Dark energy from a theorist viewpoint

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AstroParticule et Cosmologie, Paris

Blois 2011, June 3

A major challenge for fundamental physics in this century

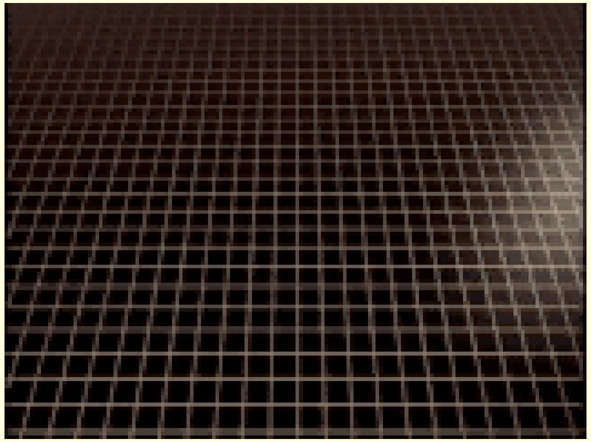
XXth century legacy :

- General relativity

A single equation, Einstein's equation, successfully predicts tiny deviations from classical physics and describes the universe at large as well as its evolution.

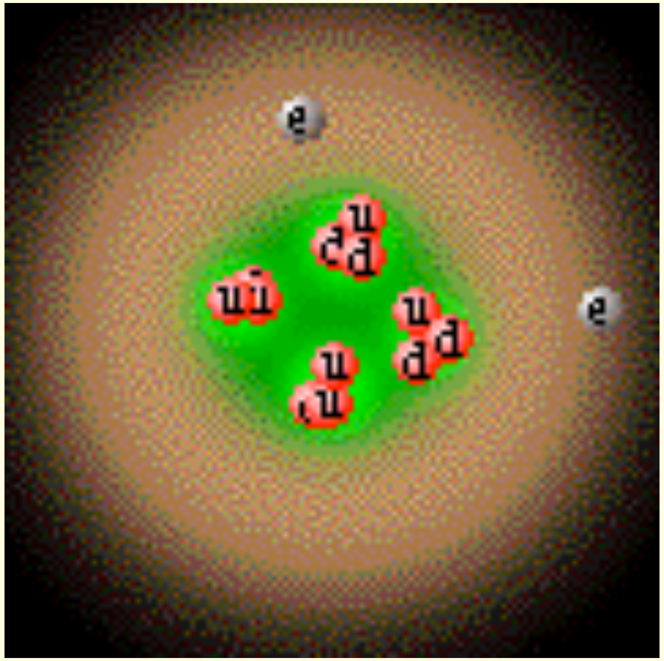
$$R_{\mu\nu} - (R/2)\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}$$

geometry matter



- Quantum theory

Describes nature at the level of the molecule, the atom, the nucleus, the nucleons, the quarks and the electrons .



The two theories are colliding in several well-known instances:

Three possible manifestations:

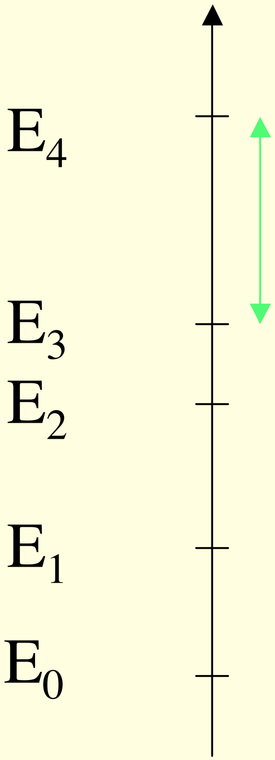
- vacuum energy

- issue of Lorentz violations

e.g. non-commutativity $[x_\mu, x_\nu] = \Theta_{\mu\nu}$ *associated with quantum gravity*

- equivalence principle : $m_{inertial} = m_{gravitational}$

Vacuum energy



Classically, only differences of energy can be measured (e.g. Casimir effect).

ground state = vacuum

The absolute energy E_0 cannot be measured experimentally

No longer true in a gravitational context!

