Particle density distributions of inclined air showers

M. T. Dova, L. N. Epele & A. Mariazzi
Departamento de Física, Universidad Nacional de La Plata
C.C. 67, (1900) La Plata
Argentina

Abstract

Transverse density distributions profiles along the longitudinal shower path for inclined showers are studied. Proton induced air showers are generated using the program AIRES, with primary energy up to 10^{20} eV. Defining appropriate coordinate transformations, the particle density at varying atmospheric depth and zenith angle is analysed.

The energies of the ultrahigh energy cosmic rays are determined with good precision by ground arrays using the particle density at a given distance of the shower core (Hillas, 1971; Dai, 1988; Yoshida, 1995). Extensive air shower particles travel as thin disk which moves, essentially at the speed of light, in the direction of the incoming primary cosmic ray particle. The air shower disk first grows and then contracts as it passes down through the atmosphere. Ground level at these energies is normally beyond the point of maximum development. The particles spread out laterally from the shower axis in a pancake-like shower front. The highest density is near the axis, falling off rapidly with increasing core distance.

The geometry of the shower has been studied by Pryke (1998), suggesting that an inverted cone should be a reasonable first approximation to the surface representing the particle isodensity in a three-dimensional space over time. In this representation, and neglecting a possible asymmetry due to the earth's magnetic field (Pryke,1998;Sciutto 1999a), the isodensity contours for vertical showers are circles at different levels in the atmosphere.

For inclined showers, an inclined cone could be a good approximation of the isodensity surface assuming an homogeneous atmosphere. The intersection of this cone with planes parallel to the ground-plane are increasingly eccentric ellipses.

Particle isodensity contours at $907 gr/cm^2$ are shown in Fig 1 for a vertical shower (1) and an inclined one (2) for the case of muons (a) and electrons (b).

Vertical showers

Proton-induced vertical showers thinned at 10^{-6} were generated with the code AIRES (Sciutto, 1999b) in a primary energies range from 10^{18} to 10^{20} eV. With the data obtained, we evaluated lateral distributions not only at ground level but also at 15 additional observing levels. The symmetry of the shower allows one to analyse these data using polar coordinates in each level. In Fig.1 (right) it is shown the muon isodensity contours in the plane (r,ϕ) and its projection on ϕ. We can clearly see that, as expected, there is no dependence on the polar angle.

Lateral distributions of muons, gammas and electrons at ground and also at the predetermined observing levels were used to determine the annular regions Δr, being r the mean distance to the shower core, corresponding to different values of particle densities $\Delta\rho$. With this information we study the dependence of the isodensity radius with the atmospheric depth z, as measured from the ground. In Fig 2, we plot $r \pm \Delta r$ vs. z for different values of particle density, for muons and electrons. The lines are to guide the eye.

The behavior of r vs. z for e^+e^- and γ seems to indicate a lineal dependence with changes in slope around the position of the depth of maximum (X_{max}), indicating that the shower development can be model by two cones (an inverted one after the X_{max}) as proposed by C. Pryke. Muons do not present any change in the slope. The possible explanation for this is that muons are less attenuated along their path in the atmosphere so they reach the maximum development at or below the ground level.

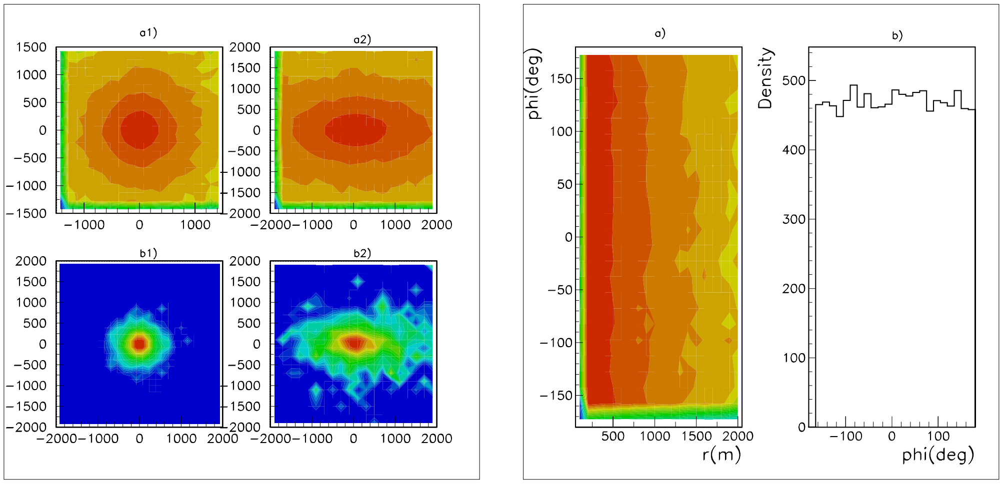

Figure 1: (left)- Particle isodensity contours for vertical and inclined showers(right)-Muon density contours in polar coordinates.

