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The Diffuse Galactic Gamma-Ray Gradient

D. Breitschwerdt[1,2], V. Dogiel[3], and H.J. Völk[4]
[1]*Heisenberg Fellow*
[2]*Max-Planck-Institut für extraterrestrische Physik, Postfach 1603, D-85740 Garching, FRG*
[3]*P.N. Lebedev Physical Institute, 117924 Moscow, Russia*
[4]*Max-Planck-Institut für Kernphysik, Postfach 103 980, D-69029 Heidelberg, FRG*

Abstract

One of the unsolved problems in cosmic ray (CR) physics is the small radial gradient of the γ-ray intensity compared to the inferred CR source distribution in the Galactic disk. In diffusive CR propagation models the most natural explanation is very efficient spatial mixing due to MHD turbulence in the interstellar medium. However, even in the most favorable case of a very large diffusive CR halo the γ-ray gradient is still larger than observed. In our view the small γ-ray gradient could be the result of strong advection by a Galactic wind. We show that a small γ-ray gradient can be obtained, if the diffusion region does **not** extend far above the Galactic plane. Important ingredients of our model are: (i) anisotropic CR diffusion, (ii) strong radial and vertical gradients of the advection velocity (due to faster winds above higher CR source density regions).

1 Introduction:

The distribution of the γ-ray emissivity in the galactic disk bears important information on the CR origin, because Galactic diffuse γ-rays result from interactions of CRs with interstellar gas. Thus, this distribution depends critically on the conditions of CR propagation in the Galaxy. For γ-ray energies above 100 MeV, the main production process is π^0-decay, resulting from nuclear collisions between high energy particles and interstellar matter. The study of diffuse γ-ray emission has shown that the nucleonic component of cosmic rays is more or less homogeneously distributed over the entire Galactic disk. If the spatial distribution of CRs were uniform, the γ-ray emission should map perfectly well the distribution of interstellar hydrogen. However, the distribution of the cosmic ray (CR) sources (most likely supernova remnants (SNRs) for particle energies below 10^{15} eV) is far from homogeneous. Taken at face-value, the discrepancy must arise during the propagation of CRs from their sources through the interstellar medium. The difficulty in interpreting the data correctly is due to the fact, that the coefficients of CR transport are not well known, and therefore models (plus their respective assumptions) are used to bridge the gap between the measured γ-ray parameters and the unknown parameters of CR origin.

The most common models for the interpretation of the spatial CR distribution in the disk are phenomenological in nature and based on CR diffusion. It is assumed that particle propagation in the interstellar medium can be described by an isotropic diffusion process due to CR scattering off magnetic field fluctuations. The value of the diffusion coefficient (or tensor) is estimateded from observational data. In many cases this model gives an acceptable interpretation of experimental results (see e.g. Berezinsky et al., 1990).

The interpretation of the γ-ray gradient in the framework of an isotropic diffusion model, in which the CRs are injected by a radially non-uniform SNR distribution (e.g. Case & Bhattacharya, 1996) leads to the following conclusion: in order to obtain a rather uniform CR distribution in the disk one has to assume that the Galaxy is surrounded by a huge halo whose vertical extension is larger than 10 kpc (Bloemen et al., 1993). Only then diffusion mixes CRs efficiently enough, so that their distribution in the disk differs appreciably from that of their sources. The numerical calculations (see Dogiel & Uryson, 1988) show however, that even in the case of a *very extended diffusion halo* the calculated gradient of CRs is larger than derived from COS-B and EGRET data. Therefore, if we believe that the observations of the SNR and γ-ray distributions are correct, we should seek alternative explanations for the gradient data.

