MUON SPECTRA AT SEA LEVEL

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Abstract

We solved the diffusion equations of hadrons and leptons in the atmosphere using the semigroup theory. We obtained the muon fluxes at sea level for different zenith angles in the energy range 1-10^4 GeV. The charge ratio of cosmic ray muons at sea level is calculated using recent experimental results on cosmic ray primaries and on hadron-nucleus collisions. Our muon fluxes at sea level for zenith angles $\theta = 0$ and $\theta = 89^o$ agree very well with the experimental data and with others analytical calculations (Lipari, Butkevich). Our vertical charge ratio of muons at sea level is also compared with experimental data and with results of several authors.

1 Introduction:

We calculated the zenithal muon flux originated by a hadronic shower in the atmosphere. These muon intensities and the muon charge ratio, μ^+/μ^-, are expressed in terms of expansional operators; these operators are decomposed into a product of some simple exponential by using the Feynman-like procedure of ordered exponential operators. With the semigroup theory, it is possible to use arbitrary forms for the parametrizations of the energy distributions of secondaries and for energy spectrum of primary cosmic radiation, N(0,E). The energy spectrum of nuclei at the top of the atmosphere, which can be given in an arbitrary form and not only in the form of a power law, was used as a boundary condition for the solution of these diffusion equations. Our calculations of the muon spectra and the charge ratio are based on the continuous energy loss approximation and for a general type of atmosphere.

We compared our results of the muon flux at sea level, with experimental data and we also made a comparison with others analytical calculations. Our vertical charge ratio of muons at sea level is also compared with experimental data and with results of the authors mentioned above.

2 Solutions of the Diffusion Equations:

In recent papers we have solved the hadronic and leptonic diffusion equations using the semigroup theory (Portella, et al., 1997; Gomes, Portella, 1999; Portella, et al., 1998). These solutions can be written in the following forms,

$$N(t,E) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\frac{t}{\lambda(E)})^n I_n(\gamma, \alpha, n) N(0,E) \tag{1}$$

$$M(t,E,\theta) = \int_0^t Exp[\int_z^t (\hat{A}_M + \hat{G}_M) dZ'] \hat{B}_N N(t,E) dZ \tag{2}$$

where $Exp[\int_z^t (\hat{A}_M + \hat{G}_M) dZ']$ is an expansional operator defined by a sum of multiple ordered integrals (Gomes, Portella, 1999; Portella, et al., 1998).

$$\mu(t,E,\theta) = \int_0^t exp[b(t-Z_1) - \int_{Z_1}^t (\frac{m_\mu c^2}{\tau_\mu c}) dZ \frac{sec\theta^*(Z)}{\epsilon((t-Z),E)\rho(Z)}] H(\epsilon, Z_1, \theta^*(Z_1)) dZ_1 \tag{3}$$

where

$$\epsilon((t-Z),E) = Ee^{(b(t-Z))} + (e^{(b(t-Z))} - 1)\frac{a}{b}$$

and

$$H(\epsilon, Z_1, \theta^*(Z_1)) = \int_{E_{min}}^{E_{max}} dE_M (BR)_M \hat{G}_M M(Z_1, E_M, \theta^*) f_{M\to\mu}(\epsilon, E_M)$$

The charge ratio $\mu+/\mu-$ can be written in the following form

$$\mu^{\pm}(t,E,\theta) = \int_0^t dZ\{\int_{E_{min}}^{E_{max}} dE_\pi (BR)_\pi \hat{G}_\pi \pi^{\pm}(Z, E_\pi, \theta^*) f_{\pi^{\pm}\to\mu^{\pm}}(E, E_\pi) +$$

$$\int_{E_{min}}^{E_{max}} dE_K (BR)_K \hat{G}_K K^{\pm}(Z, E_K, \theta^*) f_{K^{\pm}\to\mu^{\pm}}(E, E_K)\} \tag{4}$$

where $(BR)_M$ is the branching ratio of the meson M. The operators $\hat{B}_N$, $\hat{B}_M$ and $\hat{G}_M$ are defined in the references above.

We shall assume that the energy losses occur continuously, *i.e.*, fluctuations can be neglected. The values of "a" and "b" and the model of atmosphere that we have used are from Maeda (Maeda, 1964). For the primary cosmic ray, we used a power law spectrum proportional to $E^{-(\gamma+1)}(\gamma = 1.7)$ and the composition of these primaries do not change with the energy. The collision of heavy nuclei was treated with the superposition model and we assumed for the hadronic cross-section the exact **Feynman Scaling**.

