Holographic Hydrodynamics and Applications to RHIC and LHC

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Outline

Holography

Black Holes and Hydrodynamics

Relativistic Hydrodynamics

Fluid/Gravity Correspondence

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Outlook

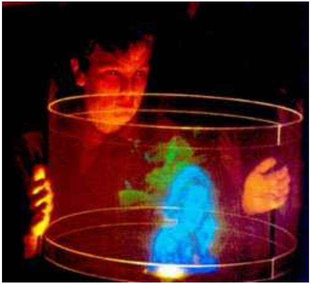
Collaborators and References

With B. Keren-Zur, C. Eling, G. Falkovich, I. Fouxon, X. Liu, Y. Neiman, M. Rabinovich.

PRL **101** (2008) 261602, JHEP **0903**, 120 (2009), PLB **680**, 496 (2009), JHEP **1002**, 069 (2010), PLB **694**, 261 (2010), , JFM **644**, 465 (2010), JHEP **1006**, 006 (2010), CMP **52**, 43 (2011), JHEP **1103**, 006 (2011), JHEP **1012**, 086 (2010), JHEP **1103**, 023 (2011), JHEP **1102**, 070 (2011), arXiv:1103.1657 and to appear.

The Holographic Principle

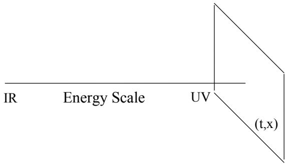
- The degrees of freedom of a quantum theory of gravity in a volume of space V are encoded on a boundary of the region A.
- In the AdS/CFT correspondence the quantum theory of gravity is string theory and the theory on the boundary is a local quantum gauge field theory (QCD-like).



The Energy Scale

- The additional coordinate is an energy scale, thus we unify in the gravity description space-time and energy.

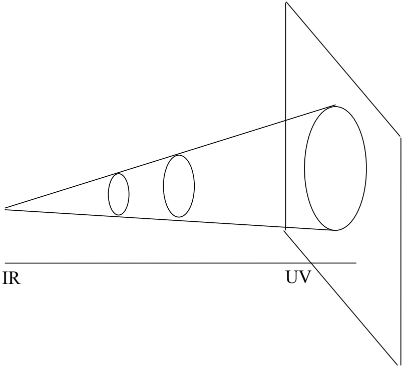
$$ds^2 = dr^2 + a(r)^2(-dt^2 + dx^i dx^i)$$



UV-IR

- The size of the object (for instance instanton) in the boundary gauge field theory is determined by its location in the energy scale direction:

$$E \sim r$$



A Strong-Weak Duality

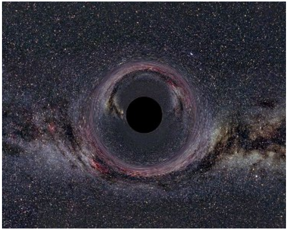
- In the limit of large gauge coupling $\lambda \to \infty$ we get a weakly coupled gravity description: small curvature.
- In the limit of small gauge coupling $\lambda \to 0$ we get a weakly coupled field theory description.

$\lambda << 1$
Perturbative
Gauge Theory

$\lambda >> 1$
Gravity

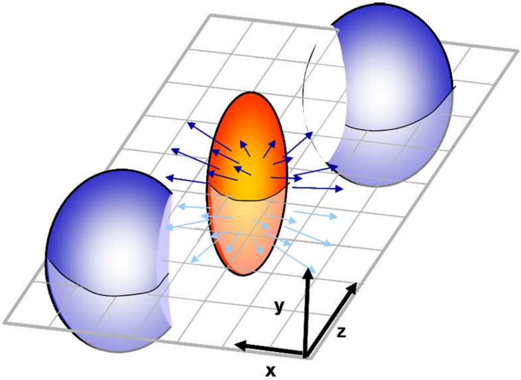
Fluid Dynamics and Gravity

- The AdS/CFT correspondence relates fluid dynamics to black hole dynamics: hydrodynamic regime of the correspondence.



Heavy-Ion Collision

- The QCD plasma produced at RHIC and LHC seems to exhibit a strong coupling dynamics $\alpha_s(T_{RHIC}) \sim O(1)$.



