Study on Multifractal Structure of the Distribution Fluctuation of Secondary Particles in the Core Region of EAS

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Abstract

The differences of space distributions and time profiles between the γ-ray and proton induced shower particles are studied using Monte Carlo simulation data. The multifractal analysis is performed with the G-moment method for the distribution fluctuation of secondary particles near the core of showers induced by γ-rays and protons. From the spectrum functions of γ and proton events, it is seen that this method can be adopted as a basis for the γ/proton separation. It is shown that the separation of γ and proton can be achieved with a good efficiency in the energy region of 1~10TeV.

1 Introduction:

The search for UHE γ-ray sources is very important for exploring the origin of cosmic ray. But to date, no definite evidence about the existence of the γ-ray sources above 10 TeV has been observed. One possible reason is that the ground based EAS array can not discriminate the γ-ray showers from the hadronic background. If we want to get some meaningful results in the search for gamma ray sources with the EAS array, the future EAS array must have lower threshold energy and could identify the γ-rays and hadrons. The YBJ-ARGO experiment (The ARGO Collaboration, 1997) to be installed at Yangbajing of china could meet the needs.

Study on the distribution fluctuation of secondary particles in the core region of EAS is meaningful to reveal the mechanisms of super high-energy interaction. A lot of analysis has been done on the distribution of secondary particles near the core and the multicore structure events (Faleiro et al. 1997; Lidvansky 1997). J.Kempa suggested to study the distribution fluctuation of secondary particles in the core region of EAS with the multifractal method (Kempa, 1994,1997).

The YBJ-ARGO experiment could give a high granularity space-time picture of the shower front, so detailed study of multifractal structure of particle densities near the core can be performed with unprecedented resolution. In the article, the multifractal analysis is performed with the G-moment method for the distribution fluctuation of secondary particles near the core of showers induced by γ-rays and protons. From the spectrum functions of γ and proton events, it is seen that this method can be adopted as a basis for the γ/proton separation.

Figure 1: The layout of RPC

2 MC Simulation:

The YBJ-ARGO experiment consists of a single layer of RPCs covering an area 5000 m^2 for the first phase of the experiment. Each RPC ($125 \times 280 \mathrm{m}^2$) is equipped with a read-out system made of strips, 6.7cm wide and 62cm long, just as shown in figure 1. Signals from the strips are OR-ed to get the
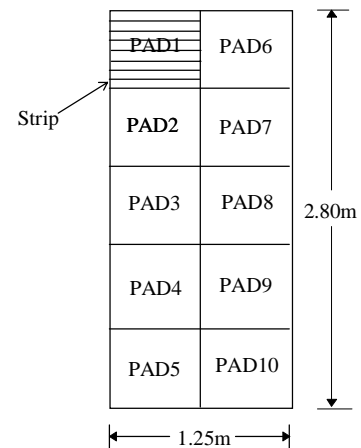
time of the first particle and the number of particles hitting each $56\times62cm^2$ PAD. A lead converter 0.5 cm thick covers uniformly the RPC plane to increase the number of charged particle and to reduce the time spread of the shower front through the shower photon conversion.

Monte Carlo simulations have been performed using the COSMOS code (Kasahara,1995) for the air shower generation and the GEANT code (Brun et al., 1991) for the shower particle detection. Primary particles are injected in vertical direction from the top of the atmosphere, and each secondary particle is followed up to 3MeV or reaching the Yangbajing observation level (a vertical atmospheric depth of 606 g/cm^2). The primary energies are sampled between 1TeV and 10TeV with a power index of -2.7. About 2×10^3 events are generated and used to simulate the detector response with cores locating in the center of the YBJ-ARGO RPC array.

3 Multifractal Analysis:

The secondary particles of EAS are produced by subsequent interactions of the primary particles with the nuclei in the atmospheric, and their distribution around the azimuthal angle have similar characters, that is to say, the distribution of particles in the core region might have multifractal structure.

