HE.1.1.10

Cross section calculations of Li^6 -nucleus at high energies

O. M. Osman
Physics Department , Faculty of Science ,Cairo University
Giza, Egypt

Abstract

Total inelastic nucleus-nucleus cross sections for Li^6 -nucleus interactions are calculated on the basis of the Glauber scattering theory. The Li^6 as a projectile is taken to be a loosely bound system possessing the cluster structure α +d. The scattering amplitude for α N and dN are used as inputs in the calculations .Results of these calculations show fine agreement with the experimental data of Li^6- emulsion interactions at 4.5 GeV/c per nucleon.

1 Introduction:

The success of the Glauber multiple scattering theory[1] in describing hadron-nucleus collisions at high energies has naturally led to its generalization to nucleus-nucleus scattering[2, 3]. The total and inelastic cross sections for the collisions between nuclei are calculated[4] by expanding the optical phase shift function in an infinite series including the effects of nuclear correlations. The first term in the series , which is the standard optical limit, is accepted only in the limit where the nuclear masses are large and the total NN cross section, σ , is weak[4,5] . In the real world ,however σ is not sufficiently small and thus the nucleons will not scatter independently but rather they will cast shadows on each other. Therefore, some higher order terms in the series should be considered in order to account for such eclipse processes[5, 6].

Earlier, it was shown by Inopin et al.[7], that the α -particle model gives a good account of the elastic and inelastic scattering of electrons from C^{12} and O^{16}. The experimental data ware interpreted by assuming that C^{12} and O^{16} nuclei consist of three and four α -particle clusters (scatterers), respectively[8-10].

In the present work, the total inelastic cross section , σ_{in}^{LiEm} for the interactions of Li^6 with the emulsion (Em) target nuclei is calculated on the basis of the glauber theory. It is known that ,Em is a heterogeneous target consisting of various nuclei ranging from hydrogen up to silver with the different percentages by weight depending on the emulsion composition. The Em constituents fall into three main groups of nuclei, namely the hydrogen H, the light (CNO), and the heavy (AgBr) group. In addition the cluster configuration (α + d) for the Li^6 projectile is considered. The internal structure of the deuterons and the α - particles are first accounted for by calculating the α N and dN scattering amplitudes in the Glauber model. They are then used as inputs in the calculations. The essential aim of this work is to verify that the (α + d) configuration is capable of reproducing the experimental data at high energies as it has been the case at low and intermediate energies. These are preliminary results which has to be supported and confirmed by more decisive parameters other than just the total inelastic cross section, $\sigma_{in}^{A_P A_T}$, for the interaction of a projectile of mass number A_P and a target of mass number A_T.

2 Total inelastic cross section

The total inelastic cross section at high energies is expressed in the form:

$$\sigma_{in}^{A_P A_T} = \int d^2b[1 - \left| e^{i\chi_{op}(\mathbf{b})} \right|^2] \qquad 1$$

Where $\chi_{op}(\mathbf{b})$ is the optical phase shift of the nucleus-nucleus scattering. We assume the collisions between the Li^6 and the different Em nuclei to proceed via elementary α N and dN collisions .Hence the nucleus-

nucleus phase shift is taken to be the sum of all the elementary α N and dN phase shifts.With the definition :

$$\Gamma_{xN}(\mathbf{b}) = 1 - e^{i\chi_{xN}(\mathbf{b})} \qquad \alpha, d \qquad x \tag{2}$$

For the elementary N and dN profile functions , one obtains:

$$e^{i\chi_{op}(\mathbf{b})} = \int d\mathbf{r}_1 ... d\mathbf{r}_{A_T} d\mathbf{r}'_\alpha d\mathbf{r}'_d \left|\psi_{Li}(\mathbf{r}'_\alpha, \mathbf{r}'_d)\right|^2 \left|\psi_T(\mathbf{r}_1, ..., \mathbf{r}_{A_T})\right|^2 \prod_{N=1}^{A_T} \{[1 - \Gamma_{\alpha N}(\mathbf{b} - \mathbf{s}_N + \mathbf{s}'_\alpha)][1 - \Gamma_{dN}(\mathbf{b} - \mathbf{s}_N + \mathbf{s}'_d)]\} \tag{3}$$