Einstein equations: $R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G\, T_{\mu\nu}$

geometry energy

Hence geometry may provide a way to measure absolute energies i.e. vacuum energy:

$$R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G\, T_{\mu\nu} + 8\pi G < T_{\mu\nu} >$$ vacuum energy

similar to the cosmological term introduced by Einstein :

$$R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G\, T_{\mu\nu} + \lambda\, g_{\mu\nu}$$

$$\lambda \equiv \ell_\Lambda^{-2}$$

Can we measure λ i.e. the associated scale ℓ_Λ?

Einstein equations → Friedmann equation

$$H = \dot{a}/a$$

$$H^2 = (8 \pi G \rho + \lambda) /3 - k/a^2$$

$c = 1$

$$\rho_\Lambda = \lambda / 8 \pi G \qquad \rho_c = 3 H_0^2 / 8 \pi G$$

$$\Omega_\Lambda \equiv \rho_\Lambda / \rho_c = (H_0^{-1} / \ell_\Lambda)^2 / 3 < 0.7 \Rightarrow \ell_\Lambda > H_0^{-1} \sim 10^{26} \text{ m}$$

A very natural value for an astrophysicist !

Introduce $\hbar$

Planck length $\ell_P = \sqrt{8\pi G_N \hbar / c^3} = 8.1 \times 10^{-35}$m

Planck	$\ell_P \sim 10^{-34}$ m	$m_P \sim 10^{27}$ eV
λ	$\ell_\Lambda \sim 10^{26}$ m	$m_\Lambda \sim 10^{-33}$eV

$$mc^2 \equiv \frac{\hbar c}{l} = \frac{200 \text{ MeV.fm}}{\ell}$$

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~~A very natural value for an astrophysicist~~ !

A very unnatural value for a Universe which presumably started in a quantum state!

Indeed, if we compute the vacuum energy, we obtain typically

$$\rho_\Lambda \sim m_P{}^4 \sim 10^{120}\, \rho_{observed}$$

Either there is an exact cancellation mechanism of the vacuum energy,
But then what is dark energy?

Or dark energy is vacuum energy.

In the latter case, note

$$\rho_\Lambda = \frac{1}{8\pi G \ell_\Lambda^2} = \frac{\hbar}{\ell_P^2 \ell_\Lambda^2} \equiv \frac{\hbar}{\ell_{DE}^4}$$

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$$\ell_{DE} = \sqrt{\ell_P \ell_\Lambda}$$

UV cut-off

IR cut-off

Cosmological constant problem : where the two ends meet…

In the latter case, note

$$\rho_\Lambda = \frac{1}{8\pi G \ell_\Lambda^2} = \frac{\hbar}{\ell_P^2 \ell_\Lambda^2} \equiv \frac{\hbar}{\ell_{DE}^4}$$

