Cosmic Ray Bounds on Violation of Lorentz Invariance

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Abstract

Recently Coleman and Glashow [Coleman and Glashow 1997] have developed a model which allows the introduction of a small violation of Lorentz invariance. Observational signatures arise because this interaction also violates flavor conservation and allows the radiative decay of the muon, $\mu \rightarrow e + \gamma$, whose branching ratio increases as b γ^4 where γ is the Lorentz factor of the muon with respect to the reference frame in which the dipole anisotropy of the universal microwave radiation vanishes. In this paper we place a bound on the Lorentz invariance violating parameter,b, of b$< 10^{-25}$ based on observations of horizontal air showers with $n_e \geq 5 \times 10^6$. Similar considerations of cosmic ray neutrinos in the atmosphere improve these bounds by twenty orders of magnitude.

To test by experiments the limits of validity of Lorentz invariance or indeed any of the fundamental principles of physics we need a theoretical model which assumes a specific form for the violation and makes predictions of physical phenomena which can be searched for by the experiments [Will 1993, Haugan and Will 1987, Fishbach et al 1985, Greene et al 1991]. The recent model of Coleman and Glashow incorporates tiny departures from Lorentz invariance which does not respect flavor conservation also [Coleman and Glashow 1997].

One of the signatures of such a flavor non conservation is the transition $\mu \rightarrow e + \gamma$ whose rate increases rapidly with the energy of the muon as measured in a preferred frame such as the one in which the 2.7^oK universal microwave background does not have any dipole anisotropy. Following their suggestion we calculate the possible contributions of such a process to the flux of "horizontal air showers" and μ-less showers which provide useful estimates for the possible strength of such an interaction and also provide a good bound on such violations[Cowsik, Sreekantan 1999]. We now extend the model to neutrinos and calculate the bound on $b_{\nu\gamma}/\tau_{\nu\gamma}$ based on the decay scheme $\nu_\mu \rightarrow \nu_e + \gamma$.

The idea on which the bound on flavor violating interactions is derived becomes clear by noting that the primary cosmic rays consist mainly of nuclei which interact strongly when they are incident on the top of the earth's atmosphere. The amount of shielding provided by the atmosphere in the vertical direction above the earth is about 1000 g cm^{-2} and increases as the secant of the zenith angle θ upto $\sim 80^o$. The total grammage in the horizontal direction is about 36500 g cm^{-2}. The primary cosmic rays interact in the atmosphere and create a 'nuclear active' cascade. Since the atmosphere is tenuous with a scale height h $\approx 7 \times 10^5$ cm, pions and kaons in the cascade decay producing the cosmic-ray muonic and neutrino component. Nuclear interactions of pions and kaons with the atmosphere compete with their decay and become increasingly dominant as the particle energy increases, so that the spectrum of the muonic and neutrino component at high energies is steeper than that of the nuclear active component by a factor E^{-1}. Also the muonic and neutrino components at high energies increases as $\sim sec\theta$, as the scale height of the atmosphere also has this dependence. Since the interaction mean free path of the hadronic components is ~ 70 g cm^{-2}, after reaching their maximum development, they are absorbed with an absorption mean free path of ~ 100 g cm^{-2}. In contrast the muons and neutrinos suffer only fewer interactions and propagate with hardly any reduction in flux. Now note that as we move away from the vertical towards the horizontal direction, with increasing sec θ the nuclear active components get severely absorbed but the high energy penetrating component increases as $\sim$ sec θ! Thus at large angles we have a nearly pure beam of high energy muons and neutrinos, traversing distances of the

order of few times the scale height $h_\theta \sim hsec\theta$. Now should the muons and neutrino decay radiatively, the decay products will induce an electromagnetic cascade which can easily be observed signalling the violation of flavor conservation, as described in the model of Glashow and Coleman. Indeed as the energy of the penetrating component increases the observability of the cascade increases as it penetrates deeper, spreads wider and produces more observable electrons and photons. The electromagnetic cascade has a very broad peak at about 500 g cm^{-2} from the point of initiation for an electron or γ of energy E $\sim 10^4$ GeV and the depth of the maximum increases logarithmically with energy. The total number of electrons at the peak of an electromagnetic cascade is approximately equal to the energy of the initiating electron or gamma ray in GeV units. Thus any array of particle detectors deployed to detect extensive air showers will be able to detect such showers generated by the radiative decay of the muon and neutrinos. There will be negligible amount of nuclear active particles and muons in these showers. The background due to showers induced by the primary cosmic ray nuclei become negligible as we go to large zenith angles. Thus 'μ-less' showers appearing in near horizontal directions constitute a signal of the new process described by Coleman and Glashow.