Where ψ_{Li} and ψ_T are the ground-state wavefunctions of the Li and the target, respectively. They depend on the coordinates of their constituents ($\mathbf{r}$ `,$\mathbf{r}_d$`) and {$\mathbf{r}$} whose projections on the impact parameter plane are denoted by ($\mathbf{s}$ ` ,$\mathbf{s}_d$`) and {$\mathbf{s}$},respectively.Considering the α -d configuration for Li^6 and changing the coordinates to the intrinsic ones ($\mathbf{r}$, $\mathbf{r}_d$) relative to the center of mass of Li ,the two-particle density of Li for single Gaussian forms ,reduces to:

$$\left|\psi_{Li}(\mathbf{r}_\alpha, \mathbf{r}_d)\right|^2 = \left(\frac{1}{\pi a_\alpha^{\,2}}\right)^{3/2} \left(\frac{1}{\pi a_d^{\,2}}\right)^{3/2} e^{-r_\alpha^{\,2}/a_\alpha^{\,2}} e^{-r_d^{\,3}/a_d^{\,2}} \tag{4}$$

Where the cm constrains is accounted for.The parameters a and a_d are related to the critical value $\langle r_c^2\rangle^{1/2}$of the relative distance between the two considered clusters by:

$$a_\alpha^{\,2} = \frac{2}{3}\left(\frac{m_d}{m_{Li}}\right)^2 \langle r_C^{\,2}\rangle$$
$$a_d^{\,2} = \frac{2}{3}\left(\frac{m_\alpha}{m_{Li}}\right)^2 \langle r_C^{\,2}\rangle \tag{5}$$

The critical distance is thought of as the distance between the clusters below which the clusters lose their identities inside the Li nucleus.It is calculated according to the relation[11] :

$$m_{Li}\langle r_{Li}^{\,2}\rangle = m_\alpha \langle r_\alpha^{\,2}\rangle + m_d \langle r_d^{\,2}\rangle + \frac{m_\alpha m_d}{m_{Li}} \langle r_C^{\,2}\rangle \tag{6}$$

Where $\langle r_{Li}^2\rangle^{1/2}$,$\langle r^{\,2}\rangle^{1/2}$, and $\langle r_d^2\rangle^{1/2}$ are the respective rms radii and m_{Li} ,m and m_d are the respective masses. The target nucleus ,on the other hand, is described as usual as a system of completely uncorrelated nucleons ;

$$\left|\psi_T(\mathbf{r}_1, ..., \mathbf{r}_{A_T})\right|^2 = \prod_{i=1}^{A_T} \rho_T(\mathbf{r}_i) \tag{7}$$

Where ρ_T is the single-particle density. For light nuclei ρ_T is taken as Gaussian:

$$\rho_T(\mathbf{r}) = \left(\frac{1}{\pi a_T^{\,2}}\right)^{3/2} e^{-r^2/a_T^{\,2}} \tag{8}$$

With

$$a_T^{\,2} = (2/3) < r_T^{\,2} > /(1 - 1/A_T) \tag{8`}$$

The factor (1-1/A_T) is introduced to correct for the cm constraint[5]. For heavy targets, the Fermi density is used:

$$\rho_T(\mathbf{r}) = \frac{\rho_o}{1 + e^{(r-c)/d}} \tag{9}$$

With $c = 1.19A_T^{1/3} - 1.61A_T^{-1/3}$ fm and $d = 0.54$ fm

The profile functions Γ_N and Γ_{dN} of the two clusters, in eq. (3), are derived from:

$$\Gamma_{xN}(\mathbf{b},\{\mathbf{t}_i\}) = 1 - \prod_{i=1}^{A_x}\left[1-\gamma(\mathbf{b}-\mathbf{t}_i)\right] \qquad (10)$$

