpha_s p^2 \frac{dL_1}{dp} > 0 \qquad \text{at} \qquad p = p^* \tag{2}$$

In a pure pair plasma the shock modification will be produced by the energetic electrons and $f(p)$ and $U_1(p)$ must be calculated self consistently. Throughout the upstream precursor and in the steady case both the mass flux, $A \equiv \rho U$, and the momentum flux, $AU + P_C$ are conserved. Here P_C is the pressure contained in energetic particles and the gas pressure is assumed to be negligible upstream. At a distance $L_1(p)$ upstream only particles with momenta greater than p remain in the acceleration region. This suggests that in the "box" model the reaction of the particles on the flow is described by the momentum flux conservation law

$$AU_1(p) + \int_p^{p_{\max}} 4\pi p^2 f \frac{pv}{3}\, dp = \text{constant} \tag{3}$$

where $p_{\max}$ is the highest momentum particle in the system and v is the particle velocity corresponding to momentum p. Differentiating with respect to p gives

$$A\frac{dU_1(p)}{dp} = 4\pi p^2 f(p) \frac{pv}{3}. \tag{4}$$

In this paper we are interested in applying this model, and a more detailed numerical scheme for the full cosmic ray transport equation, to the case where almost all of the incoming upstream kinetic energy is converted into energetic particles at p_* which then emit synchrotron radiation.

2 Synchrotron limited shocks

For the case of a synchrotron limited shock in a pure pair plasma the upper cut-off is determined not by a free escape boundary condition but by synchrotron losses. If most of the energy is radiated this way, the shock will be very compressive and the downstream velocity U_2 negligible compared to U_1. However, It is not clear that a stationary solution exists in this case. The problem is that as $U_2 \to 0$ so $L_2 \to \infty$ if a diffusion model is used for the downstream propagation. A synchrotron limited shock appears to require some form of impenetrable reflecting barrier a finite distance downstream if it is to be realised in finite time. Nevertheless we can, as an illustrative example, consider a cold pair plasma hitting an impenetrable and immovable boundary. In this case, if there is a steady solution, the upward flux due to the acceleration must exactly balance the synchrotron losses at all energies

$$\frac{4\pi p^3}{3} f(p) U_1(p) - 4\pi p^4 f(p) \alpha_s L_1(p) = \mathrm{constant} \tag{5}$$

where $L_1(p) = \kappa_1(p)/U_1(p)$. In general it appears impossible to satisfy both this condition and the momentum balance condition for $p < p^*$ unless the diffusion coefficient has an artificially strong momentum dependence. However a solution exists corresponding, in the box model, to a Dirac distribution at the critical momentum p^*. This steady population of high energy electrons has enough pressure to decelerate the incoming plasma to zero velocity and radiates away all the absorbed energy as synchrotron radiation.

We can make a rough estimate of the typical photon energy that can be radiated from a strong synchrotron shock. In the limit of $U_2 \to 0$ the cut-off momentum is $p^* \approx U_s/\alpha_s L(p_*)$ where U_s is the shock speed and we take the synchrotron emissivity to be a delta function at frequency $\nu^* = a_1 (p^*/m_e c)^2 B$ where B is the average magnetic field throughout the precursor and a_1 is a constant. If the mean free path for particle scattering if equal to the gyroradius then $\kappa \propto p/B$ so that $L \propto 1/B$ and, since $\alpha_s \propto B^2$, $p^{*2} \propto 1/B$ and the emission frequency is *independent of the magnetic field*. The energy of the emitted photons can be shown to be

$$E^* \approx \frac{m_e c^2}{\alpha} \beta_s^2 \tag{6}$$

where $\beta_s = U_s/c$ and α is the fine structure constant so that $E^* \approx 70MeV$. This extreme form of pile-up may be of interest as a means of very efficiently converting the bulk kinetic energy of a cold pair plasma into soft gamma-rays.

3 Numerical solution

While the above discussion in the context of the box model allows us to address the essential physics of synchrotron dominated shocks, it is the solution of the full, spatially dependent, cosmic ray transport equation which is needed to examine this system in more detail. To this end we have adapted a self consistent particle acceleration code (Duffy 1992) to include synchrotron losses (Webb, Drury and Biermann 1984). The gas is described by the conservation laws,

$$\frac{D\rho}{Dt} = -\rho \frac{\partial U}{\partial x} \tag{7}$$

$$\frac{DU}{Dt} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left(P_G + P_C\right) \tag{8}$$

$$\frac{DE_G}{Dt} = -\left(E_G + P_G\right)\frac{\partial U}{\partial x} \tag{9}$$

where ρ, U and P_G are the gas density, flow velocity and pressure respectively. The gas energy unit mass, E_G, is given by the equation of state $E_G = U^2/2 + P_G/(\gamma - 1)$ where $\gamma = 5/3$ for a nonrelativistic gas. The energetic particle pressure is P_C