$$m_{DE} = \sqrt{m_P m_\Lambda}$$

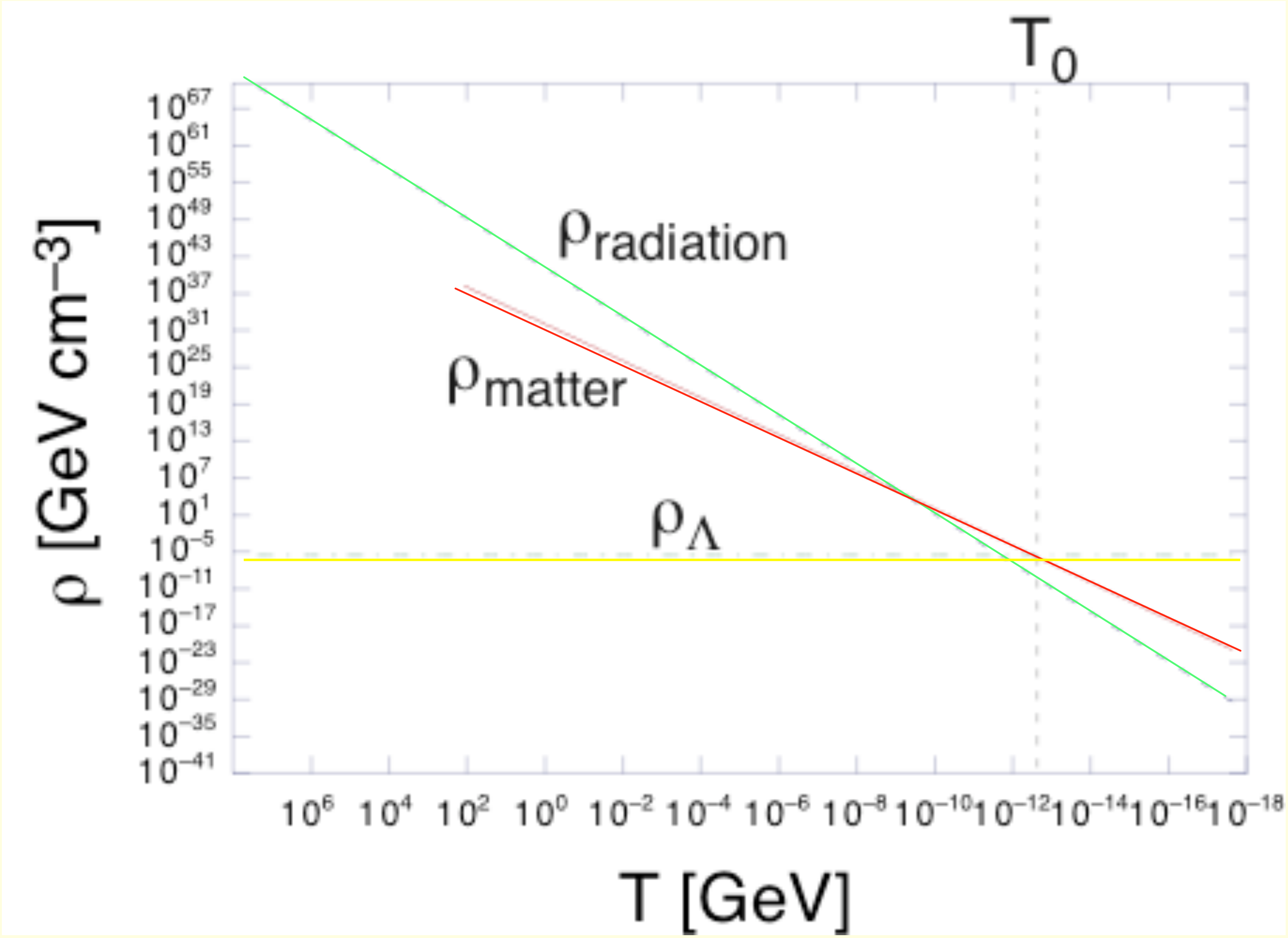
10^{-3} eV

UV cut-off

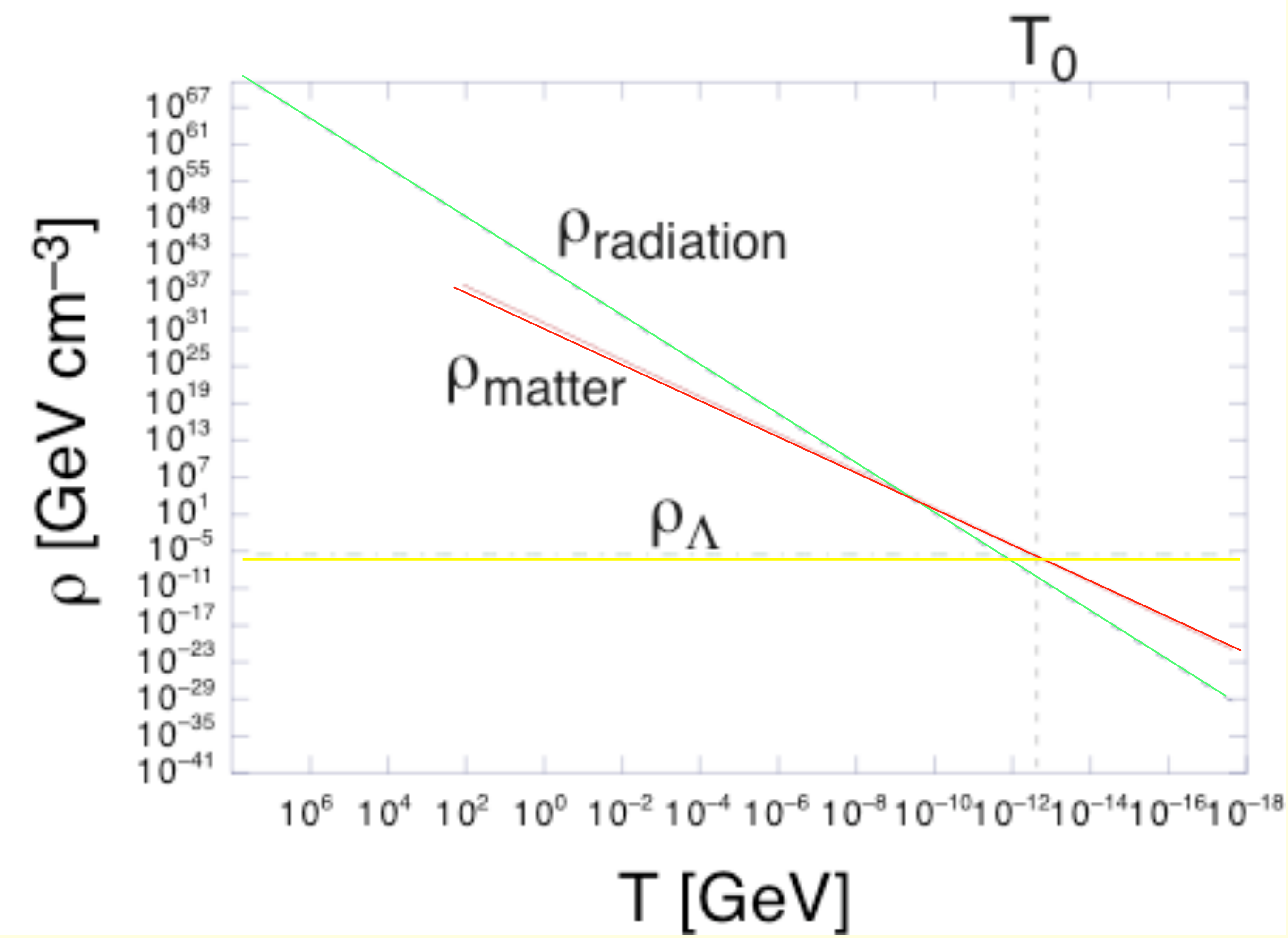
IR cut-off

Cosmological constant problem : where the two ends meet…

Central question : why now?
why is our Universe so large, so old?



Cosmic coincidence problem:
Why does the vacuum energy starts to dominate at a time t_Λ ($z_\Lambda \sim 1$) which almost coincides with the epoch t_G of galaxy formation ($z_G \sim 3$)?



Note: vacuum energy and global supersymmetry

$$H = \frac{1}{4} \sum_r Q_r^2$$

$$< 0 \mid H \mid 0 > = 0 \Leftrightarrow Q_r \mid 0 > = 0 \text{ for all } r$$

Hence supersymmetry is the space time symmetry connected with the vacuum energy.

But supersymmetry breaking scale is in the TeV range, not in the 10^{-3} eV range!

Note also that supersymmetry is also the only symmetry that controls the largest violations of Lorentz invariance (dim. 5 operators).

Are there more general ways than a cosmological constant to account for the acceleration of the expansion?

Einstein equations: $R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\pi G\, T_{\mu\nu}$

geometry matter-energy

Dark energy

modify gravity

add new effects or a new form of energy

Friedmann equation : $H^2 = 8\,\pi G\,\rho\,/3 - k/a^2$

modified Friedmann equation

new contributions to the Friedmann equation

Are the two cases so different?

Take for illustration the simplest model using a scalar field

Why scalar fields to model dark energy?