Where $\mathbf{t}_i$ is the projection of the internal coordinate of the ith nucleon inside the cluster x of mass number A_x and $\gamma(\mathbf{b})$ is the NN profile function which has the usual parameterization:

$$\gamma((\mathbf{b}) = \frac{\sigma(1-i\alpha)}{4\pi a} e^{-b^2/2a} \qquad (11)$$

σ being the total NN cross section ,a is the slope parameter of th e NN profile function and $\alpha = \mathrm{Re}\, f_{NN}(0) / \mathrm{Im}\, f_{NN}(0)$,where $f_{NN}(0)$ is the forward NN scattering amplitude. Treating all the nucleons inside both clusters as identical particles, one obtains the following closed expression for α N and dN profiles :

$$\Gamma_{xN}(\mathbf{b}) = \sum_{n=1}^{A_x} \frac{a_n A_n}{2B_n} e^{-b^2/4B_n} \qquad (12)$$

Where the parameters a_n ,A_n and B_n are listed in table (1) for both clusters.

Table (1): The parameters a_n ,A_n ,and B_n of the considered clusters, eq.(12)

x	A_x	a_1	a_2	a_3	a_4	A_n	B_n
α	4	1/2	-3/8	1/6	1/32	$(2\beta)^n/(8a+3R^2)^n$	$(8a+3R^2\alpha)/16$ n
d	2	1/2	-1/8	-	-	$\beta^n/(4a+R_d^2)^n$	$(4a+R_d^2)/8n$

Where R_x^2 is the radius parameter of each cluster x ,given by eq. (8`).The calculations are thus carried out by using eqs. (4) ,(7) and (12) into eq.(3). One gets:

$$e^{i\chi_{op}(\mathbf{b})} = \{1 - \int d^2\mathbf{s}\rho_T(\mathbf{s})\int d^2\mathbf{s}_\alpha d^2\mathbf{s}_d \left|\psi_{Li}(\mathbf{r}_\alpha,\mathbf{r}_d)\right|^2 [\Gamma_{\alpha N}(\mathbf{b}-\mathbf{s}+\mathbf{s}_\alpha) + \Gamma_{dN}(\mathbf{b}-\mathbf{s}+\mathbf{s}_d) - \Gamma_{\alpha N}(\mathbf{b}-\mathbf{s}+\mathbf{s}_\alpha)\Gamma_{dN}(\mathbf{b}-\mathbf{s}+\mathbf{s}_d)]\}^{A_T} \qquad (13)$$

Eq. (13) is used to calculate the total inelastic cross sections for the interactions of Li, in the α -d congifuration, with nuclei at high energies.

3 Comparison with experiment:

The dependence of $\sigma_{in}^{A_P A_T}$ on A_P and A_T is shown to have ,at high incident energies, a geometrical parameterization of the form[12] :

$$\sigma_{in}^{A_P A_T} = r_o^2 (A_P^{1/3} + A_T^{1/3} - \delta)^2 \qquad (14)$$

Where r_o is an interaction radius parameter and represents the nuclear transparency of nuclei at high energies.Fig.(1) shows $(\sigma_{in}^{A_P A_T})^{1/2}$, as calculated according to the present work, as function of $A_P^{1/3}+A_T^{1/3}$ for Li^6 on H,C,N,O,Br, and Ag nuclei. The fig. reveals the linear dependence (14) with the parameters r_o= 1.60 fm and δ = 1.15 which are significantly larger than the previous parameterizations ,namely r_o 1.48 fm.

Table (2) summarizes all the calculations carried out in the present work as compared with $\sigma_{in}^{A_P A_T}$ of Li-Em data at 4.5 GeV/c per nucleon [13,14] .The input data used in the calculations are[15] ; σ_{NN} = 4.27 mb ,a = 0.25 fm^2 , and α = -0.28 . The radius parameters are calculated from the rms nuclear radii obtained from the electron scattering experiments[16] . The calculated cross sections for different targets are used with the weight percentages of the Em constituents[13] to determine the cross

