Spectrum and Composition of CRs Accelerated in SNRs

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Abstract

Spectrum and chemical composition of cosmic rays accelerated in supernova remnants are studied on the basis of kinetic selfconsistent approach. It is shown that the calculated spectrum shape fits the experimental data for all available species. The theory reproduces also the required enhancement of heavy elements in the observed galactic cosmic rays.

1 Introduction:
Supernova remnants (SNRs) are considered as the main source of cosmic rays (CRs) in the Galaxy. Diffusive shock acceleration process is able to convert sufficient amount of the explosion energy into CRs and to produce CR spectrum with the necessary shape and amplitude (Berezhko et al., 1996). Investigation of CR chemical composition can give additional evidence whether SNRs are indeed the source of observed CRs. Here we use the nonlinear kinetic model for CR acceleration in SNRs (Berezhko et al., 1996; Berezhko & Völk, 1997) to study the expected CR composition.

2 Method:
Kinetic model for CR acceleration in SNRs (Berezhko et al., 1996) is based on the CR diffusive transport equation which is solved self-consistently together with the gas dynamic equations. The model includes an assumption about injection of some fraction \( \eta \ll 1 \) of suprathermal gas particles (ions) at the SN shock into the acceleration regime. By definition the gas particles become CRs when they reach some minimum velocity \( v_{\text{inj}} \) during their thermalization. Therefore, the momentum of injected particles

\[
p_{\text{inj}}^A = A p_{\text{inj}}
\]

is proportional to the mass number \( A \). Here and hereafter variables without subscript \( A \) correspond to protons. According to the hybrid plasma simulation of quasiparallel collisionless shock (e.g. Trattner & Scholer, 1993) the relative number of \( \alpha \)-particles in the power-law tail of the spectrum is essentially larger as compared with protons. One can assume that the actual injection rate is an increasing function of the particle rigidity \( R \propto A/Q \), where \( Q \) is the particle (ion) charge number. Therefore, the injection rate of ions with \( A > 1 \)

\[
\eta_A = \eta e_{\text{inj}}(A/Q),
\]

is a presumably increasing function of \( x = A/Q \), that is described by the enhancement factor \( e_{\text{inj}}(x > 1) > 1 \).

We assume that supernova (SN) shock expands into the uniform interstellar medium (ISM) with normal chemical composition. Therefore, the relative number of accelerated nuclei \( \frac{N_{\text{inj}}^A}{N_{\text{inj}}} = \left( \frac{\eta_A}{\eta} \right) \left( \frac{n_A}{n_H} \right) = \left( \frac{n_A}{n_H} \right) e_{\text{inj}}(A/Q) \) is proportional to the ISM element abundance (concentration) ratio \( n_A/n_H \).

We use the CR diffusion coefficient

\[
\kappa_A(p_A) = p_A e^2/(Q e),
\]

which at the relativistic energy \( p_A \gg Amc \) coincides with so-called Bohm diffusion coefficient. Here \( c \) is the speed of light, \( m \) and \( e \) are the mass and charge of protons. The ion charge number \( Q \) is assumed to be a function of momentum \( p_A \). At the beginning of acceleration, that corresponds to small momenta \( p_A \sim p_{\text{inj}}^A \), \( Q \) coincides with the equilibrium ion charge state in the background ISM \( Q_0 \). At nonrelativistic energies the acceleration process is extremely fast due to the small value of the diffusion coefficient. It becomes
progressively less rapid with increasing CR energy and at the sufficiently high energies \( p_A \gg Amc \) all kinds of ions become completely ionized. Therefore, we use

\[
Q(p_A) = Q_0 \quad \text{at} \quad p_A \leq Amc \quad \text{and} \quad Q(p_A) = Z \quad \text{at} \quad p_A \gg Amc,
\]

where \( Z \) is the nuclear charge number of elements with the mass number \( A \).

