ESTIMATIVE OF THE INELASTICITY COEFFICIENT USING THE ENERGY SPECTRUM OF THE HADRONS, ELECTRONS AND PHOTONS

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Abstract

The diffusion equations of hadrons in the atmosphere are integrated using the semigroup theory. The electromagnetic flux is derived from the hadronic fluxes using the approximation A of the electromagnetic cascade theory.

The integral fluxes of these components can be numerically calculated considering the following hypothesis:

a) The hadronic interaction mean free path has a power-law dependence on energy $\lambda_i(E) = \lambda_{0i}E^{-a}$ (i = n or π).

b) Mixed composition of the primary cosmic radiation. The heavy nuclei are included considering the superposition model.

c) The nucleon elasticity distribution $f(\eta)$ has the form $f(\eta) = (1 + \beta)(1 - \eta)^{\beta}$. Diffractive phenomena are also included.

1 Hadron Diffusion Equation:

The diffusion of the nucleonic and mesonic component in the earth atmosphere can be written.

$$\frac{\partial N(t,E)}{\partial t} = -\frac{N(t,E)}{\lambda(E)} + \int_0^1 \frac{N(t,E/\eta)}{\lambda(E/\eta)} f(\eta)\frac{d\eta}{\eta} \tag{1}$$

$$\frac{\partial M(t,E)}{\partial t} = -\frac{M(t,E)}{\lambda_m(E)} + \int_0^1 \frac{M(t,E/x)}{\lambda_m(E/x)} f_{mm}(x)\frac{dx}{x} + \int_0^1 \frac{N(t,E/x)}{\lambda(E/x)} f_{nm}(x)\frac{dx}{x} \tag{2}$$

with the boundary conditions

$$N(0,E) = G(E) \tag{3}$$

and

$$M(0,E) = 0 \tag{4}$$

Where $\lambda(E)$ and $\lambda_m(E)$ are the interaction mean free path of nucleons and mesons in the atmosphere. η is the elasticity coefficient of the nucleons in the atmosphere that is distribuited according to $f(\eta)$. The f_{nm}(x) and f_{mm}(x) are respectively the spectra of the mesons produced in the nucleon-air nuclei and in the meson-air nuclei interactions and x is the Feynman variable.

In order to solve the diffusion equations Eq.1 and Eq.2 we introduce the operators:

$$\widehat{A} = (-1 + \int_0^1 f(\eta)d\eta\widehat{\sigma})1/\lambda(E) \tag{5}$$

$$\widehat{B}_N = (\int_0^1 f_{nm}(x)dx\widehat{\sigma}_n)1/\lambda(E) \tag{6}$$

and

$$\widehat{B}_M = (-1 + \int_0^1 f_{mm}(x) dx \widehat{\sigma}_m) 1/\lambda_m(E) \tag{7}$$

where

$$\widehat{\sigma} F(x,E) = (1/\eta) F(x, E/\eta), \quad \text{for} \quad \eta \geq \eta_{\text{min}} > 0 \tag{8}$$

Provided that $\widehat{A}$ and $\widehat{B}_i$ (i=N or M) are bounded, the solutions of the equations Eq.1 and Eq.2 are:

$$N(t,E) = e^{-t\widehat{A}} N(0,E) \tag{9}$$

and

$$M(t,E) = \int_0^t e^{-(t-z)\widehat{B}_m} \widehat{B}_m N(z,E) dz \tag{10}$$

<u>2 Particular Case:</u>

Taking the cosmic ray primary energy spectrum $N(0,E) = N_0 E^{-(\gamma+1)}$ and $\lambda(E)$ in the form $\lambda_0 E^{-\alpha}$ and $\lambda_m(E) = \omega_m \lambda(E)$, with ω_m a constant that depends on the type of mesons "m" the solutions (9) and (10) take the forms:

$$N(t,E) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{t}{\lambda(E)}\right)^n \prod_{j=1}^{n} (1 - \langle \eta^{\gamma - j_\alpha} \rangle) \tag{11}$$