2 Effects of CR Transport by a Galactic Wind:

A completely different scenario of CR escape into intergalactic space is implied by the so-called galactic wind models. It is shown that the combined pressure of thermal plasma, CRs, magnetic fields and MHD waves can lead to a secular escape of gas and CRs in normal spiral galaxies, provided that the coupling between scattering waves and energetic nucleons is strong. In this case there is a net forward momentum transfer from CRs to the gas via the frozen-in waves as a mediator (see Breitschwerdt et al., 1991; 1993). In wind models the coupling between MHD-waves and CRs is generally assumed to be strong everywhere in space except for a rather narrow region surrounding the galactic plane where CRs propagate predominantly by diffusion. In the wind CRs are transported along with the gas and the waves to large distances from the disk. As numerical simulations have shown, these models exhibit two effects that may provide the long-sought explanation for the γ-ray gradient variation problem, both of which are not present in static halo models. The first one is that the diffusion tensor may be *anisotropic*. The reason is that the component along the magnetic field lines depends on the CR energy, as $D_{||} \propto E^{0.6}$, and the component perpendicular to the lines $D_\perp$ is energy independent. As a result in the case of a strong galactic wind the location, z_c, of the halo boundary depends on the energy as: $z_\mathrm{c} \propto E^{0.6}$ and the local CR life time $T_{cr} \sim \frac{z_\mathrm{c}^2}{D} \propto E^{0.6}$. The diffusion component $D_{||}$ rises with energy so the diffusion is one dimensional at high energies.

The second, and more important, new effect is the *radial variation of the wind velocity* (or mass flux, if the gas density is constant) of the galactic wind along the galactic plane, which may give rise to an almost uniform CR distribution in the disk, even for a strongly non-uniform CR source distribution, provided that the diffusion halo height is small. The reason is that, depending on the local CR source strength (i.e. the number of SNRs per unit volume and hence the number density of CRs at a given energy, $n(r, z, E)$), the CR energy density, ϵ_c or its pressure P_C, will vary accordingly with galactocentric radius. Thus an increase in ϵ_c will also increase the galactic wind mass flux or wind velocity, respectively, and hence decrease the distance to the advection boundary, given by $D(R_\mathrm{c}, z_\mathrm{c}, E)/(V(R_\mathrm{c}, z_\mathrm{c})z_\mathrm{c}) \sim 1$, where R and $V = u + V_\mathrm{A}$ denote the halo radius and the CR transport speed with respect to an Eulerian frame of reference. Consequently, the CR storage volume will be reduced, thus facilitating CR escape locally and thereby decreasing the number of nuclear collisions, which produce γ-rays via π^0-decay.

3 Uniform Disk Distribution of CRs in the Wind Model:

To illustrate the behaviour of the solution of the advection-diffusion equation we discuss two simple cases. The first one describes one-dimensional diffusion with the advection velocity V depending on the radial coordinate r only, as $V(r) = V_0 \cdot f_1(r)$, with V_0 being a constant and $f_1(r)$ an arbitrary function of r. The equation to solve for the CR distribution function $N(r, z)$ is

$$-\frac{d}{dz}\left(D\frac{dN}{dz} - V(r)N\right) = Q \cdot f_2(r) \tag{1}$$

with the boundary condition $N = 0$ at $z_c = 1$. Here Q is the non-radial part of the CR source distribution, and $f_2(r)$ is also an arbitrary function of r, which describes the radial SNR distribution in the disk. The solution of Eq. (1) is

$$N(z, r) = \frac{Qf_2(r)}{V_0 f_1(r)}\left(1 - e^{-(V_0 f_1/D)(1-z)}\right) . \tag{2}$$

We see that for the case of weak advection $V_0 \ll 1$ the solution is $N \approx Qf_2(1-z)/D$, and the CR gradient depends on the source distribution f_2 only. However if $V_0 \gg 1$ then

$$N \approx \frac{Qf_2(r)}{V_0 f_1(r)}\,, \tag{3}$$

and in this case the gradient is a strong function of the velocity distribution too. In the special case $f_1(r) = f_2(r)$ we obtain $N = const$.

Let us investigate a more complex case in which the velocity is also a function of z. For a velocity distribution $V(z,r) = 3\,V_0\,z\,f_1(r)$, with some suitable constant V_0, the solution of the one-dimensional diffusion equation was obtained by Bloemen et al., 1993. For strong advection the solution converges to

$$N(z=0,r) \simeq A \frac{Q f_2(r)}{\sqrt{D V_0 f_1(r)}}\,; \tag{4}$$

we retrieve $N = const$, if $f_1(r) = f_2^2(r)$, where A is a constant.

The diffusion coefficient D is assumed to be constant, although we cannot exclude its spatial variation, which can change the CR density distribution in the disk as well.