Due to the rigidity-dependent CR leakage from the Galaxy, the average CR spectrum (flux) inside the galactic volume is connected with the overall spectrum \( N_A(p_A) \) of CRs produced in SNR (source spectrum) by the relation

\[
J_A^S(\epsilon_k) \propto N_A(p_A) \tau(p_A/Z),
\]

where we assume the power-law rigidity dependent mean CR residence time

\[
\tau(R) \propto (R + R_0)^{-\alpha},
\]

\( R_0 \approx 5 \) GV (e.g. Berezinskii et al., 1990), \( \epsilon_k(p_A) \) is the CR kinetic energy. The observed CR spectrum deviates from the mean galactic one due to the solar wind modulation effect. It can be quantitatively described by the assumption that each particle reached the Earth orbit looses the energy \( \Delta \epsilon = Ze \phi \). Then the observed CR intensity as a function of their kinetic energy \( \epsilon_k \) is determined by the relation

\[
J_A(\epsilon_k) = J_A^S(\epsilon_k + \Delta \epsilon)(\epsilon^2 - \epsilon_0^2)/(\epsilon^2 + 2\epsilon \Delta \epsilon + \Delta \epsilon^2 - \epsilon_0^2),
\]

where \( \epsilon = \epsilon_k + \epsilon_0 \) is the total particle energy, \( \epsilon_0 = Amc^2 \) is the rest energy.

3 Results and Discussion:

We consider the values of the SN explosion energy \( E_{\text{sn}} = 10^{51} \) erg and the ejecta mass \( M_{\text{ej}} = 1.4M_\odot \) typical for a SN Ia, which expands into the uniform ISM.

The main fraction of the galactic volume is occupied by so-called hot and warm phases of ISM, which we consider here, with hydrogen number density, temperature and magnetic field values \( N_H = 0.003 \text{ cm}^{-3}, T_0 = 10^6 \text{ K}, B_0 = 3 \mu \text{G} \) and \( N_H = 0.3 \text{ cm}^{-3}, T_0 = 10^4 \text{ K}, B_0 = 5 \mu \text{G} \) respectively. The ISM temperature \( T_0 \) determines the equilibrium ionization state of elements: at \( T_0 = 10^4 \text{ K} \) \( K \) is close to 1 for all elements, whereas at \( T_0 = 10^6 \text{ K} \) the mean ion charge number increases from \( Q_0 \approx 1 \) for H and He to \( Q_0 \approx 10 \) for heavy ions with \( A \approx 100 \) (Kaplan & Pikelner, 1979).

The expected CR spectra produced in SNRs calculated at moderate injection rate \( \eta = 10^{-4} \) and a modulation parameter \( \phi = 600 \text{ MV} \) are compared in with the experimental data Fig.1. The value of \( \alpha \) and the proton spectrum normalization are selected to fit the experiment. The function \( \epsilon_{\text{inj}}(A/Q_0) \) is taken to fit the experiment at \( \epsilon_k \approx 10 \text{ GeV} \) for all heavier elements. The all particle spectrum

\[
J_S(\epsilon_k) = \Sigma J_A(\epsilon_k)
\]

includes the spectra of elements presented in Fig.1.

One can see that calculated spectra for all elements equally well fit the experiment at \( \epsilon_k \lesssim 10^{14} \text{ eV} \) for both considered ISM phases. Note that the maximum energy in the all particle spectrum \( \epsilon_{\text{max}} \approx 10^{14} \text{ eV} \) and \( \epsilon_{\text{max}} \approx 4 \times 10^{14} \text{ eV} \) for warm and hot ISM, respectively, only slightly exceeds the maximum proton energy.

We believe that the observed CR spectrum which has the only peculiarity, so-called knee at \( \epsilon \approx 3 \times 10^{15} \text{ eV} \), at energies \( \epsilon \gtrsim 10^{15} \text{ eV} \) is produced by some reacceleration process. In this case one need to form in SNRs the CR spectrum up to \( \epsilon_{\text{max}} \approx 3 \times 10^{15} \text{ eV} \) which is essentially higher than the calculated one. To demonstrate how CR spectrum could look like at \( \epsilon > \epsilon_{\text{max}} \) we present in Fig.1 CR spectra calculated at \( B_0 = 12 \mu \text{G} \) and extended towards higher energies according to the law \( \epsilon^{-3.1} \). This rather formal procedure gives the
prediction of CR composition at energies $\epsilon_k \gtrsim 10^{15}$ eV, which is expected to be sensitive to the value $\epsilon_{max}$. The calculated mean CR atomic number
\[ <A> = \frac{\sum A J_A(\epsilon_k)}{\sum J_A(\epsilon_k)} \]
is presented in Fig. 2. One can see that the expected value of $<A>$ increases from 10 to 20 in the energy interval $10^{15} \div 10^{16}$ eV that is in a reasonable agreement with the experimental data.