with

$$\langle \eta^{\gamma - j_\alpha} \rangle = \int_0^1 f(\eta) \eta^{\gamma - j_\alpha} d\eta \tag{12}$$

and

$$M(t,E) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \int_0^t dz \frac{(-1)^k (-1)^n}{k! n!} \left(\frac{t-z}{\lambda_m(E)}\right)^k \left(\frac{z}{\lambda(E)}\right)^n *$$

$$* Z_{nm}(\gamma, \alpha, \eta) I_n(\gamma, \alpha, \eta) I_{mm}(\gamma, \alpha, k, \eta) \frac{N_0 E^{-(\gamma+1)}}{\lambda(E)} \tag{13}$$

where

$$I_{mm}(\gamma, \alpha, k, \eta) = \prod_{i=1}^{k} (1 - \langle x^{\gamma - \alpha(k+\eta+1)} \rangle) \tag{14}$$

with

$$\langle x^{\gamma - \alpha(k+\eta+1)} \rangle = \int_0^1 f_{mm}(x) x^{\gamma - \alpha(k+\eta+1)} dx \tag{15}$$

3 Electromagnetic Component:

The γ-ray production spectrum is given by:

$$P_\gamma(E_\gamma, Z) = 2\int_{E_\gamma}^{\infty} \frac{1}{2} P_{\pi\pm}(E', Z)\frac{dE'}{E'} \tag{16}$$

The differential electromagnetic flux is:

$$F_\gamma(E,t) = \int_0^t dZ \int_E^\infty dE_\gamma P_\gamma(E_\gamma, Z)(e,\gamma)(E_\gamma, E; t-Z) \tag{17}$$

Where the expression $(e,\gamma)(E_\gamma, E; t-Z)$ is the electromagnetic component produced by an incident photon of the primary energy E_γ (approximation A) (Nishimura, 1967).

The integral electromagnetic intensity is:

$$F_{e,\gamma}(\geq E, t) = \int_E^\infty F_\gamma(E,t)dE \tag{18}$$

4 Diffractive-Type Processes:

The total inelastic cross-section is related as

$$\sigma_{in} = \sigma_{ND} + \sigma_{SD} + \sigma_{DD} \approx \sigma_{ND} + \sigma_{SD}$$

Where σ_{ND}, σ_{SD} and σ_{DD} are the cross-sections for non, single and double diffractive processes, respectively.

In the high energy region $1Tev \leq E_{lab} \leq 1000Tev$ the ratio σ_{SD}/σ_{in} is approximately 0.20 (Goulianos,1987).

The mean value of elasticity in a proton-air collision is

$$<\eta>^{p-air} = \frac{\sigma_{SD}^{p-air}}{\sigma_{in}^{p-air}} <\eta>_{SD}^{p-air} + \frac{\sigma_{ND}^{p-air}}{\sigma_{in}^{p-air}} <\eta>_{ND}^{p-air}$$

The criterium for diffractive process that we used is based on Heisenberg Relation and it results in the limitation on the mass of driffractive excitated system, $M^2 \leq 0.1s$ (s is the squared C.M. energy). At high energy, $1-\eta \approx M^2/s$ and the $d\sigma/d(M^2/s) \approx 1/(M^2/s)$.

5 Discussions and Conclusions:

Figures 1 and 2 show the comparison of our solutions with the integral hadron and electromagnetic fluxes, respectively measured at Fuji (650 g/cm^2).

We have solved the diffusion equations of cosmic-ray nucleons and mesons analytically using the semi-group theory and taking into account the rising of the cross-section with the energy in the general form. The solutions are written in the compact expressions (9) and (10). These solutions take simplified forms when we assume a power law dependence on energy for the hadron-air cross-sections and for the primary energy spectrum.

The hadron fluxes at mountain atmospheric depths decrease when we include in our calculations the rising of the cross-section and the decreasing of the average nucleon elasticity.

Through a comparison with the integral hadron and electromagnetic fluxes at mountain altitudes, we have found that the values of $\mathbf{a} = 0.06$ and the average nucleon elasticity coefficient, $<\eta> = 0.37$, give a good consistency although a change of distributions and/or parameters on various elements largely affect the hadronic and electromagnetic fluxes.