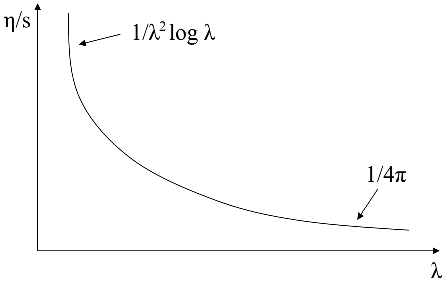
Heavy-Ion Collision

- Nonperturbative methods that can be used to study real time dynamics are largely unavailable.
- Lattice QCD methods are inherently Euclidean.
- The AdS/CFT correspondence provides a real-time nonperturbative framework.
- We can use the strong coupling properties of $N = 4$ gauge theory plasma as a reference point for describing the strongly coupled QCD plasma.

The Shear Viscosity to Entropy Density Ratio $\frac{\eta}{s}$

$$T_{\mu\nu} = T_{\mu\nu}^{Ideal} + \eta T_{\mu\nu}^{Viscous}$$
$$\partial^{\mu} T_{\mu\nu} = 0$$

- In Einstein's gravity: $\frac{\eta}{s} = \frac{1}{4\pi}$



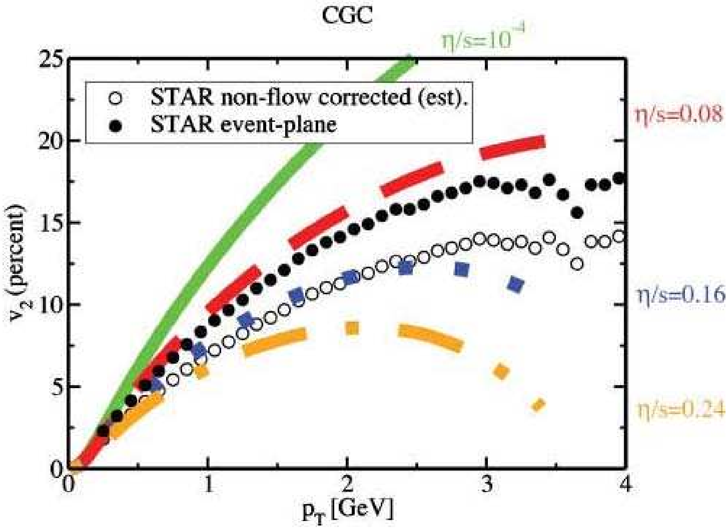
Elliptic Flow

- Hydrodynamic simulations at low shear viscosity to entropy ratio are consistent with RHIC Data.
- The elliptic flow parameter is the second Fourier coefficient $v_2 = \langle Cos(2\phi) \rangle$ of the azimuthal momentum distribution $dN/d\phi$

$$\frac{dN}{d\phi} \sim 1 + 2v_2 Cos(2\phi)$$

Elliptic Flow

- Luzum, Romatschke:2008



The Hydrodynamic Modes

- The effective degrees of freedom are charge densities $\rho(\vec{x}, t)$, and the hydrodynamics equations are conservation laws
$$\partial_t \rho + \partial_i j^i = 0 \tag{1}$$
- The constitutive relations express j^i in terms of ρ and its derivatives. For instance $j^i = -D\partial^i \rho$, and we get
$$\partial_t \rho - D\partial_i \partial^i \rho = 0 \tag{2}$$
- Writing $\rho(\vec{k}, t) = \int d^3 x e^{-i\vec{k}\cdot\vec{x}\rho(\vec{x},t)}$ we have
$$\rho(\vec{k}, t) = e^{-Dk^2 t} \rho(\vec{k}, t = 0) \tag{3}$$
This is the characteristic behaviour of hydrodynamic mode. It has a life $\tau(k) = \frac{1}{Dk^2}$ which is infinite in the limit $k \to 0$.