To quantify these ideas first consider the decay of muons into electrons and photons. We note that the spectrum of muons at high energies near the earth may be parametrized as

$$\mu(E) \quad = \quad \frac{\kappa_i sec\theta}{E^{\beta_i+1}} cm^{-2}s^{-1}sr^{-1}GeV^{-1} \tag{1}$$
with
$$\kappa_1 \quad = \quad 10, \beta_1 = 2.7 \quad for \quad 10^3 GeV < E < 10^5 \ GeV \tag{2}$$
and
$$\kappa_2 \quad = \quad 10^4, \beta_2 = 3.3 \quad for \quad 10^5 GeV < E < 3 \times 10^7 GeV \tag{3}$$

Here β_1 and β_2 are the power law exponents of the primary cosmic ray spectrum at energies of 10 to 30 times the energy of the muon.

According to Coleman and Glashow[1] the total decay probability per unit time, Γ, of a muon of Lorentz factor γ is given by:

$$\Gamma = \Gamma_w + \Gamma_r = \frac{1 + b\gamma^4}{\gamma\tau_o} = \frac{1}{\gamma\tau_o} + \frac{b\gamma^3}{\tau_o} \tag{4}$$

Here $\tau_o \approx 2.2 \times 10^{-6} s$ is the life-time of the muon and b is a very small parameter describing the violation of Lorentz invariance and flavor conservation. For a muon to decay close to the earth, say at a distance d of about 5 km ($\sim 700 gcm^{-2}$) from the air shower array, it has to survive decay during its flight though the atmosphere upto this point i.e. a distance of few times h_θ, the scale height in that direction. Thus the number of muons decaying in the 5 km stretch is given by

$$s(E) \approx \kappa sec\theta E^{-\beta-1} exp\{-jh_\theta\Gamma/c\}\Gamma d/c \tag{5}$$

where j is a number of the order of 2 to 3. Noting that Γ is a small number and that at high energies $\Gamma \sim \Gamma_r$, the exponential in eq. 5 may be set to unity and eq. 5 is rewritten as

$$s(E) \sim \kappa sec\theta E^{-\beta-1}.\Gamma_r d/c \quad \approx \quad \frac{\kappa sec\theta b d m_\mu^{-3}}{c\tau_o}E^{2-\beta} \tag{6}$$
$$\equiv \quad \kappa b\eta \ E^{2-\beta}$$

where $\eta = dm_\mu^{-3}sec\theta/c\tau_o \ GeV^{-3} \approx 5 \times 10^4 \ GeV^{-3}$, for $\langle sec\theta \rangle \approx 7$. The products of the radiative decay of the muon generate an extensive air shower which contains a large number of electrons,n_e, near the maximum, related to the muon energy through the simple relation

$$n_e \approx E/\epsilon \tag{7}$$

where $\epsilon \approx 1$ GeV for an electromagnetic shower of primary energy in the range 10^4 GeV - 10^6 GeV. The number spectrum of particles that will be seen by an air shower array is given by

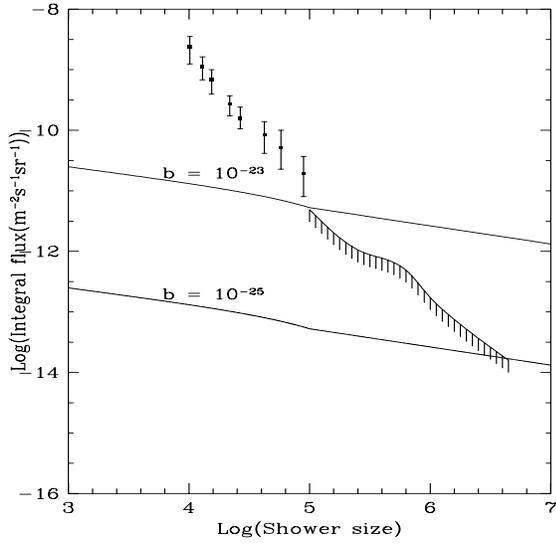$$f(n_e) \approx \epsilon^{3-\beta}.\kappa b\eta \, n_e^{2-\beta} \tag{8}$$

Or the number of showers F, of size larger than n_e is given by

$$F(n_e) = \int_{n_e}^{\infty} f(n'_e)dn'_e \tag{9}$$

$$F_2(n_e) = \frac{\epsilon^{3-\beta_2}\kappa_2 b\eta}{\beta_2 - 3}n_e^{3-\beta_2} \quad for \, n_e \geq 10^5 \tag{10}$$

$$F_1(n_e) = \frac{\epsilon^{3-\beta_1}\kappa_1 b\eta}{3 - \beta_1}\left[10^{5(3-\beta_1)} - n_e^{3-\beta_1}\right] + F_2(10^5) \quad for \, n_e < 10^5 \tag{11}$$

We compare the integral number spectrum of horizontal air showers obtained by Nagano et al [Nagano et al 1986] with the Akeno array in Fig. 1 for $b = 10^{-23}$ and $b = 10^{-25}$. The theoretically derived spectrum is very flat, $\sim n_e^{-0.3}$, in contrast with the observed spectrum of horizontal air showers which shows $\sim n_e^{-2}$ behavior. Note that $b \sim 3 \times 10^{-23}$ is excluded even by the lower energy data at $n_e \sim 10^5$ and that the bound $b < 10^{-25}$ is obtained when we consider the fluxes of horizontal air showers quoted by Nagano et al for $n_e \sim 5 \times 10^6$. Clearly these bounds are considerably more stringent than those derived by looking at the depth intensity curves for muons and as such small values of branching ratio for radiative decay will not have any detrimental effects on the functioning of muon colliders (Coleman and Glashow 1998). It is interesting to note that in the Coleman Glashow model this limit translates to $| \, 1 - c \, | \leq 6 \times 10^{-21}$

