Thinning of High-Energy Cosmic-Ray Air-Showers

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Abstract

Maximum number of particles in a 10^{20} eV proton air-shower exceeds 10^{11} which makes simulation of a complete shower impossible. Thus, tracking of a representative sample of particles has to be introduced. The thinning method incorporated in present simulation programs selects particles regarding only their energy and is mostly effective in the region of the shower core. However, surface detectors sample particle densities far from the shower core (≥ 100m). We developed a thinnig method based on the particle energies and weights, their distances from the core, and their incident angles. With the new method the same quality as in the standard thinning is obtained in the working area of the detector in approximately 20 times shorter CPU time.

1 Introduction

High-energy cosmic-rays are detected through atmospheric air-showers. By comparing measured particle densities at different sampling points on the ground with computer simulations, one is able to determine the properties of the incident cosmic ray. The Pierre Auger Observatory will detect cosmic rays with energies above 10^{19} eV and will have sampling points approximately 1.5 km apart. Therefore, simulations should be accurate also several kilometers from the shower core.

Since the number of particles in a 10^{20} eV proton air-shower exceeds 10^{11}, only a small fraction of particles can be tracked and the whole shower is reconstructed from the sample. This method is called thinning. The first thinning algorithm was introduced by Hillas [1] and was based only on particle energies. Particle densities in the shower rapidly decrease with lateral distance from the core. Since Hillas thinning uniformly reduces particle densities, some optimization was needed to improve statistics far from the shower core, where the measurements are performed. By studying air-showers [2], it was shown that thinning must be based on particle energies, but specific particles can be selected for tracking, unconditionally. Particles with large weights induce large fluctuations. This means that particles with weights above some maximum weight should be unconditionally accepted to prevent further increase of their weights. Particles traveling at larger incident angles or those at larger lateral distances produce more laterally distant secondaries [2]. Unconditional selection of such particles increases the number of tracked particles far from the core. These particles have low energies and their sub-showers rapidly fade away. This makes their tracking important only near the observation depth. Since only low-energy particles move far from the core, the statistics there can also be increased by selecting all particles with energies below some energy threshold near the observation depth. Unconditional selection of particles regarding their distance from the core, their incident angles or their energy can improve performance by factor of 10, but the selection overlaps with weight limitation [3]. Since joint use with weight limit does not improve performance significantly and weight limit is more effective and less parameter dependent, we use it stand alone.

2 Statistical thinning

Thinning is performed after each interaction at which new particles are produced. Each particle in the vertex is selected for further tracking only with a certain probability F_i, called the acceptance factor. The weights $w_i = w_i'/F_i$ are assigned to accepted particles to account for the untracked ones. w_i' is the weight of the particle which interacted. We chose F_i as [2]:

$$F_i(E_i) = \begin{cases} 1, & E_i > E_t \\ E_i^{\alpha}, / \sum_k E_k{}^{\alpha} & \text{otherwise} \end{cases} \quad . \tag{1}$$

Different values of parameter α are considered elsewhere [2]. Since thinning quality weakly depends on α, we set α to 1, which is similar to the Hillas thinning. The number of tracked particles and consequently CPU time (t_{cpu}) depend mainly on the ratio $t_f = E_t/E_0$. We will denote statistical thinning with STAT(t_f).

The shower is reconstructed from particles with weights (w_i). The number of particles N in the shower in a given region is

$$N = \sum_i w_i \pm \sqrt{\sum_i w_i^2}, \tag{2}$$

where the sum goes over all tracked particles in the interval. Fluctuations in (2) are only the Poisson part of the actual fluctuations. It was shown [3] that they represent the actual fluctuations well, particularly at larger distances from the core were also the actual fluctuations are Poissonian [4].

The thinning performance is defined by fluctuations in the reconstruction. Fluctuations σ can be estimated from (2). Fluctuations in energy distribution and lateral density on the ground for statistical thinning are shown in figure 1. When statistical thinning is used the estimated fluctuations are roughly proportional to $1/\sqrt{n}$ and the number of particles (n) is approximately proportional to t_{cpu}. For comparison of different

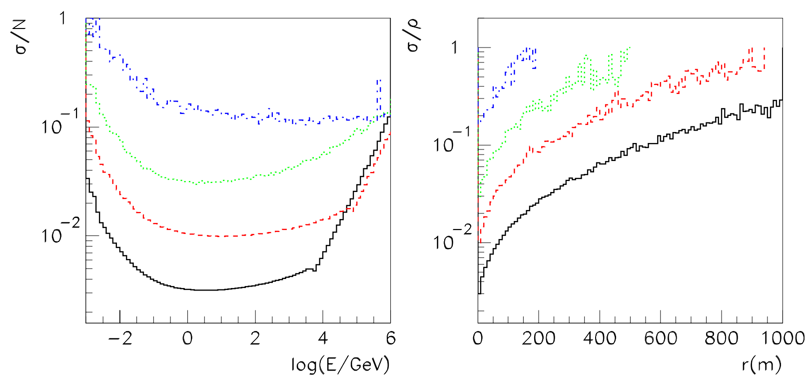

Figure 1: Relative errors (2) in energy distribution (σ/N) and in lateral density (σ/ϱ) on the ground in an 10^{19}eV vertical electron shower calculated with STAT(t_f) thinning. Solid line corresponds to t_f=10^{-7}, dashed to t_f=10^{-6}, doted to t_f=10^{-5} and dash-dot to t_f=10^{-4}. Lateral errors are plotted only in statistically significant regions.