Scalar fields easily provide a diffuse background

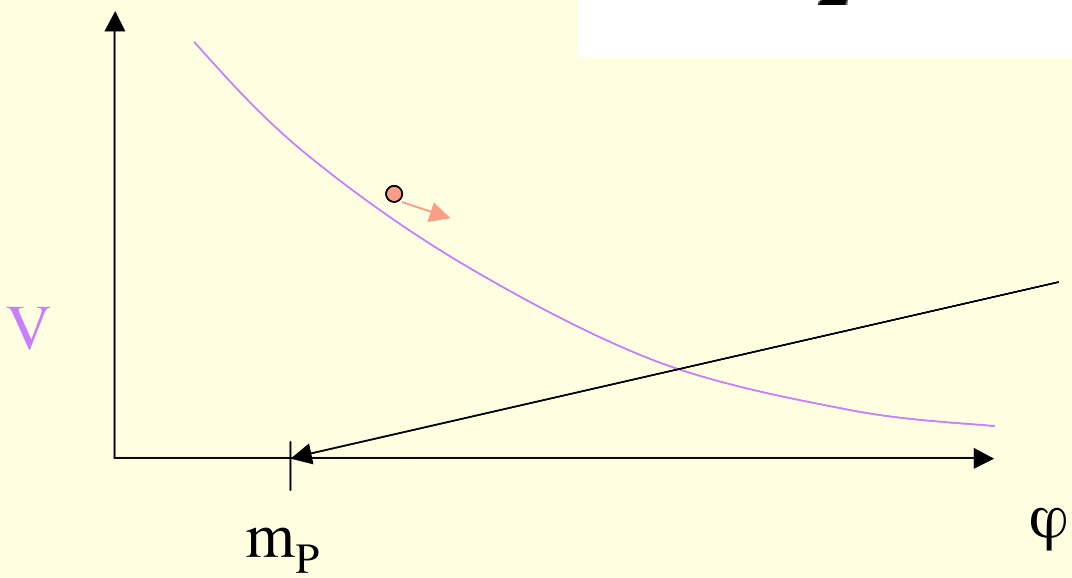
Speed of sound $c_s^2 = (\delta p / \delta \rho)_{adiabatic}$

In most models, $c_s^2 \sim 1$, i.e. the pressure of the scalar field resists gravitational clustering :

scalar field dark energy does not cluster

Quintessence

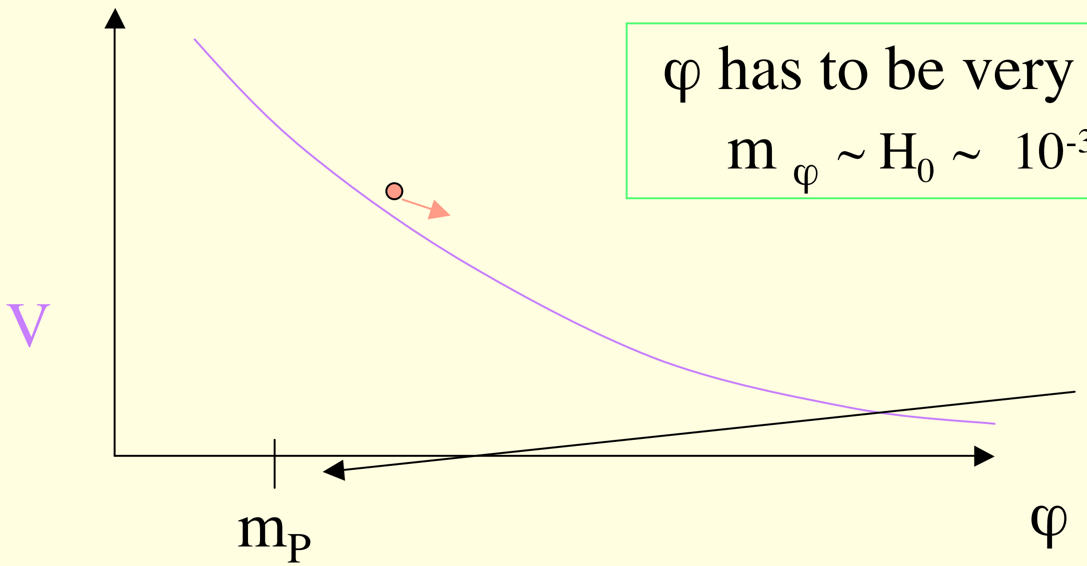
$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi)$$



$$\varepsilon = (m_P V'/V)^2/2 \ll 1$$

$$w = p_\varphi/\rho_\varphi = \frac{\dot{\varphi}^2/2 - V(\varphi)}{\dot{\varphi}^2/2 + V(\varphi)} > -1$$

A generic problem



$\varepsilon = (m_P V'/V)^2 / 2 \ll 1$

φ has to be very light :

$m_\varphi \sim H_0 \sim 10^{-33}\,\text{eV}$

φ exchange would provide a long range force similar to gravity:
φ has to be extremely weakly coupled to ordinary matter
(more weakly than gravity!)