Relativistic Hydrodybnamics

- Hydrodynamics applies under the condition that the correlation length of the fluid l_{cor} is much smaller than the characteristic scale L of variations of the macroscopic fields

$$Kn \equiv l_{cor}/L \ll 1 \tag{4}$$

- Hydrodynamics equations are conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^\mu_a = 0 \tag{5}$$

Relativistic Hydrodynamics

- The equations of relativistic hydrodynamics are determined by the constitutive relation expressing $T^{\mu\nu}$ and J_a^{μ} in terms of the energy density $\epsilon(x)$, the pressure $p(x)$, the charge densities $\rho_a(x)$ and the four-velocity field $u^{\mu}(x)$ satisfying $u_{\mu}u^{\mu} = -1$.
- The constitutive relation has the form of a series in the small parameter $Kn \ll 1$,

$$T^{\mu\nu}(x) = \sum_{l=0}^{\infty} T^{\mu\nu}_{(l)}(x), \quad T^{\mu\nu}_{(l)} \sim (Kn)^l \tag{6}$$

Ideal Hydrodybnamics

- Keeping only the first term in the series gives ideal hydrodynamics and the stress-energy tensor reads

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu + p\left(\eta^{\mu\nu} + u^\mu u^\nu\right) \tag{7}$$

  The equation of state $\epsilon(p)$ is an additional input.
- CFT hydrodynamics: $T^\mu_\mu = 0$, $\epsilon = 3p \sim T^4$ and the stress-energy tensor reads

$$T^{\mu\nu}_{(0)} = T^4[\eta^{\mu\nu} + 4u^\mu u^\nu] \tag{8}$$

Viscous Hydrodynamics

- The dissipative hydrodynamics is obtained by keeping $l = 1$ term in the series. The stress-energy tensor reads (Landau Frame)

$$T^{\mu\nu}_{(1)} = -\eta\sigma^{\mu\nu} - \xi(\partial_\alpha u^\alpha)\left(\eta^{\mu\nu} + u^\mu u^\nu\right) \tag{9}$$

where

$$\sigma^{\mu\nu} = (\partial^\mu u^\nu + \partial^\nu u^\mu + u^\nu u^\rho \partial_\rho u^\mu + u^\mu u^\rho \partial_\rho u^\nu - \frac{2}{3}\partial_\alpha u^\alpha [\eta^{\mu\nu} + u^\mu u^\nu] \tag{10}$$

- The dissipative hydrodynamics of a CFT is determined by only one kinetic coefficient - the shear viscosity η

$$T^{\mu\nu}_{(1)} = -\eta\sigma^{\mu\nu} \tag{11}$$

Gravitational Dual Description

- Consider the five-dimensional Einstein equations with negative cosmological constant

$$R_{mn} + 4g_{mn} = 0, \qquad R = -20 \tag{12}$$

- These equations have a particular "thermal equilibrium" solution - the boosted black brane

$$ds^2 = -2u_\mu dx^\mu dr - r^2 f[br] u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \tag{13}$$

where

$$f(r) = 1 - \frac{1}{r^4}, \quad P^{\mu\nu} = u^\mu u^\nu + \eta^{\mu\nu}$$

and the constant $T = 1/\pi b$ is the temperature.

Gravitational Description

- One looks for a solution of the Einstein equation by the method of variation of constants (Bhattacharya et.al.: 2007)

$$g_{mn} = (g_0)_{mn} + \delta g_{mn} \tag{14}$$

$$(g_0)_{mn} dy^m dy^n = -2u_\mu(x^\alpha) dx^\mu dr - r^2 f[b(x^\alpha) r] u_\mu(x^\alpha) u_\nu(x^\alpha) dx^\mu dx^\nu + r^2 P_{\mu\nu}(x^\alpha) dx^\mu dx^\nu \tag{15}$$

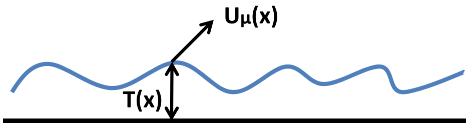
- $y = (x^\mu, r)$. As in the Boltzmann equation, the condition of constructibility of the series solution produces equations for $u^\mu(x^\alpha)$ and $T(x^\alpha) = 1/\pi b(x^\alpha)$. The series for g_{mn} is the series in the Knudsen number of the boundary CFT hydrodynamics.