In order to illustrate the effect of preferential acceleration of heavy elements we compare in Fig. 3 the calculated CR enhancement factor
\[ e(\epsilon_k) = A[J_A(\epsilon_Ak = A\epsilon_k)/J(\epsilon_k = \epsilon_Ak/A)]/(n_A/n_H) \]
at kinetic energy per nucleon $\epsilon_k = 3$ GeV/n with experimental data. In this case we use $e_{inj}(x) = 1$.

One can see from Fig. 3 that in the case of low injection ($\eta = 10^{-5}$), when nonlinear modification of the SN shock by the CR backreaction is negligible, the theory in contrast to observations predicts much less efficient production of all kinds of species relative to protons. CR enrichment by heavy elements is not essential also for the high injection rate ($\eta = 10^{-3}$) in the case of hot ISM. Only in the low temperature ISM, with a low degree of ionization for all kinds of species, the theory predicts the essential CR enrichment by heavy elements if the injection rate is relatively high. But even at $\eta = 10^{-3}$ it is not high enough as compared with the experimental requirement. At the same time, the CR spectrum at such high injection rates becomes too hard (Berezhko et al., 1995). It is clear that the observed heavy elements enhancement can be produced only if the injection mechanism also provide $e_{inj} > 1$ at $A > 1$. We present in Fig. 3 the calculation performed at $\eta = 3 \times 10^{-4}$ with the injection enhancement factor $e_{inj} = A/Q_0$ which is consistent with the collisionless shock simulation (Trattner & Scholer, 1993). On average, it is in a reasonable agreement with the data.

Fig. 3 clearly demonstrates that the CR enrichment by heavy elements during their acceleration is pure nonlinear effect. It can be easily explained taking into account that the shape of CR spectrum produced by the modified shock is essentially different at relativistic and nonrelativistic energies: $N(p) \propto p^{-\gamma_1}$ at $p \leq mc$ and $N(p) \propto p^{-\gamma_2}$ with $\gamma_2 < \gamma_1$ at $p \gg mc$ (Berezhko et al., 1996). The higher shock modification, the larger difference between $\gamma_1$ and $\gamma_2$. The CR diffusion coefficient (3) with momentum-dependent ion charge number (4) provides
\[ N_A(p_A/A)^{\alpha} \propto e_{inj}(A/Q_0)n_A(p_A/A)^{-\gamma_1} \quad \text{at} \quad p_A \leq Q_0 mc \quad \text{and} \quad N_A(p_A/A)^{\alpha} \propto e_{inj}(A/Q_0)n_A(p_A/A)^{-\gamma_2} \quad \text{at} \quad p_A \gg Q_0 mc \]
that gives at relativistic energies the enhancement factor
\[ e(A) = e_{inj}(A/Q_0)(A/Q_0)^{\gamma_1 - \gamma_2}(A/Z)^{-\alpha}. \]
Figure 2: The average CR mass number as a function of kinetic energy. Experimental points are taken from Watson (1997), the calculation corresponds to the dot-dashed line of Fig. 1.

The ratio $A/Z$ is close to 2 for all elements except hydrogen and $\alpha$ is about 0.7 in all considered cases. Therefore, the unmodified shock, taken place at low injection $\eta \lesssim 10^{-5}$, provides $\gamma_1 \approx \gamma_2$ and $e < 1$ (if $e_{inj} = 1$), i.e. the underabundance of heavy elements relative to H. The observed overabundance of heavy elements ($e > 1$) can be produced by SNRs i) in low temperature ISM with low ionization state $Q_0 \approx 1$; ii) at relatively high injection rate, which provide the efficient CR production and strong SN shock modification and iii) at preferential injection of heavy elements into the shock acceleration.

Since there are experimental (e.g. Trattner & Scholer, 1994) and theoretical (Trattner et al., 1993) evidences that the actual injection is characterized by the required properties, we conclude that the observed CR spectrum and composition at energies $\epsilon \lesssim 10^{15}$ eV can be accounted for the diffusive shock acceleration in SNRs, if they are predominantly situated in a relatively low temperature ($T_0 \sim 10^4$ K) ISM.

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References

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