Modification of gravity

Extended gravity

The Einstein action $\mathcal{S} = \int \sqrt{-g}\ R$ can be generalized into

$$\mathcal{S} = \int \sqrt{-g}\ f(R)$$

Perform a redefinition of the metric $g^{(E)}_{\mu\nu} = 2\ |df/dR|\ g_{\mu\nu}$ and write

$$\phi \equiv (\sqrt{6}/2)\ \ln\ [2|df/dR|]$$

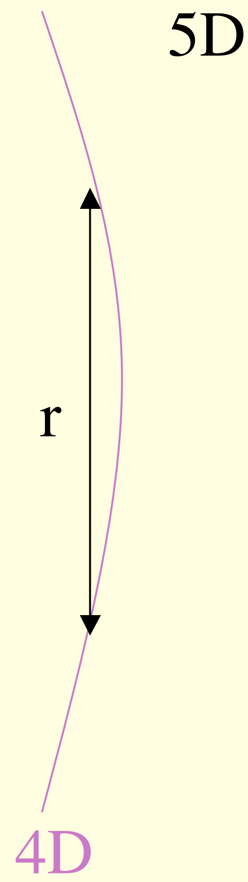
Then

$$\mathcal{L} = \frac{1}{2} R^{(E)} - D^{\mu}\phi D_{\mu}\phi - V(\phi)\ ,$$

$$V(\phi) = \epsilon e^{-2\sqrt{6}\phi/3} \left[\frac{\epsilon}{2} R e^{\sqrt{6}\phi/3} - f\right]\ , \epsilon = \text{sign of } \frac{df}{dR}\ .$$

Brane world models: induced gravity à la DGP Dvali, Gabdadze, Porrattii

$$\mathcal{S} = \int d^5x \sqrt{-g} M_5^3 \frac{1}{2} R^{(5)} + \int_{\text{brane}} d^4x \sqrt{-h} M_{\text{Pl}}^2 \frac{1}{2} R^{(4)} + \int_{\text{brane}} d^4x \sqrt{-h} \mathcal{L}_m + \mathcal{S}_{GH}$$



For distances $r > r_c$ one recovers the 5-dim $1/r^3$ behavior:

$$r_c = M_{Pl}^2 / 2\, M_5^3$$

Gravity leakage into the 5th dimension

Cosmology

Deffayet, Dvali, Gabadadze

$$H^2 = \left(\sqrt{\frac{\rho}{3M_{\text{Pl}}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2 - \frac{k}{a^2}$$

Hence acceleration at late time, without a need for a cosmological constant!

More precisely, taking flat space, this may be written

$$H^2 - \frac{\epsilon}{r_c} H = \frac{\rho}{3M_{\text{Pl}}^2}\ , \epsilon = \pm 1\ .$$

As long as $H^{-1} \ll r_c$, we have the standard Friedmann equation

$$H^2 = \frac{\rho}{3M_{\text{Pl}}^2}\ .$$

But when H^{-1} becomes larger than r_c,

- $\epsilon = +1$

the final regime is $H \rightarrow H_\infty = 1/r_c$. ⟵ similar to inflation

- $\epsilon = -1$

the final regime is

$$H^2 = \rho^2 \frac{r_c^2}{9M_{\text{Pl}}^4} = \frac{\rho^2}{36M_5^6}\ .$$

This looks like a genuine modification of gravity.

