Angular distribution of muons in rock

A. A. Lagutin[1], **V. V. Makarov**[2], **A. Misaki**[3], **A. V. Yushkov**[1]
[1] *Altai State University, Barnaul, Russia*
[2] *STANKIN, Moscow, Russia*
[3] *Graduate School of Policy Science, Saitama University, Urawa 338, Japan*

Abstract

In the given paper we present calculations of the angular distribution of muons at large depths of rock with the complete account of energy loss fluctuations in processes of bremsstrahlung, knock-on electron production, pair production and photonuclear interaction. Calculations are performed in the frameworks of the adjoint approach. Scaling properties of the obtained distributions are discussed.

1 Introduction:

In many problems of astropartical physics the deflection of muons due to Coulomb scattering is neglected, but the study of this question is required in consideration of some tasks: so, the angular spread of excessive flux of muons, registered by underground experiments in direction of Cygnus X-3 (Marshak et al. 1985, Dettore B. Piazzoli, 1987), now is not considered to be resulted by the above mentioned process (Kitamura, Nakatsuka, Kobayakawa, 1989, Perkins, 1986, Lagutin et al., 1998b). In this work we present not only the new calculations of angular distributions of single muons with different energies in standard rock, but also discuss scaling properties of the obtained distributions.

2 Adjoint approach:

The equation for importance of muon with the account of Coulomb scattering in small angle approximation has the following form (e.g. Kolchuzhkin, Uchaikin, 1978, Uchaikin, Lagutin, 1992):

$$\partial\bar{q}(t,\vartheta,E)/\partial t + (\Sigma+\Sigma_s)\bar{q}(t,\vartheta,E) - \int\limits_0^{2\pi} d\varphi' \int\limits_0^{\infty} \vartheta' d\vartheta' \, W_s(\theta)\bar{q}(t,\vartheta',E) -$$

$$-\sum_\beta \int\limits_0^{\infty} W_\beta(E,E')\bar{q}(t,\vartheta,E')dE' = \frac{1}{2\pi}[\delta(\vartheta)/\vartheta]\delta(t); \quad (1)$$

here $t = z_0 - z$, z_0 - coordinate of the observation point; Σ_s and $W_s(\theta)$ - total and differential scattering cross-sections, $\theta = \sqrt{\vartheta^2 + \vartheta'^2 - 2\vartheta\vartheta' \cos\varphi}$; Σ - total cross-section of inelastic interaction; $\beta = i, p, r, h$ for the processes of ionization, e^+e^- - pair production, bremsstrahlung and photonuclear interaction correspondingly; $W_\beta(E,E')$ - differential cross sections of these processes. For the description of cross-sections we used the results of works (Kokoulin, Petrukhin, 1969, Bezrukov, Bugaev, 1981a, Bezrukov, Bugaev, 1981b, Rossi, 1955). Applying the Bessel transformation to the eq. (1)

$$\bar{q}(t,\vartheta,E) = \frac{1}{2\pi}\int\limits_0^{\infty} \tilde{q}(t,p,E) J_0(p\vartheta) p \, dp \quad (2)$$

we get the following equation for the transform:

$$\partial\tilde{q}(t,p,E)/\partial t + \left(\Sigma + \frac{p^2\langle\vartheta_s^2\rangle}{4}\right)\tilde{q}(t,p,E) - \sum_\beta \int\limits_0^{\infty} W_\beta(E,E')\tilde{q}(t,p,E')dE' = \delta(t), \quad (3)$$

here $\langle \vartheta_s^2 \rangle = 2\pi \int\limits_0^\infty \vartheta^2 W_s(\vartheta)\vartheta d\vartheta = E_s^2/(E^2 t_0)$, $E_s = 19.3$ MeV, t_0 - radiation unit.

Solving the obtained equation numerically, as described in (Lagutin, Litvinov, Uchaikin, 1995), and performing the inverse Bessel transformation we find $\bar{q}(t,\vartheta,E)$. The normalized angular distribution function is expressed as:

$$f(t,\vartheta,E) = \frac{\bar{q}(t,\vartheta,E)}{2\pi \int\limits_0^\infty \vartheta \bar{q}(t,\vartheta,E)d\vartheta}.$$

In order to analyze the dependence of this function on energy E and depth t, similarly to (Lagutin, Plyasheshnikov, Goncharov, 1998) we introduced the scaling factor $x = \vartheta/\vartheta_{\mathrm{m.s.r.}}$, here $\vartheta_{\mathrm{m.s.r.}}(t,E)$ is mean square scattering angle:

$$\vartheta_{\mathrm{m.s.r.}}(E,t) = \left[2\pi \int\limits_0^\infty \vartheta^2 f(t,\vartheta,E)\vartheta d\vartheta \right]^{1/2}. \qquad (4)$$

The relation between $f(t,x,E)$ and $f(t,\vartheta,E)$ is: $x\, f(t,x,E) = \vartheta_{\mathrm{m.s.r.}} \vartheta f(t,\vartheta,E)$.

