Is the cosmic-ray residence time $E^{-0.6}$ (spallation) or $E^{-1/3}$ (anisotropy, turbulence)?

A.M. Hillas
Department of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, UK

Abstract

Spallation of cosmic-ray nuclei appears to imply that cosmic rays of energy E are trapped for a time $\propto E^{-0.6}$ near the galactic plane, but the spectrum of interstellar turbulence would lead us to expect a trapping time varying as $E^{-\frac{1}{3}}$. The reported anisotropy of cosmic rays is shown to agree more nearly with the latter than the former trapping time. This would require a spectrum of cosmic rays produced in their accelerator more like $E^{-2.4}$ than the usual spectrum $E^{-2.1}$ obtained from simple models of diffusive acceleration at strong shocks. (The accelerator also has to work up to much higher energies than expected from most investigations of these models.) As GeV gamma-ray observations suggest that much spallation occurs in denser (spiral-arm) regions, the spallation data may relate to trapping in special regions (inhomogeneous diffusion).

1 Problems with common assumptions about cosmic ray residence time:

Much progress has been made in modelling particle acceleration by supernova remnants (SNR) expanding supersonically into a surrounding medium, especially with the work done on expansion into a uniform medium by Ellison et al. (1997) and Berezhko et al. (1996, 1997), though different aspects of a complex problem are the focus of attention in different papers. A feature of the work by Ellison et al., and some of the results of Berezhko et al., is a spectrum emerging from the SNR of a form approximating to $E^{-2.1}$ up to a maximum energy near $10^{14} \times Z$ eV (Z being the nuclear charge), though the maximum energy can be somewhat greater in certain circumstances (and Berezhko finds the spectrum considerably harder – the magnitude of the exponent less than 2 – at the highest energies in the 1997 model). At the maximum energy, the spectral intensity is predicted to fall off extremely rapidly, but below this expected cut-off energy, if the effect of the diffusion through the galaxy on the observed spectrum is allowed for by the authors, the predicted spectrum that we should observe looks very close to what is seen. It is the main purpose of this paper to suggest that the model still requires more changes than might be apparent from this comparison, in part because the spectral modification due to trapping must be different from what has been assumed in the quoted works.

The interplay of the rate of production of cosmic rays and their trapping time can be demonstrated by simplifying a complex process as follows: considering the dependence of numbers on energy:

if rate of injection of cosmic ray particles into galaxy, $Q(E) \propto E^{-2.1}$ to $\sim 10^{14} Z$ eV, (1

and residence time in galaxy, $t_{residence}(E) \propto E^{-0.6}$, (2

the resulting number of cosmic rays in the galaxy is $N(E) = Q(E) \times t_{residence}(E) \propto E^{-2.7}$, (3

which is in close agreement with observation.

The assumption (2), that the residence time in the galactic gas disc varies as $E^{-0.6}$, is taken from the proportion of secondary nuclei in the cosmic ray flux arriving at Earth, which indicates that the amount of spallation varies in this way. Ptuskin et al. (1997) have developed a transport model in which cosmic rays are trapped by self-generated waves, excited by cosmic-ray streaming, and predict from this a trapping time dependence of $E^{-0.55}$. However, the spectrum of turbulence in the local interstellar medium follows a power law (e.g. Armstrong 1981) over a very wide scale range, that corresponds to a Kolmogorov spectrum, and leads to the expectation that trapping time $\propto E^{-\frac{1}{3}}$ if these irregularities map magnetic field irregularities which scatter cosmic rays. Biermann has argued for this form of turbulence-determined cosmic-ray diffusion, and has proposed a different model of acceleration of relativistic particles (1993) in which postulated details of diffusion of particles within the SNR during acceleration alter the spectrum $Q(E)$ generated, which becomes $E^{-2.42}$ for protons rather than $E^{-2.1}$. It is the purpose of this paper to make some quantitative (though approximate) use of cosmic-ray isotropy to give estimates of the residence time, supporting Biermann's figures.