However, define the scalar field

$$\pi(\mathbf{x},t) = -\frac{H}{4r_c}|\mathbf{x}|^2 + \frac{1}{4r_c}(\dot{H}/H+H)\,t^2 + bt + c$$

Then the generalized Friedmann equation can be recast into:

$$6\,\Box\pi - 4r_c^2\,(\partial_\mu\partial_\nu\pi)^2 + 4r_c^2\,(\Box\pi)^2 = -\,T^\mu{}_\mu = \rho - 3p$$

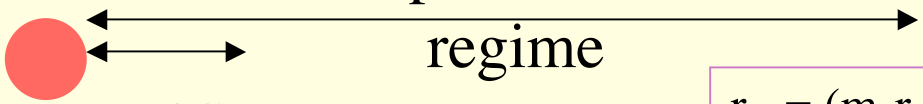
Hence this can be described by an effective scalar field
(a brane-bending mode)

Note two problems in this approach:

- one solved (Vainshtein mechanism)

massive graviton 5 d.o.f. $\rightarrow$ massless graviton 2 d.o.f.
massless graviphoton 2 d.o.f.
real scalar 1 d.o.f.
contributes to one – particle exchange

Mass M — non perturbative regime — perturbative regime: expansion in powers of G_N



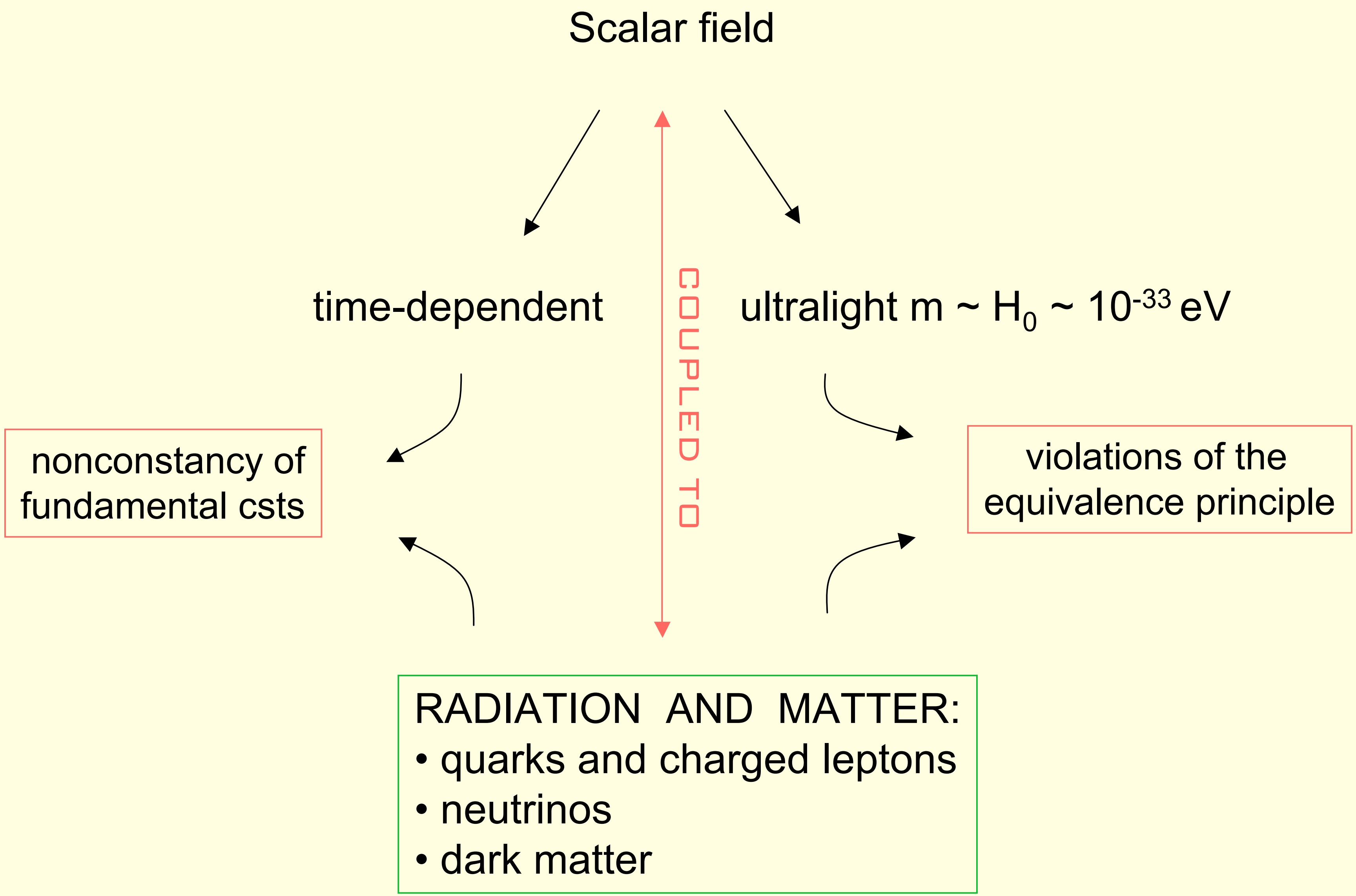
$r_M = 2G_N M$

Schwarzsch. radius

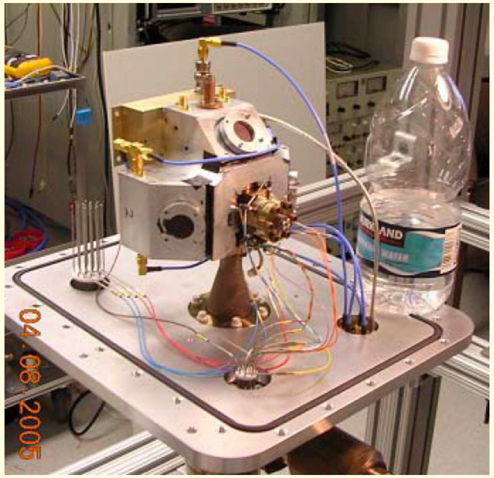