In the following, the multifractal analysis is performed with the G-moment method for fired strips in the core region ($R_1 < R < R_2$). Figure 2 shows .the distribution of fired strips for a 500GeV γ shower in the YBJ-ARGO experiment.
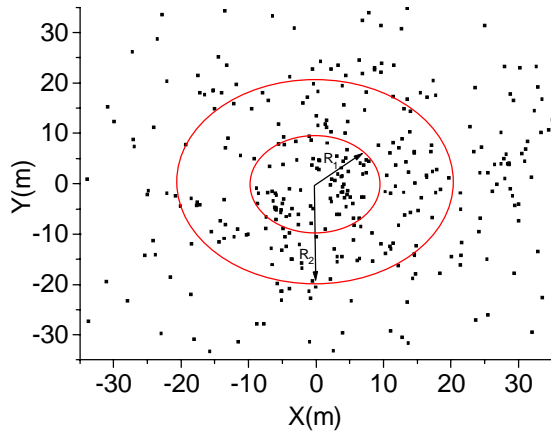

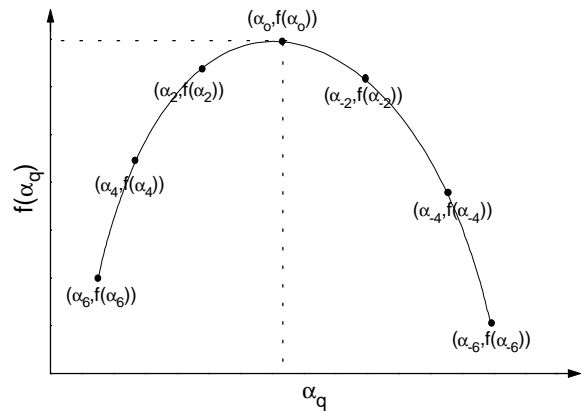**Figure 2:** The distribution of fired strips for a 500GeV γ shower in the YBJ-ARGO experiment

Figure 3: The $f(\alpha_q) \sim \alpha_q$ spectra

The azimuthal angle $\Delta\phi(\Delta\phi = 2\pi)$ is divided into M bins of width $\delta = \Delta\phi / M$, let K_m be the number of fired strips in the m th bin. Since there may be bins which have no fired strips at all, M is defined as number of non-empty bins which constitute a fractal set.

The multifractal moments are defined as follows:

$$G_q = \sum_{m=1}^{M}\left(\frac{K_m}{N}\right)^q = \sum_{m=1}^{M} P_m^q$$

where $P_m = K_m / N$, N is the number of fired strips in the core region of an events, q is a real number, and the summations is carried over non-empty bins only.

If the detected particles exhibit self-similar behavior, then the moments show a power law relation:

$$G_q \approx \delta^{\tau(q)} \approx M^{-\tau(q)}$$

Generalized fractal dimensions are used as characteristics of multifractal, it is defined as (Hwa, 1990):

$$D_q = \tau(q)/(q-1)$$

where D_0 is the fractal dimension, D_1 the information dimension, and D_2 the correlation dimension.

For monofractal structure all D_q have equals values. For multifractal, however, the following hierarchy conditions are fulfilled: $D_q > D_p$ if $p > q$.

The multifractal spectrum function $f(\alpha_q)$ characterized the distribution fluctuation of secondary particles near the core, it is calculated by the Legendre transformation (Halsey, 1986; Paladin 1987):

$$f(\alpha_q) = q\alpha_q + \tau(q) , \quad \text{where} \quad \alpha_q = d\tau(q)/dq$$

The $f(\alpha_q)$ is smooth, concave downwards function with a peak at α_0, just as shown in figure 3. It give a quantitative description of the distribution fluctuation of secondary particles in both dense and sparse regions, corresponding to the $\alpha_q < \alpha_0$ and $\alpha_q > \alpha_0$ regions of $f(\alpha_q)$ (Chiu et al., 1991).

To determine the event-averaged $< f(\alpha_q) >$, it is essential that the procedure guarantees the determination of the event-averaged $< \tau(q) >$. This means that the average should not be performed on G_q, but on $\ln G_q$. In practice, we let $M = 2^\nu$, and calculate

$$< \tau(q) > = -\Delta(< \ln G_q >)/\Delta(\ln M) = -(\ln 2)^{-1} \Delta < \ln G_q >/\Delta\nu$$

where angular brackets denote events averaging, and $\Delta\nu$ is the range of ν in which self-similarity exists, $< \alpha_q >$ and $< f(\alpha_q) >$ could be obtained from the following equations:

$$< \alpha_q > = d < \tau(q) >/dq$$

$$< f(\alpha_q) > = q < \alpha_q > - < \tau(q) >$$

4   Results and Conclusions:

In order to study the multifractal structure, the multifractal analysis is performed with the G-moment method for the distributions of detected particles in the rings between 10m and 20m radii.

The event-averaged $< \ln G_q >$ are calculated for 400 γ events and 400 proton events respectively with the order q from -6 to +6 in step of 0.1. Figure 4 shows that $< \ln G_q >$ is a function of q and ν. The slopes $< \tau(q) >$ of $< \ln G_q >$ dependencies on ν are determined in intervals of ν from 0 to 4.