The charge ratio was calculated neglecting energy loss and muon decay. For the Z-factors and for the excess of protons over neutrons in the primary cosmic ray we have used the values tabulated (Lipari, 1993).

3 Numerical Results:

The calculation of the muon spectra and the ratio μ^+/μ^- are made with the parameters and distributions described above and are showed in the figures 1, 2 and 3.

The figure 1 shows a comparison of our calculated vertical muon flux with the analytical calculation made by Lipari (Lipari, 1993) and with the experimental data (Allkofer, 1971). The figure 2 shows the same comparison of the zenithal muon flux for zenith angle $\theta = 89^o$. For the experimental data we have used the values tabulated by Matsumo (Matsumo, 1984). The figure 3 shows a comparison of our ratio μ^+/μ^- for $\theta = 0$ with others analytical calculations (Lipari, 1993; Thomson and Whalley, 1977; Bhattacharyya, 1983) and with the experimental data (Allkofer, 1971; Baxendale, 1975; Ashley II, 1975).

Our calcultaions of the muon fluxes agrees very well with the experimental data mentionated above and with the analytical calculations (our fluxes are 3% greater than Lipari's for $\theta = 0$ and 5% for $\theta = 89^o$). We also made a numerical comparison with the analytical results of Butkevich (our fluxes are 5% smaller for $\theta = 0$ and 7% for $\theta = 89^o$).

Our calculated vertical charge ratio is somewhat higher than the observed values, though the experimental error is large. The so-called charge-symmetry relations assumed heve been modified for the kaon production by neutron. However, the discrepancy between our calculated charge ratio and the experimental data has not been removed. In this way we think that the heavier nuclear species in cosmic ray primary at high energy is greater than the used in this paper. Another possible solution is consider the so-called EMC effects in lepton-nucleus collisions.

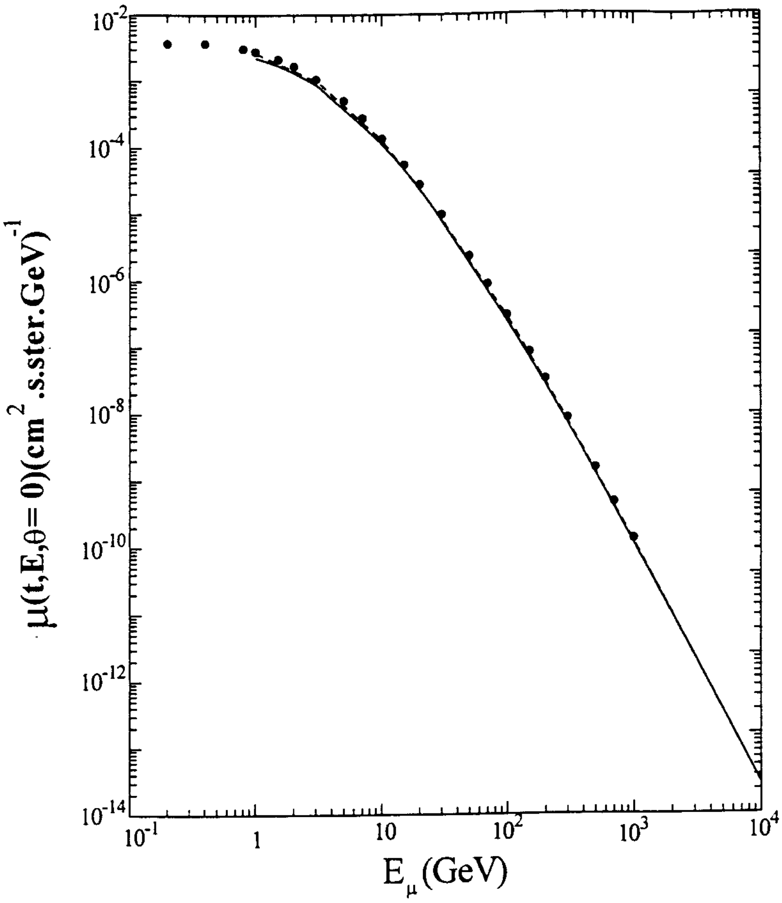

Figure 1. Vertical muon fluxes at sea level. The solid line is the flux calculated by Lipari (Lipari, 1993) and the dashed line is our calculated flux. Data are from (Allkofer, et al., 1971).