Horizon Dynamics

- The way the black brane horizon geometry encodes the boundary fluid dynamics is reminiscent of the *Membrane Paradigm* in classical general relativity, according to which any black hole has a fictitious fluid living on its horizon.
- The dynamics of the event horizon of a black brane in asymptotically AdS space-time (Gauss-Codazzi equations) is described by the Navier-Stokes equations.
- Recently the two approaches are related by an RG flow (Bredberg et. al: 2010).
- The apparent horizon emerges as a useful object that captures time dependent properties of the plasma.

The Horizon Geometry

- Thermalization in field theory is the process of black hole creation in gravity.
- Hydrodynamics is the deformation of the black hole geometry.



The Bulk Viscosity ζ

- (Eling,Y.O.:2011) The bulk viscosity is captured by the horizon dynamics.
- Consider $(d+1)$-dimensional gravitational backgrounds holographically describing thermal states in strongly coupled d-dimensional field theories. The $(d+1)$-dimensional gravitational action reads

$$I = \frac{1}{16\pi}\int \sqrt{-g}d^{d+1}x\left(\mathcal{R} - \frac{1}{2}\sum_i(\partial\phi_i)^2 - V(\phi_i)\right) + I_{gauge}\ , \tag{16}$$

$V(\phi_i)$ represents the potential for the scalar fields and I_{gauge} represents the action of gauge fields (abelian or non-abelian) A^a_μ.

The Bulk Viscosity ζ

- The null horizon focusing equation obtained by projecting the field equations of (16) on the horizon is equivalent via the fluid/gravity correspondence to the entropy balance law of the fluid.
- Using this equation we derived

$$\frac{\zeta}{\eta} = \sum_i \left(s\frac{d\phi_i^H}{ds} + \rho^a \frac{d\phi_i^H}{d\rho^a} \right)^2$$

η is the shear viscosity, s is the entropy density, ρ^a are the charges associated with the gauge fields A_μ^a, and ϕ_i^H are the values of the scalar fields on the horizon.

- $\frac{\zeta}{\eta}$ rises in the vicinity of T_c.

Anomalies

- The hydrodynamics description exhibits an interesting effect when a global symmetry current of the microscopic theory is anomalous

$$D_\mu J^\mu_\alpha = \frac{1}{8} C_{\alpha\beta\gamma}\epsilon^{\mu\nu\rho\sigma}F^\beta_{\mu\nu}F^\gamma_{\rho\sigma}$$

- The form of an anomalous symmetry current is modified in the hydrodynamic description by a term proportional to the vorticity of the fluid

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho$$

- This has been first discovered in the context of the the fluid/gravity correspondence (Erdmenger et.al, Banerjee et.al.:2008).

Anomalies

- The global symmetry current takes the form

$$j_a^\mu = \rho_a u^\mu + \sigma_a{}^b \left(E_b^\mu - T P^{\mu\nu} D_\nu \frac{\mu_b}{T} \right) + \xi_a \omega^\mu + \xi_{ab}^{(B)} B^{b\mu}$$

where ρ_a, T, μ_a and σ_a^b are the charge densities, temperature, chemical potentials and the conductivities of the medium.

Anomalies

- The anomaly coefficients are (Son,Surowka; Neiman, Y.O)

$$
\begin{aligned}
\xi_a &= C_{abc}\mu^b\mu^c + 2\beta_a T^2 - \frac{2n_a}{\epsilon + p}\left(\frac{1}{3}C_{bcd}\mu^b\mu^c\mu^d + 2\beta_b\mu^b T^2\right) \\
\xi_{ab}^{(B)} &= C_{abc}\mu^c - \frac{n_a}{\epsilon + p}\left(\frac{1}{2}C_{bcd}\mu^c\mu^d + \beta_b T^2\right)
\end{aligned}
$$

C_{abc} is the coefficient of the triangle anomaly of the currents j_a^μ,j_b^μ and j_c^μ.

QCD Axial Anomaly in HIC

- In very energetic collisions the hot dense QCD matter can go through a phase transition to a deconfined phase described by a fluid-like collective motion of quarks and gluons. We consider a deconfined QCD fluid phase, with three light flavors and chiral symmetry restoration.
- We consider the experimental implications of the axial current triangle diagram anomaly in a hydrodynamic description of high density QCD.