In general, the total energy injected into cosmic rays seems not unreasonable for shock acceleration models, and the proportion of different nuclei can be accounted for well (Ellison, 1997), and although the spectrum (equations 1 to 3, above) seems satisfactory up to a point using the more well-investigated models which these summarise, there are problems which become apparent at the higher energies.

The observed spectrum extends smoothly (though with a modest steepening at the "knee") to not less than $10^{17}Z$ eV, rather than ending suddenly at $\sim\ 10^{14}$ eV. And, as argued in Section 2, the residence lifetime is surely wrong, falling much less with increasing energy, or it would become unphysically short at energies where there is no observational sign of rapid escape. This implies that the injection spectrum of cosmic rays must fall more steeply than $E^{-2.1}$. This may soon be supported by TeV gamma-ray observations of supernova remnants, which have not yet found the initially expected evidence of cosmic ray hadrons with the flux and energy spectrum corresponding to the models like that of Berezhko and others. Whilst this is still preliminary, as it may turn out that in all cases there is less gas in the remnants to form a target for gamma-ray production than had been believed, the observations would be consistent with the steeper spectrum just suggested.

Whereas it is hard to find a substitute for supernovae to inject the cosmic rays into the galaxy, the current models of supernova remnants and their acceleration of relativistic particles thus still require changes.

2 Trapping time and anisotropy:

Not only are cosmic ray density and amount of spallation proportional to the cosmic ray residence time, but the anisotropy varies inversely with this time, and if $t_{residence} \propto E^{-0.6}$ the anisotropy would be huge at energies at which it has proved too small to measure easily. This can be shown quantitatively for simplified models. Thus, consider the average time that any cosmic ray particle spends in a slab of area A and thickness h, conveniently (but not necessarily) taken to contain much of the galactic gas. (The area may refer to some large part of the Galaxy.) It is assumed here that almost all cosmic rays are produced within the thickness of this slab. Then the particle density in the gas disc may be written

particle density $= production\ rate \times t_{residence}\ \ / A \times h$.

The outflow at each surface of the slab is $\frac{1}{2} \times production\ rate$, from which the anisotropy of the cosmic rays there can be calculated under certain assumptions. If the cosmic rays are generated in a thin layer near the galactic plane, and diffuse within a thicker slab, being lost at a distance around 1500-2000 pc from the plane (to judge from the spread of particles from source regions – Section 3), the outflow results in an anisotropy particularly immediately outside the main production slab. If the cosmic rays can follow helical motions around local field lines in the galaxy, with frequent scattering, the density of particles per unit solid angle moving at an angle θ to this local field line is proportional to $(1 + a\cos\theta)$, where a measures the all-sky anisotropy. To estimate a, one may take a simple model of diffusive motion. In this, the degree of restraint on the directions in which particles may travel, imposed by the direction of the magnetic field, plays a part. If, to take a reasonable example, the field lines prefer the tangential direction (towards galactic longitude $90°$) in the galactic plane, but have random directions in a cone extending to $45°$ from this preferred direction, the average anisotropy found in places with different local field directions, just away from the galactic plane, near the edge of the slab containing the main sources (probably massive supernovae) was found to be be $a = kt_{crossing}/t_{residence}$, where $k = 3.4$, and $t_{crossing} = h/c$. (h, taken as 200 pc to obtain illustrative figures, is not well defined, but the ratio of times depends only weakly on h.) For free random diffusion, the factor $k = 1.5$. This is not the observed "anisotropy", however, for three reasons: (a) The anisotropy reported by air shower experiments refers to changes in flux as the viewing direction sweeps round a small circle (declination near to latitude) in the sky: in the variation $(1+A\cos\phi)$, ϕ being the right ascension, A will be less than the true anisotropy a by a factor which depends on the latitude and on the true axis of the cosmic ray flow (local galactic field direction).