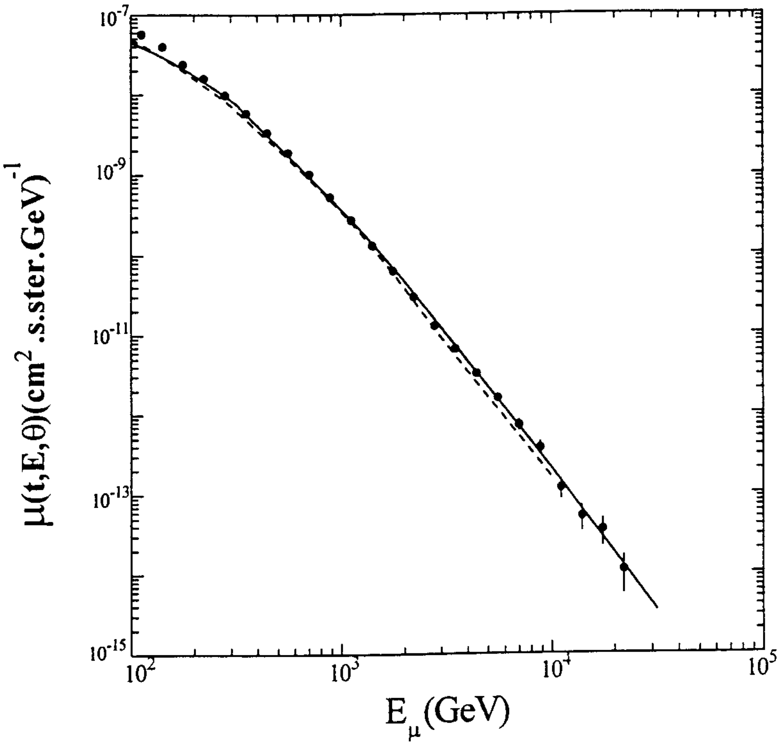

Figure 2. Zenithal muon fluxes at sea level. The solid line is the calculated flux by (Lipari, 1993) and the dashed line is our calculated flux. Data are from (Matsumo, 1984).

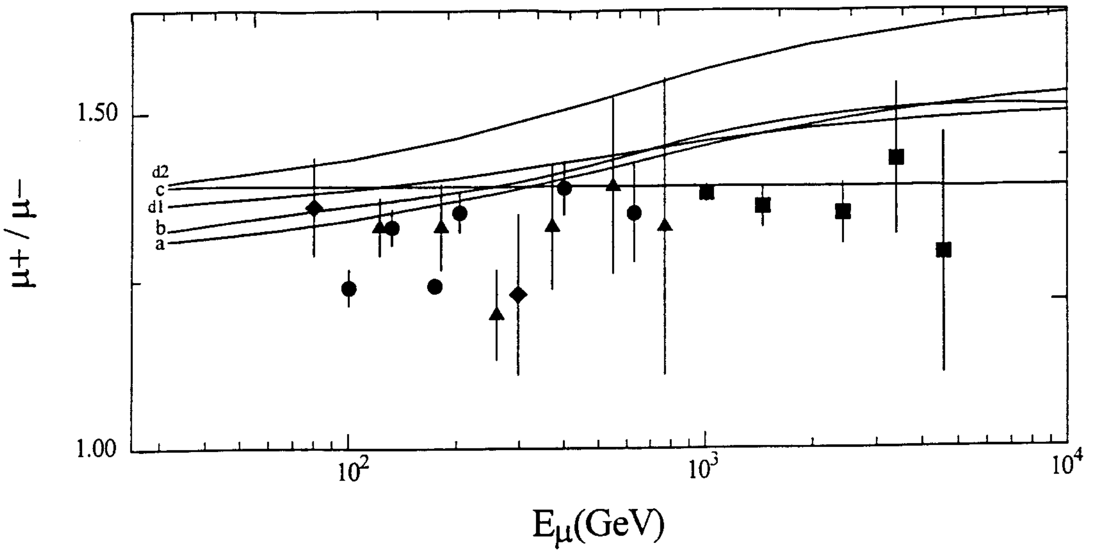

Figure 3. $\mu+/\mu-$ charge ratio at sea level for $\theta = 0$. a - calculated flux by (Lipari, 1993); b- our results; c- calculated flux by (Bhattacharyya, 1983); d1 and d2- results from (Thomson and Whalley, 1977), alternative 1 and 2 respectively. Data are from (● MARS, 1980), (▲ Allkofer, et al., 1971), (◆ Baxendale, et al., 1975) and (■ Ashley II, et al., 1975).

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