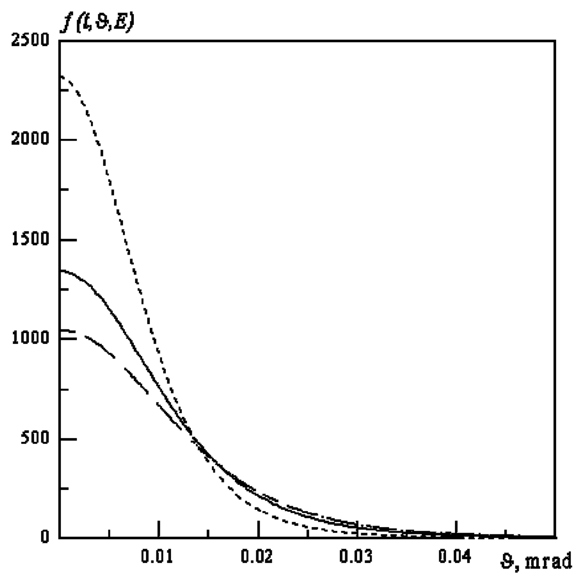

Figure 1: The normalized angular distribution function $f(t,\vartheta,E)$. $E = 1$ TeV. Dotted line — $t =$ 4 km w.e.; solid line — $t = 5$ km w.e.; dashed line — $t = 6$ km w.e.

3 Results:

We carried out the calculations of normalized function $f(t,\vartheta,E)$ for different depths of standard rock (e.g. Fig. 1) and considered the dependence of $x\, f(t,x,E)$ on energy and depth. The analysis shows that $x\, f(t,x,E)$ does not practically depend on t and E in region of $0.03 \le x \le 3.5$. This scaling property is illustrated in Fig. 2. The similar deduction was earlier obtained in investigation of lateral distribution functions of electrons in EAS and pure electromagnetic cascades (see (Lagutin et al., 1998a)).

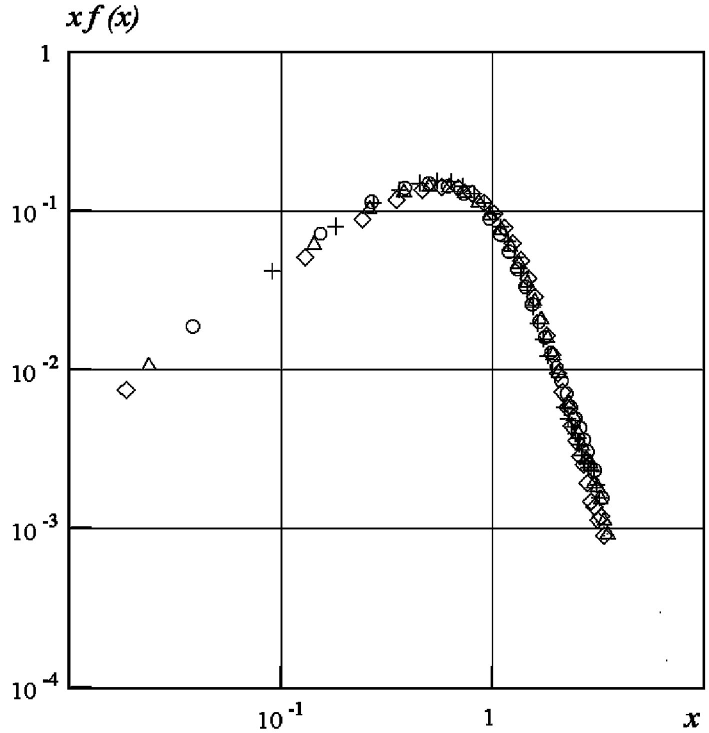

Figure 2: The dependence of the invariant part of the angular distribution function of muons on the scaling variable $x = \vartheta/\vartheta_{\text{m.s.r.}}(t, E)$. $\diamond$ – $E = 1.75 \cdot 10^2$ GeV, $t = 0.5$ km w.e.; $+$ – $E = 1$ TeV, $t = 1$ km w.e.; $\bigcirc$ – $E = 1$ TeV, $t = 2$ km w.e.; $\triangle$ – $E = 1$ TeV, $t = 3$ km w.e.

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