Typically, $A \sim 0.2a$, and though the factor depends on the actual local field direction, this estimate is probably within a factor 1.6. (b) In fact our observing position is not at the surface of the slab, but close to its centre, and a will be reduced by some factor f. Although in principle one might by symmetry have $a = 0$ on the galactic plane, in such a situation there would be a considerable two-way flow, and the second harmonic would not vanish, but in the models, this is about 1/4 of the first harmonic, a, quoted above. Also, the actual location of sources is not expected to form a smooth layer, so one does not in reality expect a reduction to zero. Thus if we use the second harmonic coefficient in place of the first harmonic if the latter is appreciably smaller, it seems a reasonable estimate (based on simplified simulations) to take $f \approx \frac{1}{4}$.

Hence $\quad A \approx 0.2kf\frac{t_{crossing}}{t_{residence}}, \quad$ where $0.2kf \approx 0.2 \times 3.4 \times 0.25 \approx 0.17.$ (4

At 4 GeV rigidity, the residence time in the gas disc is about 5 million years, so if one is taking the residence time to vary as $E^{-0.6}$, as is often assumed on the basis of the fragmentation data, figure 1 shows the values that follow at higher energies. At 10^{16} eV the residence time has fallen to the direct crossing time of 650 years, and at higher energies it would be unphysical. (Two continuations are shown above the knee, depending on whether the observed increase in magnitude of ~ 0.4 in the spectral exponent is due to a more rapid escape from the galaxy or to a change in the injection spectrum. The knee is here placed at 0.5Z PeV.) One sees that this formula cannot be accepted at such high energies. Figure 1 also shows the consequences of supposing this $E^{-0.6}$ or the alternative $E^{-\frac{1}{3}}$ variation. Deriving the anisotropy from equation (4), the values are shown on the right hand axis. In plotting the residence time derived by equation (4) from the observed anisotropy of 1.7 % at 1.5×10^{17} eV – averaging over amplitudes observed at Haverah Park, Akeno and Yakutsk, tabulated by Clay et al. (1997) – and 0.036 % at 1.5×10^{14} eV (Aglietta 1996), the effective charge Z has been taken as 3 at the lower energy and 8 at the higher energy, using probable elemental compositions, in order to convert to rigidity, E/Z. The large error bars reflect the uncertainty in the conversion factor.

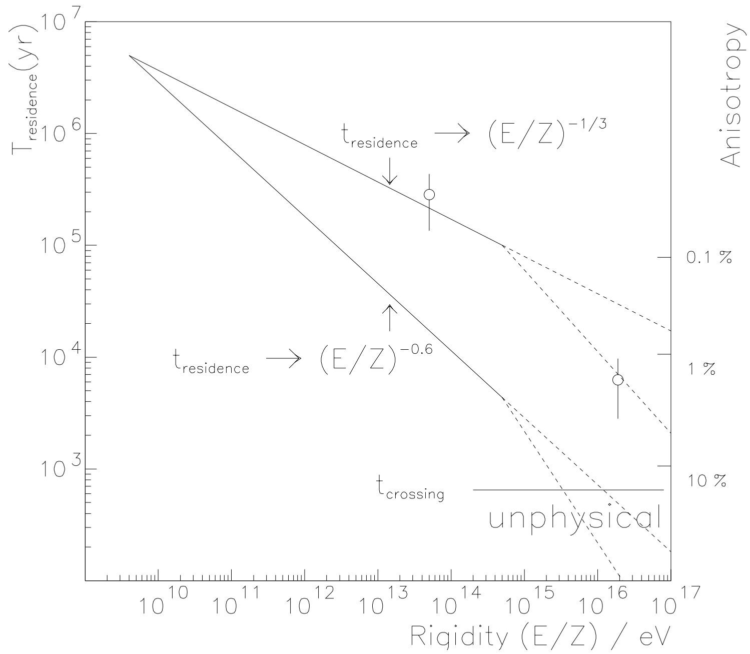

Figure 1: Energy dependence of anisotropy and residence time. The time is measured, for the simple model described, in the central 200pc of the galaxy. Points refer to observed anisotropies, plotted taking an effective Z of 3 below 1 PeV, and 8 above 10 PeV. Anisotropies of 0.02% and less are not measurable because of the Compton-Getting effect.