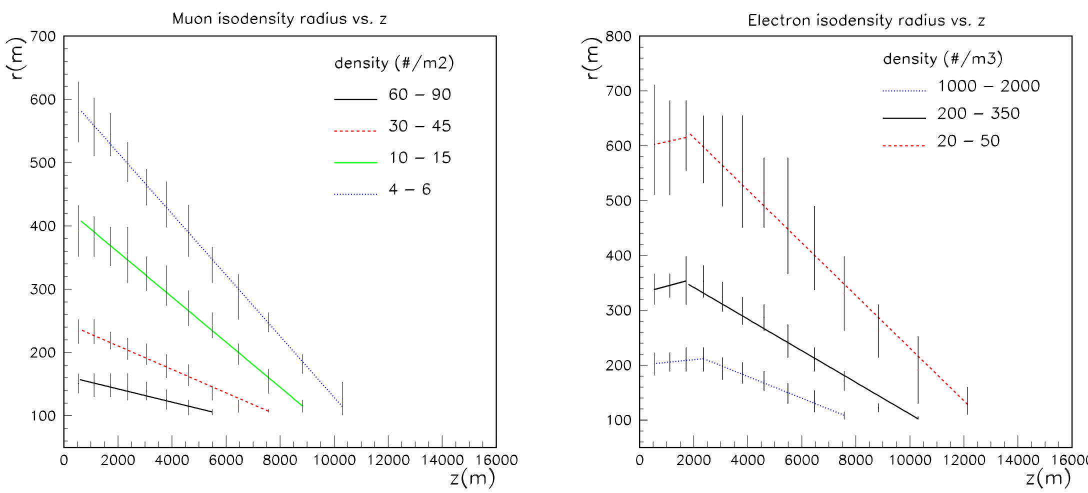

Figure 2: Isodensity radius vs atmospheric depth.

Inclined showers

The natural coordinates for studying inclined showers seem to be the elliptical coordinates:

$$x = c\cosh(h)\cos(t) = cuv \tag{1}$$

$$y = c\sinh(h)\sin(t) = c\sqrt{u^2-1}\,\sqrt{1-v^2} \tag{2}$$

where c is the focal distance:

$$c = \sqrt{a^2-b^2} \tag{3}$$

a and b are the major and minor axis of the ellipse. The condition $u = cte$ corresponds to an ellipse.

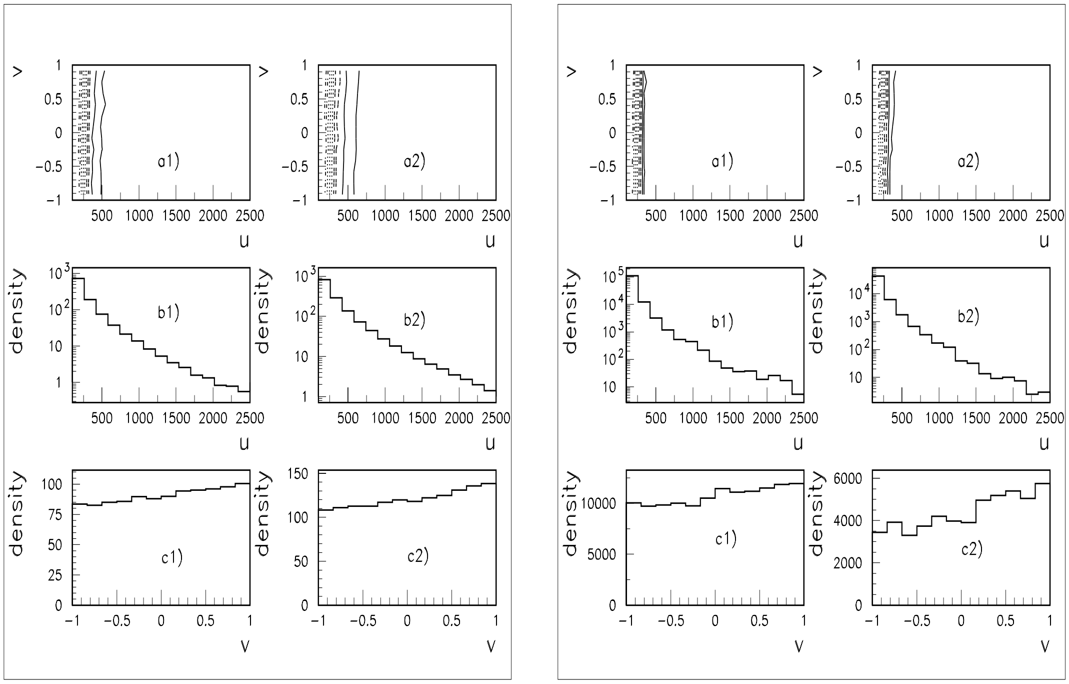

Figure 3: Muon (left) and electron (right) isodensity contours at 323.78 gr/cm^2 and 647.56 gr/cm^2.