Chiral Magnetic and Vortical Effects

- (Kharzeev, Son :2011) Chiral magnetic effect: charge separation. Chiral vortical effect: baryon number separation.

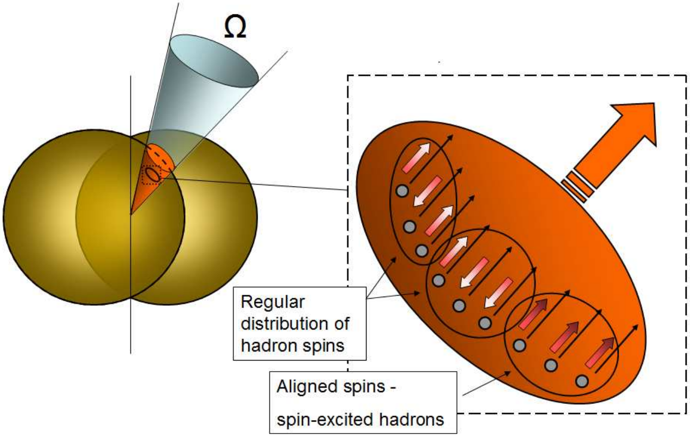
$$\vec{J} = \frac{N_c \mu_5}{2\pi^2}\left(tr(VAQ)\vec{B} + tr(VAB)2\mu_B\vec{\omega}\right)$$

  The electromagnetic current corresponds to V = Q and the baryon current to V = B.
- The ratio between the baryon asymmetry and the charge asymmetry increases when the center of mass energy is lowered.

QCD Axial Anomaly in HIC

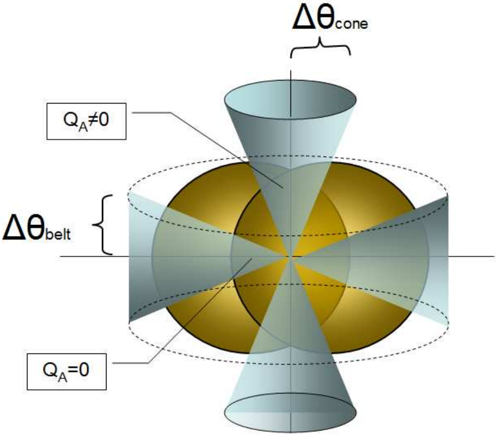
- (Keren-Zur,Y.O.:2010): The basic idea is that the the axial charge density, in a locally uniform flow of massless fermions, is a measure of the alignment between the fermion spins.
- When the QCD fluid freezes out and the quarks bind to form hadrons, aligned spins result in spin-excited hadrons. The ratio between spin-excited and low spin hadron production and its angular distribution may therefore be used as a measurement of the axial charge distribution.
- We predict the qualitative angular distribution and centrality dependence of the axial charge.

QCD Axial Anomaly in HIC

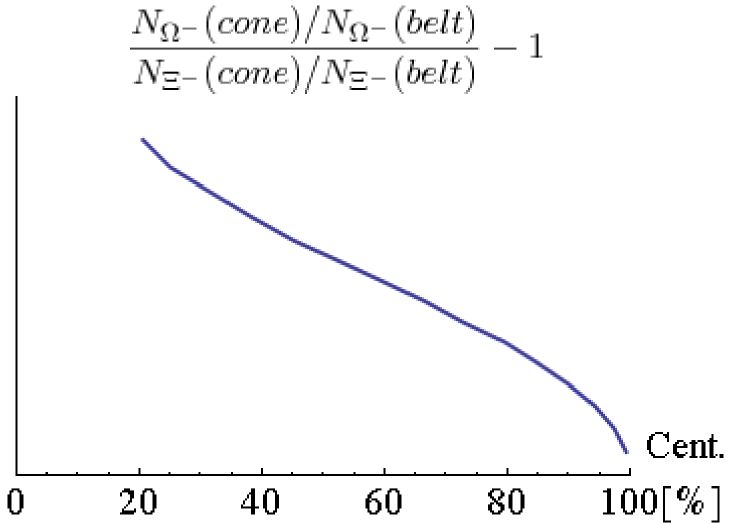


QCD Axial Anomaly in HIC

- Our main proposal is that for off-central collisions we expect enhancement of Ω^- production along the rotation axis of the collision

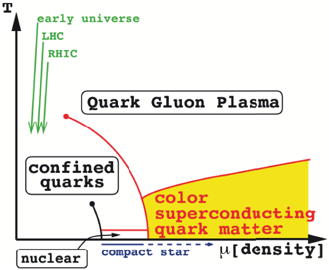


QCD Axial Anomaly in HIC



Superfluid Hydrodynamics - CFL Phase

- See Yasha Neiman's talk.



