Nuclear Interactions in $10^{16} \sim 10^{20}$ eV

A. Ohsawa[1], E. H. Shibuya[2] and M.Tamada[3]

[1] *Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo, 188 Japan.*
[2] *Inst. de Física, Univ. Estadual de Campinas, 13083-970 Campinas, São Paulo, Brasil.*
[3] *Faculty of Science and Technology, Kinki University, Higashi-Osaka, Osaka, 577 Japan.*

1   Introduction:

A primary cosmic ray of high energy, incident upon the top of the atmosphere, initiates nuclear and electromagnetic cascade processes in the atmosphere to produce a number of particles, which is observed as a high energy cosmic-ray phenomenon, such as an extensive air shower, a family observed by the emulsion chamber experiment, a muon bundle, etc.

Study of high energy cosmic-ray phenomena is important and interesting, because they bear the information on the primary cosmic rays and on the high energy nuclear interactions.

The current method of analyzing the high energy cosmic-ray phenomena is to compare the experimental data with the outputs of the simulation. (And the most of the simulation assume a QCD-inspired model for nuclear collisions.) The method is very powerful and straightforward, but we cannot say that it is complete. For example,

(1) The high energy nuclear interaction which is assumed in the simulation is not established well yet. The energy distribution of produced particles in the forward region, which is the most important for the propagation of cosmic rays in the atmosphere, is not easy to be observed by the accelerator experiments and is under discussion. That is, the violation of the Feynman scaling law in the forward region is observed by several sets of experimental data[1] except one[2], while the QCD-inspired models predict the validity of the law[3]. Furthermore, several models all of which are based on QCD theory have different predictions even in the energy region of the accelerators.

(2) The hadron-air collisions which are assumed in the simulation depends on the geometrical model of nucleus structure (*i.e.* Glauber theory) or on the QCD-picture. However, the examination of the model is made only in low energy region of *several* 100 GeV.

(3) Simulation could bring description of the phenomena, which is essential at the first step, but could not bring understanding of the phenomena. That is, when the simulation can describe the phenomenon, it is not easy to show that the processes, assumed in the simulation, are the unique ones. When not, one can say only that the assumed processes are not appropriate ones, but cannot say anything more because the simulation does not make clear how the respective processes, assumed in the simulation, affect the phenomenon.

Hence, on the considerations above, we try to explore an analytical description of the high energy cosmic-ray phenomena. It is important, in particular, in the present situation in which even the qualitative understanding on the phenomena has not been attained yet. Our main goals are,

(i) to establish the relationship between the assumed model of nuclear collisions, ingredients of which are the energy spectrum of the produced particles in the collisions, inelasticity, inelastic collision mean free path, etc., and the high energy cosmic-ray phenomena to be observed.

(ii) to construct a model of the high energy nuclear interactions which describes the cosmic-ray data by the air shower arrays, emulsion chambers, muon detectors, etc., simultaneously and consistently. It may be worthy to mention that there are various sets of cosmic-ray data which are not described well by the current simulations. In particular, emulsion chamber experiments at high mountain altitudes indicate several points.[4]

(iii) to examine the air shower data which claim that their primary energies exceed GZK cut-off

energy. The data are interesting for the nuclear interactions, too, because the change of the nuclear interaction characteristics becomes more distinct in higher energies.

It is mentioned commonly that the disadvantage of the analytic calculation exists in the points that only the average value is available and that the discussion of the fluctuation is not easy. In this sense one should be interested in the relative dependence of the physical quantities by changing the values of the parameters, leaving discussion of the absolute values of them to the future complete calculation by simulation.

2 Cosmic-ray components:

2.1 Elementary processes   We assume the following for the elementary processes.

(1) Mean free path of $N - air$ and $\pi - air$ collisions

$$\lambda_N(E_0) = \lambda_N \left(\frac{E_0}{B}\right)^{-\beta} \qquad \lambda_\pi(E_0) = \lambda_\pi \left(\frac{E_0}{B}\right)^{-\beta}$$

which means that the inelastic cross section increases with the incident energy.