The intersection of an inclined cone with a plane parallel to the ground results [1]:

$$(\frac{x - x_0}{a})^2 + (\frac{y}{b})^2 = 1 \tag{5}$$

x and y are the coordinates measured on the plane perpendicular to the shower axis or shower plane. The coordinates used in AIRES are measured in planes parallel to the ground, centered where the shower axis strikes each plane. There is an important difference between the ellipses in the ground plane as compared with the ones projected into the shower plane: in the shower plane x_0 is exactly the focal distance c while this condition is not satisfied in the ground plane. As a result, it is convenient the following re-scaling:

$$x' = \frac{x}{c} = uv + \frac{x_1}{c} = uv + 1 \tag{6}$$

$$y' = \frac{y}{c} = \sqrt{u^2 - 1}\,\sqrt{1 - v^2} \tag{7}$$

In what follows, we use (u, v) coordinates for the study of the structure of inclined showers.

Proton-induced showers thinned at 10^{-6} and 10^{-7} were simulated arriving at a zenith angle of 60° and primary energies range from 10^{18} to 10^{20} eV. As in the vertical case, we have done an analysis including 15 observing levels along the shower path. For each particle type, two-dimension density distributions were generated. Fig 3 shows muon and electron isodensity contours in the (u, v) plane and its projections in u and v. The u coordinate behaves as the r coordinate for vertical showers. If the isodensity contours were ellipses, we do not expect any dependence with the v coordinate ($v = \cos\phi$).

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[1]If L is the distance along the axis between the center of the cone and the plane, θ is the zenith angle and α is the cone angle, then x_0 is given by :

$$x_0 = \frac{L\tan(\theta)\tan^2(\alpha)}{1 - \tan^2(\alpha)\tan^2(\theta)} \tag{4}$$

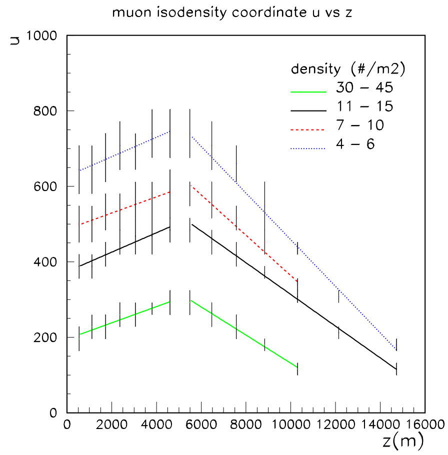

Figure 4: Dependence of u vs. atmospheric depth.

However, as it is shown in Fig 3, there are deviations at 0° and 180° where the shower reaches the maximum and minimum development in each observation level. Clearly, this effect is enhanced at ground level. In the case of electromagnetic particles, the effect is even more important than for muons, due to the fact that muons are less attenuated in the atmosphere. We can conclude that the shape of the shower has deviations from the conic approximation when one considers a non homogeneous atmosphere.

Using the same procedure described for vertical showers and using the u coordinate instead of r, we analyse, neglecting the deviations from ellipses, the space-time structure of the shower. We present here the results corresponding to muons, where the conic approximation is rather good. Fig 4 shows the dependence of u with z, which is lineal in first approximation with changes in slope around the depth of maximum. Notice that for inclined showers, it is expected a different behavior near the X_{max}, because the x_0 changes it sign after reaching the maximum development. It is worth noticing that the asymmetry observed in the isodensity contours increases for observing levels close to the ground. On the other hand, this effect diminishes as the energy of the primary proton increases.

In this work we have studied the structure of the shower along its path through the atmosphere using Monte Carlo techniques. The conic shape is, at this level of analysis, a reasonable approximation. However, we observed several effects that can be explained by the influence of the atmosphere, mainly in those regions where the particles travel different distance due to the shower inclination. Work is continuing on the space-time structure of the shower in order to be able to find a good parametrization of the particle density at ground level valid at any zenith angle.

References

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