- (Neiman,Y.O.) A Chiral Electric Effect:

$$J^{a\mu} = C^{cgd}(\delta^a_c - \frac{n^a \mu_c}{\epsilon + p})(\delta^b_d - \frac{n^b \mu_d}{\epsilon + p})\epsilon^{\mu\nu\rho\sigma} u_\nu \xi_{g\rho}(E_{b\sigma} - T\partial_\sigma \frac{\mu_b}{T})$$

Relativistic Turbulence

- The Reynolds number is

$$\mathcal{R}_e \sim \frac{TL}{\eta/s} \tag{17}$$

When $\mathcal{R}_e$ is large we expect turbulence.

- Is there a universal structure in relativistic turbulence?

Relativistic Turbulence

- In relativistic hydrodynamics we consider the hydrodynamics equation with a random force term

$$\partial^{\nu} T_{\mu\nu} = f_{\mu} \tag{18}$$

  and derive the exact scaling relation (Fouxon,Y.O.:2010)

$$\langle T_{0j}(0,t) T_{ij}(r,t) \rangle = \epsilon r_i$$

  where $d \langle T_{0j}(0,t) f_j(0,t) \rangle \equiv \epsilon$.

- In charged hydrodynamics with a conserved symmetry current J^{μ}:

$$\langle J_0(0,t) J_i(r,t) \rangle = \epsilon r_i$$

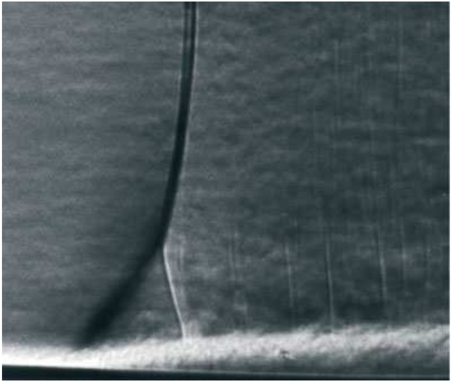
RHIC and LHC

- Does turbulence show up in RHIC and LHC?
- For gold collisions at RHIC, the characteristic scale L is the radius of a gold nucleus $L \sim 6$ Fermi, the temperature is the QCD scale $T \sim 200$ MeV, and $\frac{\eta}{s} \sim \frac{1}{4\pi}$ is a characteristic value of strongly coupled gauge theories.
- With these R_e is too small for an experimental realization of the steady state relativistic turbulence.

Singularities in Hydrodynamics

- (Y.O.,M.Rabinovich:2010) The basic question concerning singularities in the hydrodynamic description, is whether starting with appropriate initial conditions, where the velocity vector field and its derivatives are bounded, can the system evolve such that it will exhibit within a finite time a blowup of the derivatives of the vector field.
- Physically, such singularities if present, indicate a breakdown of the effective hydrodynamic description at long distances and imply that some new degrees of freedom are required.

Shock Wave



Singularities in Gravity

- The issue of hydrodynamic singularities has an analogue in gravity. Given an appropriate Cauchy data, will the evolving space-time geometry exhibit a naked singularity, i.e. a blowup of curvature invariants and the energy density of matter fields at a point not covered by a horizon.
- Penrose type inequality:

$$E \geq \frac{d-1}{16\pi}(4S)^{\frac{d}{d-1}}$$

  E is the energy and S is the entropy.
- Initial conditions violating the inequality lead to a creation of singularities.

Outlook

- The heavy ion dynamics forces us to deal with strong coupling. The holographic description provides a weakly coupled description of this dynamics using gravitational variables. This leads to a large number of applications and surprising discoveries.