The single diffractive contribution in the value of $<\eta^\gamma>$ is about 3% only. The calculated hadronic flux with diffractive contribution is about 9.0% greater than the same flux calculated without diffractive phenomena.

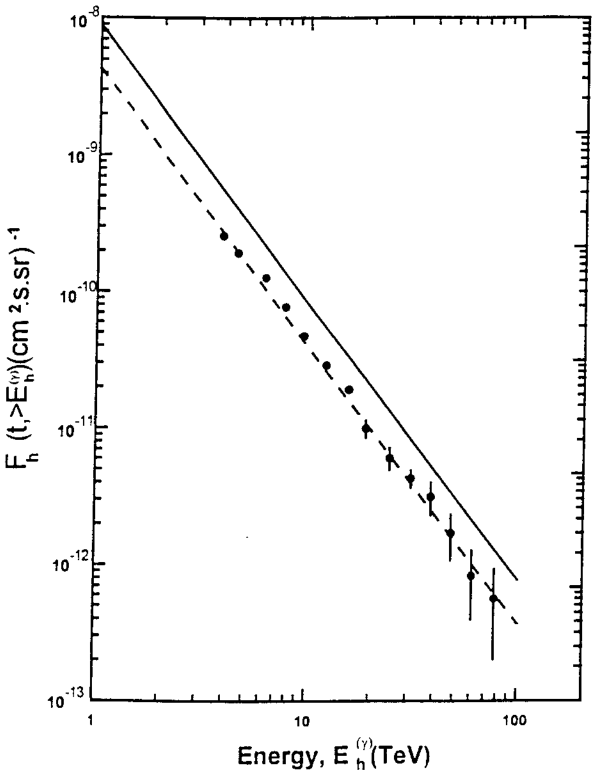

Figure 1: Integral hadron spectrum at 650 g.cm^{-2}. (● from [17]). The full line represents the calculated flux for <K> = 0.5 and the dashed line is the same flux for <K> = 0.63. Both fluxes are for a = 0.06.

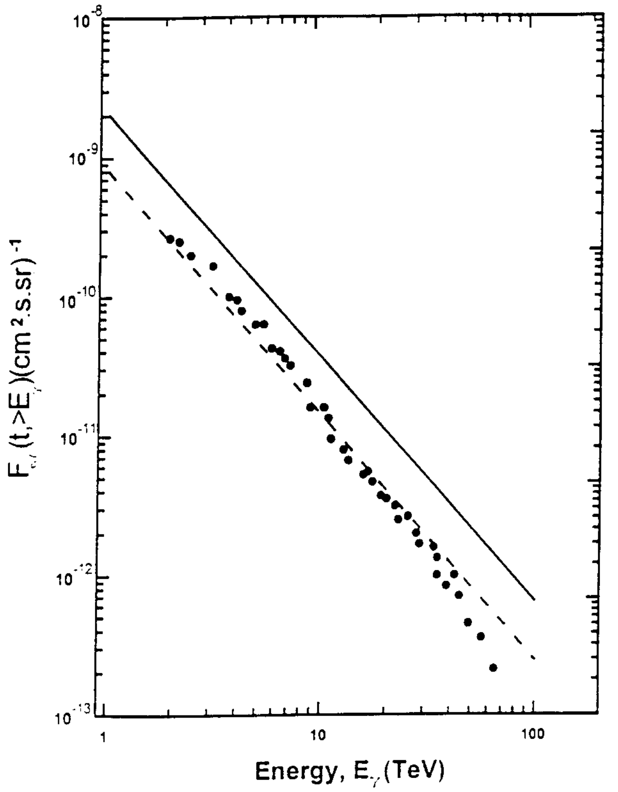

Figure 2: Integral energy spectrum of eletromagnetic showers at 650 g/cm^2. The blue line represents the calculated flux for <K> = 0.5 and the red line is the same flux for <K> = 0.63. Both fluxes are calculated for a = 0.06.

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