

Electron - Proton Scattering as a Probe of Nucleon Structure

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Abstract

The problem of electron-proton scattering is handed over both the elastic and inelastic scattering. Two models are presented in this sense. The first, depends on the multi photon exchange ladder diagram, where the transition matrix is expanded in multi steps form. The second model uses the multi peripheral mechanism developed for the electromagnetic field. It allows the particle production in the inelastic scattering processes. An iterative procedure is applied to Monte Carlo program to reproduce the differential cross section of the reaction. The comparison with the experimental data shows bid fair in most cases.

1 Introduction

Recent experiments for electron-proton (ep) scattering (Arnold, 1986),(H1,1994) (Rock, 1992), (Gomez, 1994), played important role in probing the nucleon structure and to reveal the dynamic mechanism of the electron interactions inside the nucleon bag. In this article we present two pictures for the (ep) scattering. The first is a scattering in view of multi-photon exchange mechanism (MPEM) which is very convenient for the elastic scattering in a wide range of momentum transfer square Q^2 . On the other hand, the (MPM) model concerns the inelastic and consequently relevant to the problem of multi particle production.

2 Multi photon exchange model (MPEM)

It is assumed that elastic scattering of electrons on protons proceeds via multi step process based on the formalism of the Feynman diagrams. The amplitude of two particles interacting via the exchange of photons is determined by the sum of ladder diagrams (Aznauryan,1980). Expanding the transition matrix T of the (ep) scattering in terms of transition ladder diagrams $T^{(n)}$ so that,

$$T = \sum_n c_n T^{(n)} \quad (1)$$

c_n being the coefficients of expansion. $\{T^{(n)}\}$ are the electromagnetic transition matrices of the ladder diagram of order n , and may be expressed as,

$$T^{(n)} = \int \dots \int \prod_{j=1}^{n-1} d\bar{k}_j \frac{V_{j+1,i} V_{i,i-1}}{k - k_j + i\varepsilon} \quad n \geq 2 \quad (2)$$

The factor, $\frac{1}{k - k_j + i\varepsilon}$ stands for the Green's propagator of the virtual intermediate state number j . $V_{j,j-1}$ is the transition probability from the state $j-1$ to the state j , assuming free particle wave function. Consider that the electromagnetic field working at each vertex has the form, $V = -V_o \frac{e^{-\alpha r}}{r}$ to represent the screening effect, so that each internal integration in Eq.(2) has the form;

$$\langle k | V | k' \rangle = -V_o [2\pi^2 (\alpha^2 + |k - k'|^2)] \quad (3)$$

Hence, the transition matrices of the first three steps of the ladder diagram are,

$$T^{(1)} = -V_o [2\pi^2 (\alpha^2 + |k_i - k_f|^2)] \quad (4)$$

k_i and k_f are the momentum of the initial and final states.

$$T^{(2)} = (2\pi^2)^{-1} V_o^2 \int \frac{d\bar{k}_1}{(k_1 - k^2 - i\varepsilon)(\alpha^2 + |k_i - k_1|^2)(\alpha^2 + |k_1 - k_f|^2)} \quad (5)$$

$$T^{(3)} = \frac{V_o^3}{4\pi^4} \int d\bar{k}_1 \int \frac{d\bar{k}_2}{(k - k_2^2 - i\varepsilon)(\alpha^2 + |k_2 - k_1|^2)(\alpha^2 + |k_2 - k_f|^2)} \cdot \frac{1}{(k - k_1^2 - i\varepsilon)(\alpha^2 + |k_i - k_1|^2)} \quad (6)$$

The integrals in Eqs(5,6) are carried out by the Dalitz integrals (Dalitz,1951).

According to the usual formalism for the dynamics of the particle reactions (Hussein,1993), the phase space integral is an integration of the square of the transition matrix $T(\{k_i\})$ over a set of allowed values $\{k_i\}$.

$$I_n(s) = \int \prod_{i=1}^n \frac{d^3 \bar{k}_i}{2E_i} \delta^4(k_a + k_b - \sum_{i=1}^n k_i) |T(k_i)|^2 \quad (9)$$

For the problem under consideration of the elastic scattering, where only two particles are in the final state $n=2$, the reaction cross section for $k_a + k_b \rightarrow k_1 + k_2$ is

$$d\sigma / d\Omega_1^* = \frac{1}{64\pi^2 s} \frac{K_1^*}{K_a^*} |T|^2 \quad (10)$$

3 The multi peripheral model (MPM)

In this model we use a factorizable transition matrix T with the form, $T = \prod_{i=1}^{n-1} T_i$

to describe the n -particle final state reaction. Each particle in the final state is produced at a specific peripheral surface with a transition matrix T_i of

electromagnetic form, $T_i = \frac{1}{\alpha_i + t_i}$ where t_i is the four vector momentum transfer square at the i^{th} peripheral surface. α_i is the electromagnetic peripheral parameter characterizing the surface number i . The set $\{\alpha_i\}$ depend on the multiplicity n and the total energy \sqrt{s} and they are determined to conserve the total energy of the reaction. The phase space integral $I_n(s)$ is then calculated as,

$$I_n(s) = \frac{(2\pi)^{n-1}}{2M_n} \prod_{i=3}^n \frac{1}{4P_a^{(i)}} \int_{\mu_i}^{M_i - m_i} dM_{i-1} \int_{t_{i-1}^-}^{t_{i-1}^+} \frac{1}{(t_{i-1} + \alpha_{i-1})^2} dt_{i-1} \cdot \frac{1}{4P_a^{(2)}} \int_{t_1^-}^{t_1^+} \frac{1}{(t_1 + \alpha_1)^2} dt_1, \quad \mu_i = \sum_{j=1}^i m_j \quad (10)$$