(2) Energy spectrum of produced *charged* particles

$$\varphi(E_0, E)dE = D\left(\frac{E_0}{A}\right)^{\alpha} \left[1 - \left(\frac{E_0}{A'}\right)^{\alpha'} \frac{E}{E_0}\right]^{d} \frac{dE}{E} \qquad (d = 4.0, \ D = \frac{d+1}{3})$$

which converges to one of the empirical formulae of the Feynman scaling law in low energy region ($\sqrt{s} < 63$ GeV) as $\alpha, \alpha' \to 0$. The formula expresses that the particle density increases in the central region and decreases in the forward region, *i.e.* the violation of the Feynman scaling law.

(3) Inelasticity distribution

The above energy spectrum of the produced particles results in the average inelasticity of

$$< K > = \frac{3}{2} \frac{D}{d+1} \left(\frac{E_0}{A}\right)^{\alpha} \left(\frac{E_0}{A'}\right)^{-\alpha'} \equiv K_0 \left(\frac{E_0}{K}\right)^{\kappa}$$

where $\kappa = \alpha - \alpha'$. It means that the inelasticity increases with the incident energy.

Furthermore we assume that the inelasticity is distributed uniformly between $2 < K > -1$ and 1.

$$g(K)dK = \frac{dK}{2(1- < K >)} \qquad \text{with} \qquad K = [2 < k > -1 \ : \ 1]$$

which is valid when $< K > \geq 0.5$.

2.2 Nucleon component   Let the number of nucleons with the energy between E and $E+dE$ at the depth t in the atmosphere be

$$F_N(E, t)dE$$

The diffusion equation of nucleons is

$$\frac{\partial F_N}{\partial t} = -\frac{1}{\lambda_N} \left(\frac{E}{B}\right)^{\beta} F_N(E, t) + \int \delta(E - (1 - K)E_0) \frac{1}{\lambda_N} \left(\frac{E_0}{B}\right)^{\beta} F_N(E_0, t)dE_0 \ g(K)dK$$

We will use $z = t/\lambda_N$ hereafter. Applying Mellin transformation to E (See Appendix.), we have

$$\frac{\partial f_N}{\partial z} = -\frac{1}{B^{\beta}} f_\pi(s + \beta, z) + \frac{1}{B^{\beta}} \int_0^{\infty} E_0^{s+\beta} F_N(E_0, z)dE_0 \frac{(2 - 2 < K >)^s}{s + 1}$$

Because $(2 < K > -1) < 1$, we approximate the inelasticity part in the following way.

$$(2 - 2 < K >)^s = [1 - (2 < K > -1)]^s \simeq 1 - s(2 < K > -1) = s + 1 - 2s < K >$$

The approximation is reasonable when $< K > \sim 0.5$ and when $s \simeq 1$ which is the most important region of s. Then we have

$$\frac{df_N}{dz} = -\frac{2sK_0}{s+1}\frac{1}{B^\beta K^\kappa}f_N(s + \beta + \kappa, z)$$

The solution of the above equation is

$$f_N(s, z) = \frac{1}{2\pi i}\int dq \, E_0^s \left[-\left(\frac{E_0}{B}\right)^\beta\left(\frac{E_0}{K}\right)^\kappa \mu_N(s)z\right]^q \Gamma(-q)b_N(s, q)$$

with

$$a_N(s, q) = a_N(s, 0)[\mu_N(s)]^q \prod_{n=0}^{\infty}\left\{\frac{\mu_N(s + n\sigma)}{\mu_N(s + (q + n)\sigma)}\left(\frac{\mu_N(s + (n+1)\sigma)}{\mu_N(s + n\sigma)}\right)^q\right\}$$

2.3 Pion component

The charged pions are produced by $N - air$ and $\pi - air$ collisions. Consequently the diffusion equation of pions is

$$\frac{\partial F_\pi}{\partial t} = -\frac{1}{\lambda_\pi}\left(\frac{E}{B}\right)^\beta F_\pi(E, t) + (1 - b)\int \delta(E - (1 - K)E_0)g(K)dK \frac{1}{\lambda_\pi}\left(\frac{E_0}{B}\right)^\beta F_\pi(E_0, t)dE_0$$

$$+ \int \varphi(E_0, E)\frac{1}{\lambda_\pi}\left(\frac{E_0}{B}\right)^\beta F_\pi(E_0, t)dE_0 + \int \varphi(E_0, E)\frac{1}{\lambda_N}\left(\frac{E_0}{B}\right)^\beta F_N(E_0, t)dE_0$$

where $\pi - \mu$ decay is neglected.