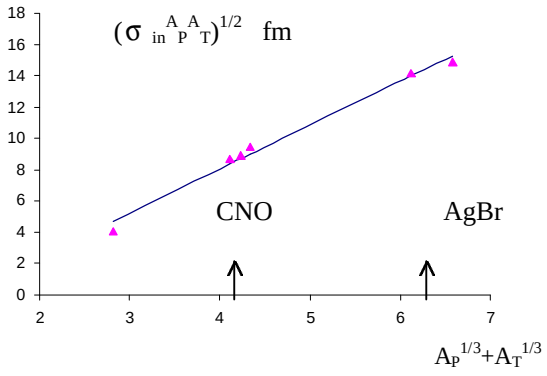

Fig. 1: The square root of total inelastic cross sections for the interactions of Li with nuclei. The sraight line is the best fit to the calculated values (▲) the present work, eq. (3).

sections for Li-CNO , Li-AgBr ,and the overall Li-Em interactions. The present calculations are listed in column 2 . Calculations considering Li^6 as independent six-nucleon system are also carried out according to[6] . Values of $\sigma_{in}^{A_P A_T}$ with only the first (optical) term are listed in column 3 ,and those with the eclipse correction terms (up to the fourth term) are in column 4. Calculations based on Karol s soft sphere model[17] are also displayed in column 5. Although the calculations of $\sigma_{in}^{A_P A_T}$, based on the present approach give good agreement with the experimental values , however it seems that the total cross section is not a decisive probe for testing the cluster structure of the nuclei. Alternatively, the calculations of projectile fragment cross sections may be a good test for such a structure.

Table (2) :Total inelastic cross section values (in mb) for Li-Em at 4.5 GeV/c per nucleon Comparisons with different approaches.

Target	+d cluster structure present work	Six-nucleon System with optical limit[6]	Six-nucleon system with eclipse corrections[6]	Soft sphere model by Karol[17]	Experiment[13]
H	164 22	184 21	185 33	171.22	145 5
CNO	812 45	885 31	818 71	842 02	798 27
Em	907 57	971 65	905 65	925 10	885 30
AgBr	2106 69	2221 59	2060 41	2039 74	2048 71

Acknowledgement

The author would like to express his deep gratitude to Prof. Dr. M.K.Hegab for encouragement, continuous help and useful discussions during the preparation of this work.

References

1.Glauber ,R.J., Lectures in Theoretical Physics , edited by W.E.Britten and L.C.Durham (Wiley Interscience, New York ,1959)Vol 1 ,P 315

2.Franco , V.,1968 , Phys.Rev.175,1376

3.Czyz ,W. & Maximon, L.C. ,1969 ,Ann.Phys.(N.Y.)52,59

4.Franco , V. & Varma , G.K., 1978 ,Phys.Rev.C18,349

5.Faldt ,G. , et al. ,1974 , Ann.Phys. 82, 326

6.Hegab , M.K. , et al. ,1990 ,Z. Phys. A326 ,345

7.Inopin , E.V. & Tishehenko , B.I. ,1960 ,Sov.Phys. JETP, 11,840

8.Ahmad , I. ,1971 ,Phys.Lett. B36,301

9.Wenying , R.& Youyan , L. ,1994 ,Z.Phys.A349 ,155

10.Matero, J., 1995, Z.Phys.A351, 29

11.Matsuse, T., et al., 1982 , Phys. Rev. C26, 2338

12.Bradt , H.L. & Peters , B. 1950 , Phys. Rev. 77,54 ; Jakobson , B. & Kullberg , P. ,1976 ,Phys. Scr.13,327

13.Sherif ,M.M. ,et al., 1992,Egypt J.Phys. 23,55;Towfik, A.M ,1993, M.Sc.Disseration , Assuit University , Sohag, Egypt

14.Jilany , M.A. , 1994 , Nucl.Phys. A579, 627

15.Giamell,G.,Nal 1972 , Proceeding of the 16th International Conference on High Energy Physsics , Batavia

16.Barette , R.C. & Jackson , D.F. , Clarendon 1977 , Nuclear sizes and structure. Oxford.

17.Karol , P.J. ,1975 ,Phys. Rev. C11,1203 \\