Now, let us suppose that the radiative decay process for the muon suggested by Coleman and Glashow is also applicable to the neutrinos leading to the decay scheme: $\nu_\mu \rightarrow \nu_e + \gamma$, then the horizontal showers may be used to set a strict bound on this process as well. The calculations of the high-energy cosmic-ray neutrino fluxes in the near horizontal direction (Cowsik et al. 1966, Gaisser et al. 1988, Volkova 1980, Volkova and Zatespin 1983) may be approximated to a single power law

Figure 1: *The integral flux of horizontal air showers given by Nagano et al. is compared with the expectation from the Coleman-Glashow process for the two values of b, 10^{-23} and 10^{-25} respectively.*

$$\nu_\mu(E) \approx \frac{\kappa_\nu \sec\theta}{E^{\beta_\nu+1}} \qquad cm^2 \, sec^{-1} \, sr^{-1} \, GeV^{-1} \tag{12}$$

for $10^3 GeV < E_\nu < 10^6 GeV$, with $\kappa_\nu \approx 10$ and $\beta_\nu = 3$. As before its decay according to the equations (14) and (4), leads to a horizontal air shower flux,

$$F_\nu(n_e) = \frac{\epsilon^{3-\beta_\nu}\kappa_\nu \sec\theta d}{(3 - \beta_\nu)c}\frac{b_{\nu\gamma}}{\tau_{\nu\gamma}}\left(\frac{2m_2^2}{m_2^2 - m_1^2}\right)^{3-\beta_\nu}\frac{1}{m_2^2}n_e^{3-\beta_\nu} \tag{13}$$

where m_2 is the mass of the muon neutrino, m_1 is the mass of the electron neutrino, $b_{\nu\gamma}$ is the branching ratio and $\tau_{\nu\gamma}$ the life time for the process given in equation (14). Notice that equation (16) is weakly sensitive to

$\Delta m^2 = m_2^2 - m_1^2$ and highly sensitive to m_2. Comparison with the data on horizontal showers as in figure 1 yields the following strict bound for $b_{\nu\gamma}/\tau_{\nu\gamma}$,

Table 1:
Bound on $b_{\nu\gamma}/\tau_{\nu\gamma}$ for various values of Δm^2 and m_2 obtained by camparing the expected integral flux from Caloman-Glashow process with the integral spectrum given Nogano et al. as in Fig. 1.

Δm^2 (eV^2)	m_2 (eV)	bound on $b_{\nu\gamma}/\tau_{\nu\gamma}(sec^{-1})$
10^{-5}	10^{-2}	1×10^{-47}
10^{-5}	10^{-1}	3×10^{-44}
10^{-5}	1	1×10^{-40}
10^{-5}	10	4×10^{-37}
10^{-3}	10^{-2}	1×10^{-48}
10^{-3}	10^{-1}	1×10^{-44}
10^{-3}	1	3×10^{-41}
10^{-3}	10	1×10^{-37}
10^{-2}	10^{-2}	1×10^{-48}
10^{-2}	10^{-1}	5×10^{-45}
10^{-2}	1	1×10^{-41}
10^{-2}	10	5×10^{-38}
1	10^{-2}	3×10^{-48}
1	10^{-1}	1×10^{-45}
1	1	5×10^{-42}
1	10	1×10^{-38}

If we assume $b_{\nu\gamma}$ to be one, this will yield $\tau_{\nu\gamma} \sim 10^{41}$s!

References

S.Coleman and S.L.Glashow, 1997, Phys. Lett. B 405, 249; Harvard University Theoretical Physics Preprint 98/AO76 (pvt. comm).
R. Cowsik et al., 1966, Proc. Ind. Acad. Sci. 63, 217.
R.Cowsik, B. V. Sreekantan, 1999, Phys. Lett. B 449, 219.
T.K. Gaisser, T.Staner and G. Barr, 1988,Phys. Rev. D 38,85.
G.L.Greene etl. al., 1991, Phys. Rev. D 44, 2216.
M.I. Haugan and C.M.Will, 1987,Physics Today 40 .
E. Fischbach et. al., 1985, Phys. Rev. D 32, 154.
M. Nagano et. al., 1986, J.Phys. G: Nucl. Phys. 12,69.
L. V. Volkova, 1980, Sovt. J. of Nucl. Phys. 31, 784.
L. V. Volkova and G. T. Zatespin, 1983, Sovt. J. of Nucl. Phys. 37, 212.
C.M. Will, 1993,Theory and Experiment in Gravitational Physics. (Reading:Cambridge University Press).