Muon Pair Production by High Energy Muons and Muon Bundles Underground

S.R. Kelner, R.P. Kokoulin, and A.A. Petrukhin
Moscow Engineering Physics Institute, Moscow 115409, Russia

Abstract

Cross section for electromagnetic production of muon pairs by high energy muons taking into account atomic and nuclear formfactors has been considered. A convenient approximate formula for a differential cross section is given. The ratio of the flux of double and triple muon events produced in this process to flux of single muons has been calculated for various observation depths. It is shown that earlier performed calculations seriously overestimated the contribution of muon pair production to the intensity of narrow muon bundles underground.

1 Introduction:

Direct production of muons pairs by muons

$$\mu + Z \to \mu + Z + \mu^+ + \mu^- \tag{1}$$

has a low cross section in comparison with those for bremsstrahlung and electron pair production. Therefore, the influence of the process on the formation of muon spectrum underground is negligible. However, the process (1) may lead to events when a group of 2 or 3 muons is observed in the detector.

The first calculations of the fluxes of double and triple muon events generated due to muon pair production in the overburden were performed by Kelner, Kotov, & Logunov (1970, 1975). For these estimates, the cross section evaluated with a logarithmic accuracy for a point-like nucleus and a limiting case of the complete screening on the basis of Kelner's results (1967) was used. This approximate formula was included into well known monography (Eq. 1.51 in Bugaev, Kotov, & Rozental, 1970; hereafter, BKR), and was later used by other authors.

However, approximations of point-like nucleus and complete screening used in the derivation of the formula mentioned above are not appropriate in this case. Firstly, transferred momenta of the order of muon mass are important in this process, and therefore the nucleus cannot be considered as Coulomb center. Secondly, the complete screening regime is approached very slowly with the increase of energy, and the use of this approximation may also lead to overestimation of the cross section.

In the present work, the cross section of the process (1) for ultrarelativistic muons taking into account the finite nuclear size and atomic screening is calculated. A simple approximation for the cross section has been found. New estimates of the fluxes of double and triple muon events are presented.

2 Cross Section:

In calculation of the cross section, the intermediate results concerning the consideration of electron pair production by muons (Kelner, 1967) have been used. For the Coulomb center, the distribution in pair particle energies $E_\pm$ and momentum q transferred to the target is given by

$$d\sigma = (Z\alpha r_\mu)^2 \, \Big(f_a(E, E_+, E_-, q) + f_b(E, E_+, E_-, q)\Big) \, dE_+ \, dE_- \, dq \, , \tag{2}$$

where E is initial muon energy, r_μ is the classical muon radius, and f_a, f_b correspond to the contribution of two basic types of diagrams (a, b in notations used by Kelner, 1967). Functions f_a, f_b are expressed as two-fold integrals and are rather bulky. If the mass of the pair particles is taken to be muon mass μ, equation (2) immediately describes the cross section of the process (1) but without the interference between direct and exchange diagrams.

The interference term $\sigma_{\rm int}$ may be evaluated on the basis of results obtained by Kelner, 1998. At muon energy 10 GeV, $\sigma_{\rm int}$ decreases the total cross section by about 5%, whereas for $E > 100$ GeV the contribution of $\sigma_{\rm int}$ is less than 1%. Therefore for high energy muons the interference term may be neglected.

In order to take into account atomic screening and finite nuclear size, expression (2) has to be multiplied by $(F_n(q) - F_a(q))^2$, where $F_n(q)$ and $F_a(q)$ are nuclear and atomic formfactors. In numerical integration, Thomas-Fermi atomic model and Fermi function for nuclear charge density have been used.

The total cross section for muon pair production in standard rock ($Z = 11$, $A = 22$) is presented in Fig. 1. Cross section is calculated assuming that final energy of each muon is greater than $E_{\rm th} = 1$ GeV. The solid curve corresponds to the cross section on the Coulomb center. The nuclear formfactor significantly decreases the value of the cross section at all energies. On the contrary, the influence of atomic screening is negligibly small up to $E = 10^5$ GeV (dotted curve in the Figure). Cross section calculated with BKR-formula is presented by dash-dotted curve. It is seen that this formula heavily overestimates the cross section: at $E = 10^2 - 10^3$ GeV the difference is about 5 times in comparison with the results of calculations for a finite nucleus.

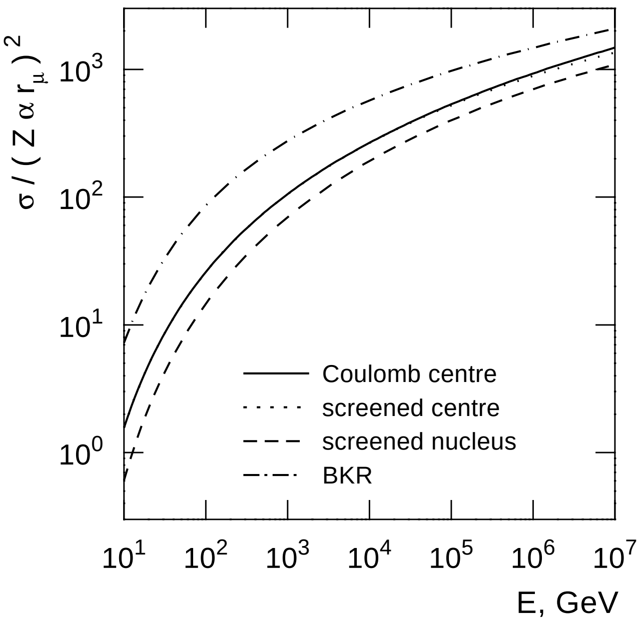

Figure 1: Total cross section for various models.