$$P_C = \frac{4\pi}{3}\int p^3 v f\, dp \tag{10}$$

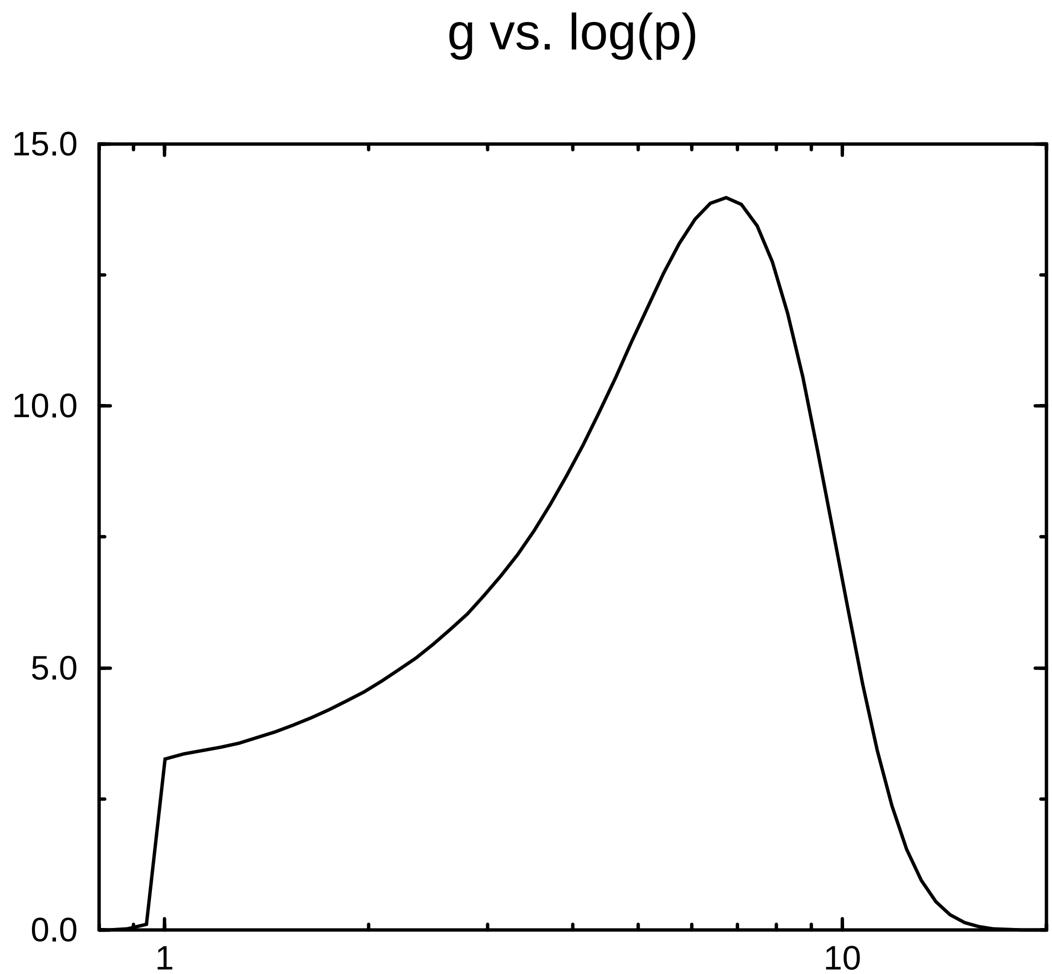

Figure 1: Enhancement of the distribution function, $g = p^4 f$ at the synhcrotron cut-off for $M_p = 75$ and $\eta = 2 \times 10^{-2}$

and is calculated from the phase space density $f(x, p, t)$ which satisfies

$$\frac{Df}{Dt} = \frac{\partial}{\partial x}\left(\kappa(x,p)\frac{\partial f}{\partial x}\right) + \frac{1}{3}\frac{\partial U}{\partial x}p\frac{\partial f}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(\alpha_s p^4 f\right) \tag{11}$$

with the terms on the right hand side describing sptial diffusion, compression and synchrotron losses. The diffusion coefficient is taken to scale with the particle gyroradius, $\kappa \propto p$.

Initially, with no energetic particles present, a piston is driven at supersonic speed into the gas which is uniform and at rest. Particles are injected at the shock front with a momentum $0.1mc$ so that two extra terms are included into the above equations; the extraction of an energy flux from the gas and the injection of a mono-energetic population of particles. This injection is scaled so that a fixed fraction , η, of gas particles incident on the shock are injected into the energetic particle population. When the shock is sufficiently smoothed out, with a Mach number below 1.2, the injection is switched off. Numerically the equations are discretised with the Riemann problem solved at cell ineterfaces and a Godunov method is used to resolve the shock. For the purposes of running a test case the synchrotron losses are scaled, through a choice of α_s, to provide a cut-off at roughly a decade in energy above injection.

We have run the code for a piston Mach number of $M_p = 75$ and an injection level of $\eta = 0.02$ and have plotted out the results when the solution reaches a quasi steady state. We stress, however, that these are initial results and future work with a range of parameters is necessary. In this case the gas subshock is almost completely smoothed out with a broad precursor extending into the upstream region. The particle distribution function is enhanced at the "pile-up" momentum where we have plotted the distribution function averaged throughout the acceleration region. The synchrotron power per unit frequency $J(\nu) = -4\pi p^2 \dot{p} f(p)(dp/d\nu)$ will therefore display a peak at the "pile-up" frequency just before the cutoff. Furthermore, although it is not plotted in this paper, the downstream energetic particle pressure is an order of magnitude greater than the postshock gas pressure so that the upstream kinetic energy of the gas is being efficiently converted into energetic particles.

4 Conclusions

We have used the box model to motivate the phenomenon of a synchrotron dominated shock where practically all of the ram pressure of the incoming gas is converted into synchrtron emission dominated at the frequency ν^*. Simple estimates put the emitted photon energies at about 70 MeV. The numerical results, although preliminary and for just one set of parameters, support the physical predictions of the box model but do not display the same level of "singularity" at p^*. However, as with paper OG.3.3.13 we stress that variations in the time spent by particles in the acceleration region will smear out the sharp pile-ups predicted by the box model. Nevertheless what is clear from the numerical result above is that enhanced emission in soft gamma rays, if observed in astrophysical objects, may be a signature of a strong synchrotron dominated shock.

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