The first and the second terms on the right-hand side are the loss and the gain of pions due to the collisions, where b is the charge exchange probability of the surviving pions (*i.e.* from the charged state into the neutral). The third and the fourth are the produced charged pions due to $\pi - air$ and $N - air$ collisions, respectively.

Applying Mellin transformation we have

$$\frac{df_\pi}{dz} = -\frac{\xi}{B^\beta}f_\pi(s + \beta, z) + (1 - b)\frac{\xi}{B^\beta}\left\{f_\pi(s + \beta, z) - \frac{2sK_0}{K^\kappa}f_\pi(s + \beta + \kappa, z)\right\}$$

$$+\xi\frac{A'^{\alpha's}}{A^\alpha B^\beta}\phi(s)f_\pi(s^*, z) + \frac{A'^{\alpha's}}{A^\alpha B^\beta}\phi(s)f_N(s^*, z)$$

where $s^* = (1 - \alpha')s + \alpha + \beta$. The derivation above is in the similar way to the case of $f_N(s, z)$.

We expand $f_\pi(s + \beta, z)$, $f_\pi(s + \beta + \kappa, z)$ and $f_\pi(s^*, z)$, because $\alpha, \alpha', \beta, \kappa \ll s$. Then we can solve the equation by iteration.

2.4 Electromagnetic component

The electromagnetic component, electrons and/or photons, are produced through the cascade processes, initiated by the γ-rays which are due to $\pi^0 \to 2\gamma$ decays. And π^0 are produced by $N - air$ and $\pi - air$ collisions.

2.4.1 Production spectrum of π^0's

π^0's are produced by $N - air$ and $\pi - air$ collisions with the energy spectrum of

$$\frac{1}{2}\varphi(E_0, E)dE$$

And one should take into account the charge exchange of the leading pion in $\pi - air$ collisions. Consequently we have the production spectrum of π^0's at the depth t in the atmosphere

$$P_{\pi^0}(E, t)dEdz = dE \int_E^\infty \frac{1}{2}\varphi(E_0, E)\frac{dt}{\lambda_N(E_0)}F_N(E_0, t)dE_0$$

$$+dE \int_E^\infty \left[\frac{1}{2}\varphi(E_0, E) + b\delta(E - (1 - K)E_0)\right]\frac{dt}{\lambda_\pi(E_0)}F_\pi(E_0, t)dE_0$$

2.4.2   Production spectrum of γ-rays   The π^0's, produced at the depth t, decay into γ-rays immediately. Hence we have

$$P_\gamma(E, z)dE = dE \int_E^\infty \frac{2}{E_0} P_{\pi^0}(E_0, z)dE_0$$

2.4.3   Electromagnetic component   An emulsion chamber detects electrons and photons of high energy, while an air shower array does the electrons with energy ≥ 0. Hence the cascade function, to be used, is different depending on the detector concerned. It is expressed in the following way generally.

$$\pi(E_0, E, t) = \frac{1}{(2\pi i)2} \int dsdq \left(\frac{E_0}{E}\right)^s \frac{1}{E} \left(\frac{\epsilon}{E}\right)^q \mathcal{M}_{1,0}(s, q)e^{\lambda_1(s)t}$$

which is a cascade function for the primary particle of a γ-ray under Approximation B. All the notations are familiar in the cascade theory.[5] Hence the number of electrons and/or photons is given by

$$F_{e/\gamma}(E, t) = \int_0^z dz' \int_E^\infty dE_0 P_\gamma(E_0, z')\pi(E_0, E, z - z')$$

3   Discussions

One can see above that the numbers of the nucleon, pion and electromagnetic components are given analytically. The numerical evaluation will be presented.

4   Appendix

Mellin transform of $F(E)$ with respect to E is defined as

$$f(s) = \int_0^\infty E^s dE \ F(E)$$

The inverse-Mellin transform is given by

$$F(E) = \frac{1}{2\pi i} \int \frac{ds}{E^{s+1}} \ f(s)$$

where the integral is the complex one and the path is from $s_0 - i\infty$ to $s_0 + i\infty$, *i.e.* parallel to the imaginary axis with all the singularities in the left-hand side.

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