3 Possible explanations:

Three possible approaches to this conflict of trapping times will be mentioned.

(1) $t_{residence}$ may be as deduced from spallation at GeV to TeV energies, but falling much more slowly, or not at all, above, say 10^{14} eV. But in this case, equation (3) shows that the cosmic ray density (and hence the observed cosmic ray spectrum) would at this point start falling much less steeply: the spectrum would change to $E^{-2.1}$ from $E^{-2.7}$, which is ruled out by observation.

2) The effect of reacceleration of the lower-energy cosmic rays during propagation has been considered by other workers, as this is capable of upsetting the secondary to primary ratio as a function of energy, and possibly producing an apparent $t_{residence} \propto E^{-0.6}$, for a true $t_{residence} \propto E^{-\frac{1}{3}}$. This may be the explanation, but there is not yet general agreement.

(3) Another possibility discussed here is that one has an inhomogeneous diffusing medium, the trapping characteristics of *different regions* being responsible for (i) retention in the region of spallation, and (ii) trapping in our part of the Galaxy, determining anisotropy. In support of this may be cited the gamma-ray surveys, which indicate that the density of GeV cosmic rays is higher in spiral-arm regions, which may be supposed to contain many sources, and lower in our location. Analyzing EGRET sky maps, Hunter et al. (1998) deduce a spread of cosmic rays from sources (or actually, from regions of high gas density) ≈ 1.8 kpc. If the cosmic rays propagate in a wandering magnetic field like that already discussed, this would suggest (a) that particles diffuse in a slab of 1.5-2 kpc half-thickness before escaping, and (b) that most cosmic rays originate in dense regions of the Galaxy (e.g. in massive supernovae), and have to diffuse out of these regions before we observe them. Such a diffusion picture would be related to a "nested leaky box" model of propagation. The spallation occurs mainly not far from the sources, and the scattering mean free path varies in a different manner from its behaviour outside this region, where the scattering mean free path varies as $E^{\frac{1}{3}}$, controlled by turbulence. We reside in this external region, and this scattering determines the residence time and the anisotropy in our locality. A numerical model has been examined, modelling the diffusion by sudden scatterings, and more results will be presented at the conference.

4 Consequences for the acceleration process:

The main importance of these arguments is that a source spectrum is required which is softer than the simplest strong-shock spectrum.

If $t_{residence} \propto E^{-\frac{1}{3}}$, one can deduce the injection spectra from the observed ambient spectra:

protons : $E^{-2.75}$ observed $\longrightarrow E^{-2.42}$ at source,

C,O,Fe.. : $E^{-2.62}$ observed $\longrightarrow E^{-2.29}$ at source.

Thus the sources have to produce steeper spectra than $E^{-2.1}$, and *much* higher maximum energies, as there is evidence for a further fall-off in the cosmic-ray flux only near $10^{17} \times Z$ eV (Bird et al., 1990). The higher energies will require stronger magnetic fields in the supernova remnants than assumed in models such as those of Ellison and Berezhko, and faster acceleration to reach these energies while the remnant is sufficiently active. Biermann (1993) has put forward a model for such production spectra (and $t_{residence}$), but this has not received much discussion by other workers.

References

Aglietta, M., et al. 1996, ApJ 470, 501
Armstrong, J.W., Cordes, J.M. and Rickett, B.J. 1981, Nature 291, 561
Berezhko, E.G. et al. 1996, J.E.T.P. 82, 1
Berezhko, E.G. et al. 1997, 25th ICRC, Durban, 4, 417
Biermann, P.L. 1993, A&A 271, 649
Bird, D.J. et al. 1993, Phys. Rev. Lett. 71, 3401
Clay, R.W., McDonough, M.A. and Smith, A.G.K.1997, 25th ICCR, Durban, 4, 185
Ellison, D.C., Drury, L.O'C and Meyer, J-P. 1997, ApJ 487, 197
Ptuskin, V.S., et al. 1997, A &A 321, 434