4 Analytical Solution of the Diffusion-Advection Equation:

We now want to work out a general solution of the two-dimensional CR transport equation for nucleons, which reads

$$-\nabla\left(\underline{\underline{D}}(\vec{x},E)\nabla N - \vec{u}(\vec{x})N\right) - \frac{\partial}{\partial E}\left(\frac{1}{3}\nabla\vec{u}(\vec{x})\,E\,N\right) - \frac{\partial}{\partial E}\left(\frac{dE}{dt}\,N\right) = Q(E,\vec{x})\,, \tag{5}$$

where $\vec{x}$ denotes the spatial coordinates, and the diffusion tensor $\underline{\underline{D}}$ in cylindrical coordinates with axial symmetry is given by

$$\underline{\underline{D}} = \begin{pmatrix} D_{rr} & D_{rz} \\ D_{zr} & D_{zz} \end{pmatrix} = \begin{pmatrix} \kappa_r & 0 \\ 0 & D_z \cdot E^{\alpha} \end{pmatrix}. \tag{6}$$

Our prime interests here are the anisotropic diffusion and advection, rather than the energy dependence and so, for convenience, we set $\alpha = 0$. For the nucleon component other than adiabatic losses are negligible ($dE/dt = 0$); thus Eq. (5) in axial symmetry becomes

$$-D_z \frac{\partial^2 N}{\partial z^2} - \kappa_r\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r}\right) + 3\frac{V_0}{r^2}\frac{\partial\,(N\,z)}{\partial z} - \frac{V_0}{r^2}\frac{\partial\,(N\,E)}{\partial E} = Q_1(r)\,\delta(z)\,E^{-\gamma_0}\,, \tag{7}$$

where we have chosen a spatial variation for the velocity field as

$$\vec{u}(\vec{x}) = \begin{pmatrix} u_r \\ u_z \end{pmatrix} = \begin{pmatrix} 0 \\ 3V_0 z/r^2 \end{pmatrix}, \tag{8}$$

and we have used a power law spectrum for particle injection by the disk sources. The choice of $\vec{u}(\vec{x})$ seems rather arbitrary at first glance, and it can indeed be quite a complicated function of r and z. However, numerical calculations of galactic winds (Breitschwerdt et al., 1991) show a similar spatial variation of the velocity field like the one we have chosen.

The full analytical solution od Eq. (7) is rather cumbersome, and will be discussed in detail elsewhere (Breitschwerdt et al., 1999). Here we present the result for the CR distribution function in the disk ($z = 0$), subject to the boundary conditions $N(r, z = \pm\infty) = 0$ and $N(r = \pm\infty, z) = 0$:

$$\begin{aligned} N(r,z=0,E) = \mp\frac{1}{4\pi^3 D_z}E^{-\gamma_0}\Bigg\{\int_{\bar{r}}^{+\infty} Q_1(r')2\pi r'dr' \sum_{n=0}^{\infty}\frac{(-1)^n\Gamma\left(\frac{1-\beta_n}{2}\right)}{n!\Gamma\left(\frac{1-2n}{2}\right)\Gamma\left(1-\frac{\beta_n}{2}\right)} \times \\ \times \left\{\frac{(-1)^{\tilde{\gamma}_n}\cos\left(\frac{\pi\beta_n}{2}\right)}{(-1)^{\tilde{\gamma}_n}\sin\left(\frac{\pi\beta_n}{2}\right)+(-1)^n}\right\}\left(\frac{\bar{r}}{r'}\right)^{-K_n}\frac{dK_n}{d\alpha} + \int_0^{\bar{r}} Q_1(r')2\pi r'dr' \sum_{m=0}^{\infty}\frac{(-1)^m\Gamma\left(\frac{\alpha_m}{2}\right)}{m!\Gamma\left(\frac{1-2m}{2}\right)\Gamma\left(1+\frac{\alpha_m}{2}\right)} \times \\ \times \left\{\frac{\sin\left(\frac{\pi\alpha_m}{2}\right)}{(-1)^{\tilde{\gamma}_m+m}\sin\left(\frac{\pi\beta_n}{2}\right)+\cos\left(\frac{\pi\alpha_m}{2}\right)}\right\}\left(\frac{\bar{r}}{r'}\right)^{-K_m}\frac{dK_m}{d\beta}\Bigg\}. \end{aligned} \tag{9}$$