Muon pairs may be produced also in collisions of muons with atomic electrons if the initial muon energy is greater than $\mu(4\mu + 3m)/m = 87.7$ GeV. However, numerical calculations show that relative contribution of this process is small: less than $0.2/Z$ for $E < 10^5$ GeV. Another correction, related with the excitation of the nucleus in the process of muon pair production by muon may be evaluated similar to Andreev et al. (1994) consideration for muon bremsstrahlung. For standard rock, this correction amounts to about 5% at $E = 100$ GeV and slowly decreases with energy.

3 Approximate Formula:

For practical purposes (e.g., double and triple muon fluxes calculations) formula (2) is not convenient, and simple analytical expressions are desirable. Such formulae for electron pair production by muons taking into account atomic screening and nuclear size was obtained by Kokoulin, & Petrukhin (1971). By analogy, the following approximate formula for the double differential cross section of the process (1) may be written as:

$$\sigma(E, v, \rho)\, dv\, d\rho = \frac{2}{3\pi}\,(Z\alpha\, r_\mu)^2\, \frac{1-v}{v}\, \Phi(v,\rho)\, \ln(X)\; dv\, d\rho\,. \tag{3}$$

Here $v = (E - E')/E = (E_+ + E_-)/E$ is the fraction of the energy transferred to the particles of the pair; E, E' are initial and final energy of the parent particle; E_+, E_- are energies of the produced particles; $\rho = (E_+ - E_-)/(E_+ + E_-)$ is the pair asymmetry parameter. Final energies are related with v, ρ by

$$E' = E\,(1-v)\,, \quad E_\pm = Ev\,(1 \pm \rho)/2\,. \tag{4}$$

Kinematic region is defined by

$$2\mu/E \le v \le 1 - \mu/E\,, \quad |\rho| \le \rho_{\rm max} \equiv 1 - 2\mu/(vE)\,. \tag{5}$$

Function Φ may be expressed as

$$\Phi(v,\rho) = \Big((2+\rho^2)(1+\beta) + \xi\,(3+\rho^2)\Big)\ln\left(1 + \frac{1}{\xi}\right) - 1 - 3\,\rho^2 + \beta\,(1 - 2\,\rho^2) +$$

$$+\left((1+\rho^2)\left(1+\frac{3}{2}\beta\right)-\frac{1}{\xi}(1+2\beta)(1-\rho^2)\right)\ln(1+\xi)\,, \tag{6}$$

with

$$\xi=\frac{v^2(1-\rho^2)}{4\,(1-v)}\,,\quad \beta=\frac{v^2}{2\,(1-v)}\,. \tag{7}$$

Argument X of the logarithm in (3) is defined as follows. Let us denote as $U(E,v,\rho)$ the function

$$U(E,v,\rho)=\frac{0.65\,A^{-0.27}B\,Z^{-1/3}\,\mu/m}{1+\dfrac{2\sqrt{e}\,\mu^2B\,Z^{-1/3}(1+\xi)(1+Y)}{m\,E\,v\,(1-\rho^2)}}\,, \tag{8}$$

where $B=183$, $e=2.718\ldots$, A is atomic weight, $Y=10\sqrt{\mu/E}$. Then

$$X=1+U(E,v,\rho)-U(E,v,\rho_{\max})\,, \tag{9}$$

with $\rho_{\max}$ defined by (5). The function U is chosen in such a way to reproduce the main logarithmic factor in the limiting cases of absence of screening and complete screening. Function Y and numerical constants serve to improve the description of the total cross section. Cross section $\sigma(E,v,\rho)$ is non-negative in the kinematic region (5) and comes to zero at $\rho=\pm\rho_{\max}$. Comparison with numerical integration shows that the accuracy of (3) is better than 10% for $E>10$ GeV and final particle energies E', $E_+, E_->1$ GeV, the total cross section being reproduced better than 3% for $E>30$ GeV.

Formulae (3)-(9) describe the distribution of final particles in $(v,\,\rho)$ variables. To obtain the distribution in the energies of the particles of the pair $E_+,\,E_-$, it is sufficient to use

$$\sigma(E,E_+,E_-)\,dE_+\,dE_-=\frac{2}{E^2v}\,\sigma(E,v,\rho)\,dE_+\,dE_-\,. \tag{10}$$

For the comparison, BKR formula differs from the cross section (3) by the argument of the main logarithm ($X_{BKR}=B\,Z^{-1/3}\mu/m$) and factor $Z(Z+1)$ instead of Z^2. BKR expression significantly overestimates the cross section in the whole kinematic region and even distorts qualitative dependencies on v and ρ.