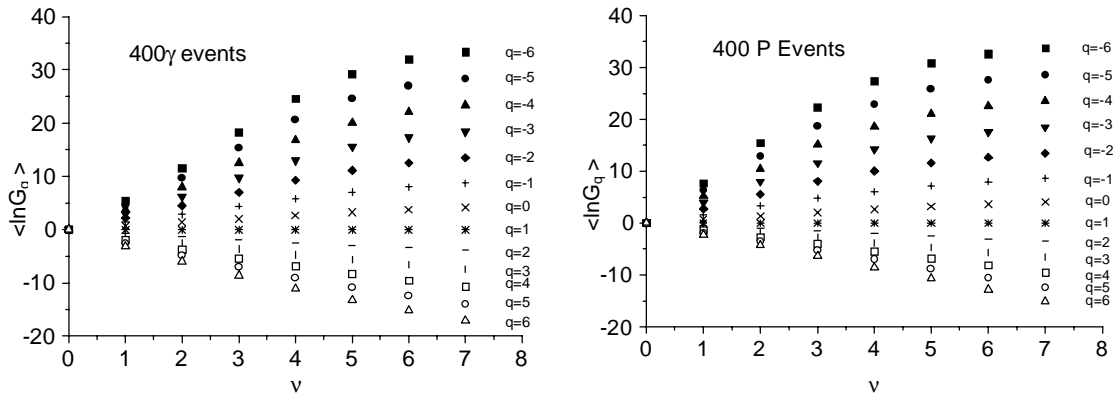**Figure 4:** Plots of the event-averaged $< \ln G_q >$ vs ν for some typical values of q

The calculated values of the fractal dimensions (D_0, D_1, D_2) are presented in Table 1. They satisfy the hierarchy conditions $D_0 > D_1 > D_2$ which indicate that the γ events and proton events have complicated

spatial structure and could be described by multifractals. The discrepancy of fractal dimensions between the γ events and proton events exhibits that they have different interaction mechanisms.

Table 1: Fractal parameters of γ events and proton events

Event type	D_0	D_1	D_2
γ event	0.990	0.933	0.891
proton event	0.962	0.803	0.712

In figure 5 the spectra $< f(\alpha_q) > \sim < \alpha_q >$ is presented for gamma events and proton events. An analysis of the spectra give the chance of identify the γ and protons events. The range in the spectra of γ events is much narrower than in proton events. This is because the distribution fluctuation of the γ induced showers is much smaller than that of the proton induced showers. We could identify the γ events and proton events according to the left-hand side and the right-hand side of the spectra. Let $\omega = \alpha'' - \alpha'$ and $\varepsilon = f(\alpha'') - f(\alpha')$. Figure 6 gives the correlation of ω and ε. If we use the condition of $0 < \omega < 0.78$ and $\varepsilon < 0.26$ as the cut value for the γ events, the identification power for γ events amounts to 80% γ and for proton events around 70%.
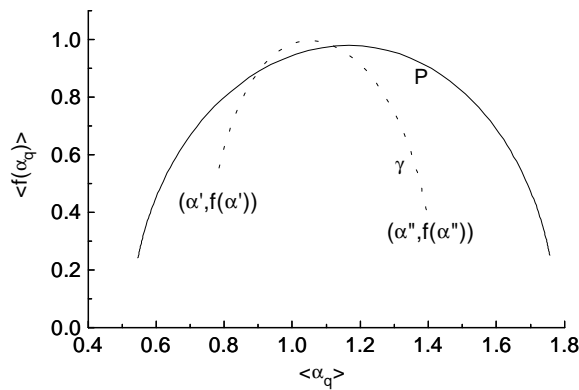

Figure 5: The $< f(\alpha_q) > \sim < \alpha_q >$ spectra
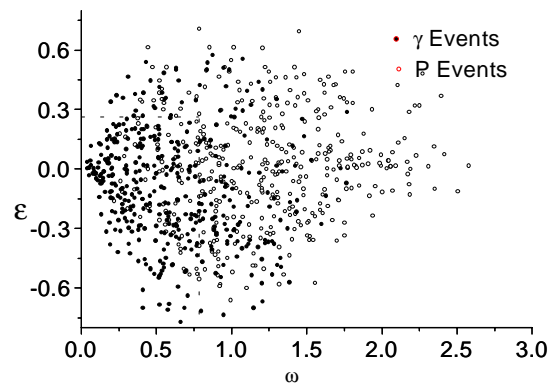
Figure 6: The ω - ε correlation

5 Acknowledgements:

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