4 Monte Carlo

The Monte Carlo program GENE2 is designed to simulate events according to the MPM. It includes 3-generators. The generator $G_n(s)$ for the multiplicity of particles, $G_M(s,n)$ for the invariant masses produced at the $n-1$ possible peripheral surfaces and finally the dynamic generator $G_t(s,n,M)$ which generates the values of the momentum transfer square t_i and is sensitive to the working field

5 Results and discussion

5-1 The elastic scattering

The transition matrix T of the ep scattering is expanded in terms of the multi step diagrams. The expansion coefficients c_i are determined by fitting with the SLAC experimental data in the range $t \sim 1-3$ (Rock,1992) and $t \sim 3-31$ $(\text{GeV}/c)^2$ (Gomez, 1994). The values of c_i are given in Table (1). Figures (1) and (2) show good agreement between the experimental data and the model prediction. It is clear that the reactions of high momentum transfer needs more terms of ladder diagrams.

5-2 The inelastic scattering.

Following the diagram Fig.(3) which assumes the production of the particles in the form of a jet of effective mass M and total center of mass energy W . A parametric relation is obtained for α_i as a function of the electron lab energy E_e and n as,

$$\begin{aligned} \alpha_i &= (0.226 - 0.0155 E_e + 0.000207 E_e^2) n + 0.4408 \text{Log}(E_e) - 2.2995 \quad n \leq n_c \\ \alpha_i &= (0.00188 E_e - 0.86848) \exp\{-0.0006875 E_e + 0.327276 n\} \quad n > n_c \\ n_c &= 0.4846 E_e + 1.0769 \end{aligned}$$

The critical value n_c corresponds to the peripheral surface at which enough energy is transferred, that is sufficient to make phase transition from the nuclear matter to the quark gluon state. The differential cross section $d\sigma/d\Omega$ as a function of W^2 is compared in Fig.(4) with the experimental data at energies 9.744, 12.505 and

15.730 GeV. Fair agreement is obtained for the first reaction only. The deviation increases as the electron energy increases. This may be due to ignoring the relative motion of the core inside the nucleon target.

References

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Table (1) The branching ratios of multi photon exchanged in ep collisions.

Reaction	1-ph-ex	2-ph-ex	3-ph-ex	4-ph-ex	5-ph-ex
Ref. [3]	1	12	7	-	-
SLAC-E-136	0	0.05	6	10	5

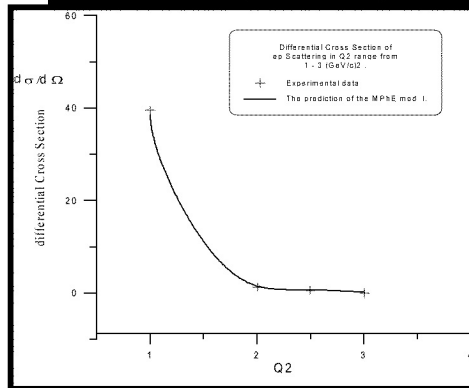


Fig.(1)

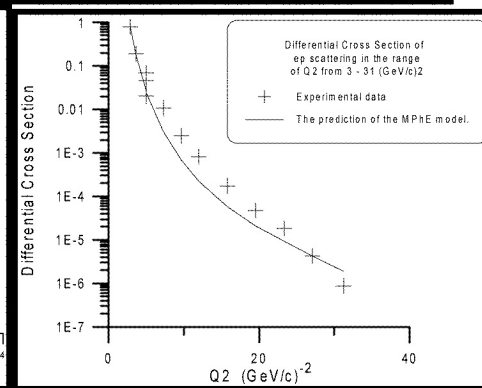


Fig.(2)

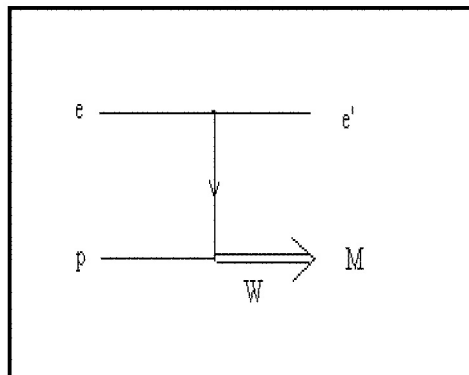


Fig.(3)

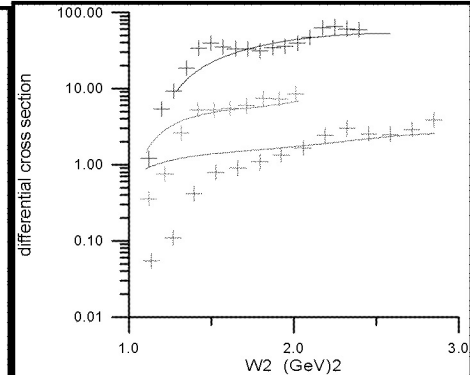


Fig.(4)