Here, $\beta_n = -2n - 3A - 2\sqrt{A(\gamma+6n)}$, $\tilde{\gamma}_n = \frac{1}{2}(1 - 4n - 3A - 2\sqrt{A(\gamma+6n)})$, $\tilde{\gamma}_m = \frac{1}{2}(2(2m+1) - 3A + 1 + 2\sqrt{A(\gamma - 3(2m+1))})$, $K_n = -2n - \sqrt{A(\gamma+6n)}$, $dK_n/d\alpha = 1 + 3A/(2\sqrt{A(\gamma+6n)})$, $\alpha_m = 2m + 1 - 3A + 2\sqrt{A(\gamma - 3(2m+1))}$, $K_m = 2m + 1 + \sqrt{A(\gamma - 3(2m+1))}$, $dK_m/d\beta = 1 - 3A/(2\sqrt{A(\gamma - 3(2m+1))})$, where $n, m = 0, 1, 2, ...$, $D_z = D_\perp$, $A = V_0/\kappa_r$, $B = \sqrt{D_z/\kappa_r}$, $\bar{r} = Br$, and $\gamma = \gamma_0 + 2$.

It is instructive to look at the asymptotic behaviour of the solution at different radii. If the CR sources occupy a limited volume of the disk bounded by a radius a, and we let $Q(r') = Qr'^{-q}\,\Theta(a - r')$ (where q is a fit to the observed SNR distribution, and Θ denotes the step function), then for $r > a$ the function N is completely determined by the second integral which is a constant. Indeed from contour integration we find that the first integral $\int_r^{+\infty} = 0$ and the second integral $\int_0^r = \int_0^a = const$ for $r > a$. Then from a simple analysis we see that $N(r) \propto r^{-1-\sqrt{A(\gamma-3)}}$ and $N(r) \propto r^{-1}$ if $A \ll 1$. This is just what is expected for purely diffusive particle propagation. In the case of strong advection, i.e. $A \gg 1$, we have $N(r) \propto r^{-\sqrt{A(\gamma-3)}}$; such a strong drop of CR density far away from the sources is due to adiabatic energy losses, since, if we neglect them, $\gamma = 3$ formally, and $N(r) \propto r^{-1}$, almost independent of the value of A.

In the vicinity of the source region ($r \ll a$) the function N is determined by the two integrals (cf. Eq. (9)) which can be written as

$$N(r) \propto r^{\sqrt{A\gamma}} \int_r^{+\infty} r'^{(1-q-\sqrt{A\gamma})}\, dr' - r^{-1-\sqrt{A(\gamma-3)}} \int_0^r r'^{(1-q+\sqrt{A(\gamma-3)})} dr' \,. \tag{10}$$

For the case of strong advection $A \gg 1$ the first integral is determined by the lower limit and the second one by the upper limit, *independent* of the source distribution. Then we have $N(r) \propto r^{2-q}$ and the CR density is almost in the disk region close to the sources if $q = 2$.

If $A \ll 1$ then the values of the integrals are determined by the source distribution. If the CR sources are concentrated towards the centre of the disk, $q > 2$, then the CR distribution is determined by the second integral and $N(r) \propto r^{-1-\sqrt{A(\gamma-3)}}$. If the sources are uniformly distributed in the disk ($q = 0$) then $N \propto (a^2 - r^2)$. Thus from these analyses we see that a more or less uniform CR distribution can be expected in the disk, if advection is strong, i.e. $A \gg 1$.

5 Conclusion:

If the advection velocity is proportional to the CR source density in the disk, then the propagation characteristics of the CRs at each point of the disk are strongly determined by local conditions. In this case we can obtain a radially uniform CR distribution, if the halo boundary is close to the disk. Therefore local CR characteristics should vary strongly from point to point in the disk. Moreover, any attempt to estimate global parameters of CRs (such as the halo height or diffusion coefficient) from distribution of their density in the disk is misleading, since mixing of CRs in the Galaxy may be rather weak. Therefore one may conclude that in the case of strong CR advection, the halo extension derived from CR nuclear data reflects only a local halo extension near Earth and the value derived from γ-ray data and pure diffusion models may be an artifact.

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