thinning methods it is useful to define the thinning quality parameter Q.

$$Q = \frac{\sigma_0}{\sigma}\sqrt{\frac{t^0_{cpu}}{t_{cpu}}}. \tag{3}$$

Quantities of reference thinning are denoted by index 0. By rewriting equation (3) we obtain:

$$t_{cpu} = \left(\frac{1}{Q}\right)^2 \left(\frac{\sigma_0}{\sigma}\right)^2 t^0_{cpu}. \tag{4}$$

If the fluctuations are kept at the same level, the value of Q^2 directly compares t_{cpu} of both thinning methods. The value Q^2 is referred to as thinning quality. If not stated otherwise STAT(10^{-7}) is used for reference thinning. Thinning qualities for statistical thinning differ by less than 40%.

3 Weight Limit

Particle weights can be limited by w_{max} if particles which would obtain larger weight than w_{max} are unconditionally accepted. When a secondary particle is produced with an energy of a few orders of magnitude smaller than its parents energy, its acceptance factor is very small (1). Suppose its new weight exceeds w_{max}. When weight limit is used such particle is accepted. But this is redundant and just takes t_{cpu}, since the particle most probably would not be accepted even when only statistical thinning was used. To eliminate such cases only particles with acceptance factors greater than some preselected acceptance factor F_w were checked for weight limit. It happens that some weights exceed w_{max}, but the probability for accepting such a particle is only F_w. We denote the use of weight limit with statistical thinning with STAT*WLIM($n_w = E_o/(w_{max} \cdot \mathrm{GeV}), \mathrm{F_w}$). It is more practical to use n_w, since t_{cpu} depends mainly on this quantity.

Distributions of tracked particles obtained using WLIM are shown on figure 2. Weights of low-energy particles exceed w_{max} more easily since they scatter more often and have smaller F_i. Therefore, WLIM mainly increases the number of low-energy particles. The reconstructed energy distributions of different thinnings agree, as expected.

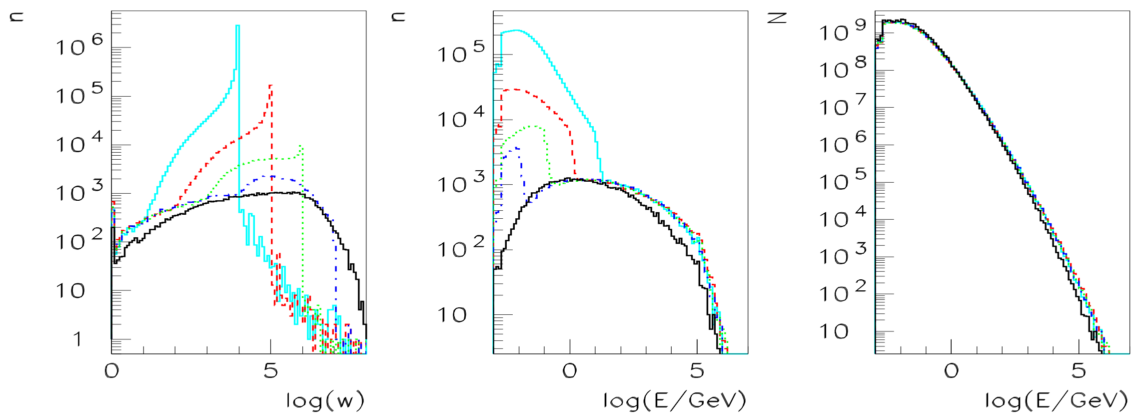

Figure 2: Weight (left) and energy (middle) distributions of tracked particles and reconstructed energy distribution (right) on the ground in an $10^{19}eV$ vertical electron shower using STAT(10^{-5})*WLIM(n_w,10^{-3}) thinning. Different curves namely solid, dashed, dotted and dash-dotted correspond to $n_w = 10^6$, $n_w = 10^5$, $n_w = 10^4$ and $n_w = 10^3$, respectively. Thinning without WLIM is represented with the lowest solid line.