4 Equilibrium Flux of Double and Triple Muons:

Integral flux of double and triple muon events produced in the process (1) may be estimated as

$$J(>E_h,h)=\frac{N_A}{A}\int\limits_0^h dh'\int\limits_{E_{\min}}^{\infty}dE\,N_\mu(E,h')\,\tilde\sigma(E,E_{h'})\,. \tag{11}$$

Here N_A is the Avogadro number; E_h is threshold muon energy at the observation depth h; $N_\mu(E,h')$ is the differential spectrum of single muons at the interaction depth h'; $E_{h'}$ is the minimal muon energy after interaction to reach the observation level with energy greater than E_h. In continuous energy loss approximation, the relation between E_h and $E_{h'}$ is given by

$$E_{h'}=(E_h+a/b)\exp(b\,(h-h'))-a/b\,, \tag{12}$$

where a, b are the coefficients in muon energy loss relation.

The cross section $\tilde\sigma(E,E_{h'})$ is integrated over the final particles energies, all three energies being greater than $E_{h'}$ for triple muon events, whereas for double muons two energies should be greater than $E_{h'}$ while the third particle energy is less than $E_{h'}$. Lower integration limit $E_{\min}$ in parent particle energy for double and triple muon events equals to $(2\,E_{h'}+\mu)$ and $3\,E_{h'}$, respectively.

It is necessary to underline, that here we take as double muon events only those cases when one muon is stopped above the observation level; the finite sizes of the setup are not considered. Energy spectrum of muons arising from π, K decays with asymptotic slope $\gamma=2.7$ is adopted in calculations; corrections for the influence of energy loss fluctuations on muon spectrum formation at great depth are taken into account.

The calculated ratios of the fluxes of double and triple muon events, and of the sum of these fluxes to the integral single muon flux is shown in Fig. 2 (solid lines). Calculations are performed for the standard rock, muon detection threshold $E_h = 1$ GeV. For the comparison, the results obtained with BKR formula for the cross section are presented (dashed curves). The points correspond to recent Monte Carlo calculations performed by Kudryavtsev and Ryazhskaya (1998). The latter calculations were made with BKR cross section, therefore the points should lie near the dashed curves. There is a good agreement with present calculations (with BKR formula) for 3 km w.e. depth, whereas at 10 km w.e. the results of Kudryavtsev, & Ryazhskaya are somewhat lower. The possible reason is that in their Monte Carlo simulation only one (the first) interaction with muon pair production was considered. Hence, appreciable part of muons (15 – 20% for 10 km depth) dropped out of triplet production process near the observation level. Another source of the difference is a limited Monte Carlo statistics (only 53 triple muon events were obtained at 10 km w.e.).

As a whole, results presented in Fig. 2 show that the use of BKR formula for the cross section leads to about 3 times overestimation of the fluxes of triple and double muons produced in muon pair production process at great depth. For shallower depth, the difference is even greater.

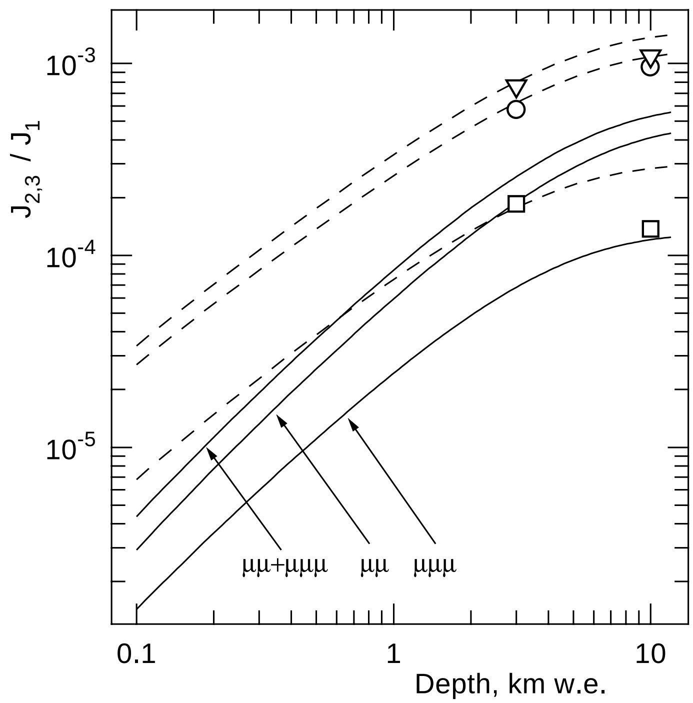

Figure 2: Relative flux of double and triple muon events (see text for explanations).

5 Conclusion:

Cross section of muon pair production by ultrarelativistic muons taking into account atomic screening and finite nuclear size is calculated. A simple approximate formula for the differential cross section describing the distribution of pair particle energies is found. Estimates of the equilibrium flux of double and triple muon events produced in this process underground have been obtained. It is found that earlier calculations seriously overestimated the role of muon pair production in the generation of narrow muon bundles because of the use of improper approximations in the cross section.

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