Scattering angles of low-energy particles are larger and their weights are increased more than those of high-energy particles which have smaller scattering angles. Particle weight is therefore a good indicator of the particle's angle and consequently of its lateral position. With weight limit one indirectly improves statistics far from the shower core (fig. 3). Since particle weights are smaller when WLIM is used, the same fluctuations are obtained with smaller number of tracked particles (fig. 3). WLIM thinning characteristics with different values of n_w and $F_w = 10^{-3}$ are shown in table 3. The thinning quality Q^2 rises with higher values of n_w and reaches 27 for $n_w = 10^6$ case.

n_w	t_{cpu}	$M(\frac{dn}{dw})$	$m(\frac{dn}{dw})$	n_l	n_l/n_t	$\frac{\sigma}{\sigma_0}$	Q^2
$\mathbf{10^3}$	314 s	0.14	0.00012	8	9.7×10^{-5}	5.5	2
$\mathbf{10^4}$	421 s	1.6	0.042	37	2.2×10^{-4}	2.5	7
$\mathbf{10^5}$	1120 s	27	3.4	194	3.1×10^{-4}	0.85	22
$\mathbf{10^6}$	7233 s	4500	130	2403	5.5×10^{-4}	0.3	27

Table 1: WLIM($n_w, 10^{-3}$) thinning characteristics obtained when simulating $10^{19}eV$ vertical electron shower on HP C160 workstation. M and m denote maximum and minimum of the weight distribution dn/dw. n_l is the number of particles with higher weights than w_{max} and n_t is the number of all tracked particles.

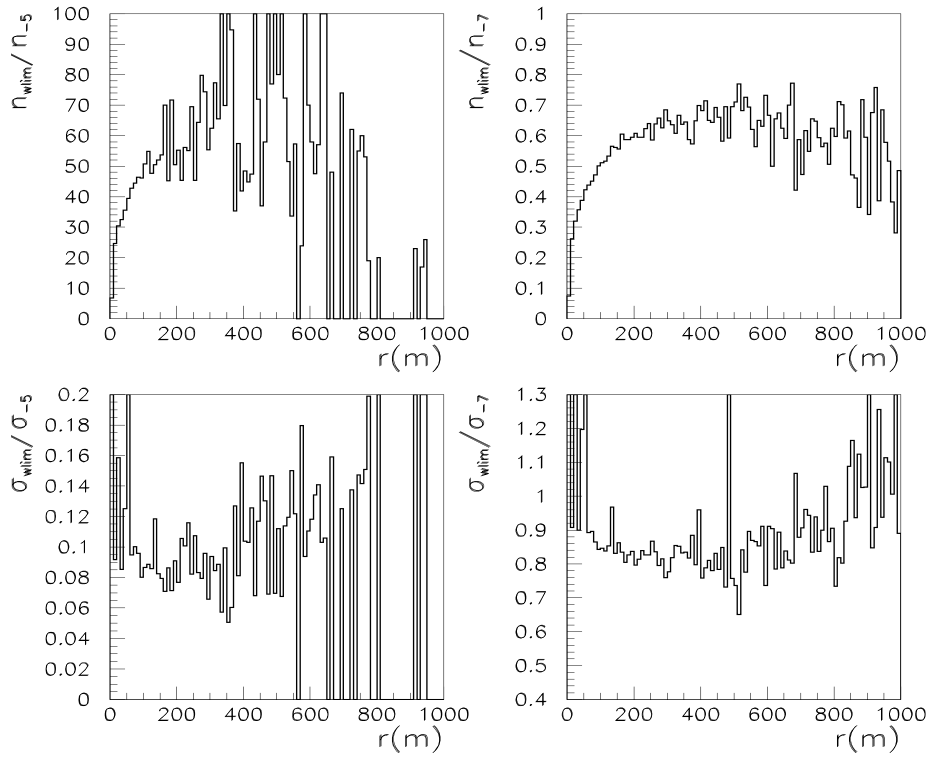

Figure 3: Ratios between number of tracked particles on the ground (above) and ratios between fluctuations in their number (below) for $10^{19}eV$ vertical electron shower calculated with different thinnings. Thinning methods STAT(10^{-5})*WLIM($10^5, 10^{-3}$), STAT(10^{-5}) and STAT(10^{-7}) are represented by indices wlim, -5 and -7, respectively.

Parameter F_w influences the number of particles that undergo the weight limit test and consequently t_{cpu} [3]. For F_w low enough (10^{-2}), average fluctuations are equal within 10% and thinning quality by a factor of 2. At larger values of F_w, larger number of tracked particles have weights above w_{max} and more bins, especially at the shower core, are polluted.

4 Conclusions

Calculations showed that weight limitation superimposed on statistical thinning reduces fluctuations and improves thinning performance. With the new thinning method the same accuracy at distances larger that 100 m is achieved in approximately 20 times shorter computer times as when Hillas thinning is used. The reconstructed results agree with the ones obtained with Hillas thinning [3].

References

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