$r_m = (m_g r_M)^{1/5}/m_g$

- one unsolved: presence of a ghost

More about the couplings of dark energy

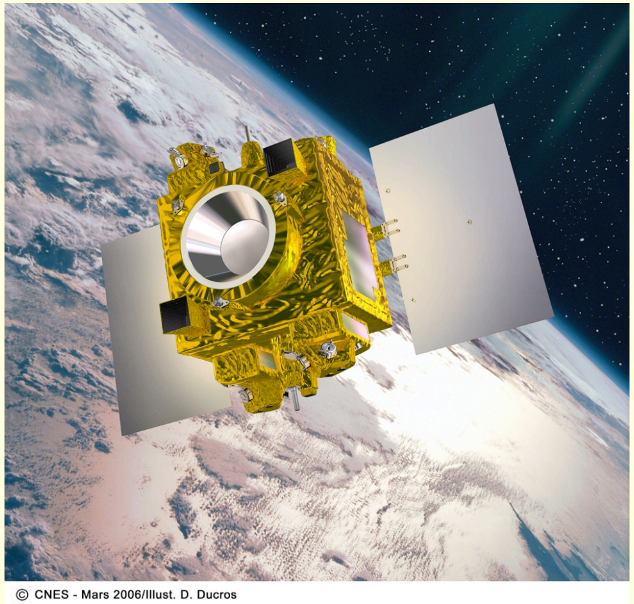

Scalar field
time-dependent
ultralight m ~ H_0 ~ 10^{-33} eV
COUPLED TO
nonconstancy of fundamental csts
violations of the equivalence principle
RADIATION AND MATTER:
• quarks and charged leptons
• neutrinos
• dark matter

A rich experimental program will allow to test the models of dark energy through tests of the theory of gravity



Atomic clocks…

MICROSCOPE



Space missions

A simple example

Wetterich, 02

φ quintessence field

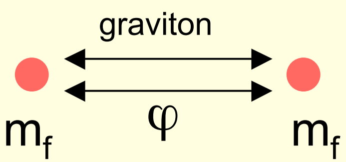
$$\mathcal{L}_{kin} = \frac{1}{2} k^2 \, \partial^\mu \varphi \, \partial_\mu \varphi$$

Fermion masses: $m_f(\varphi)$

Quintessence charge : $\beta_f \equiv \frac{m_{Pl}}{m_f} \frac{\partial m_f}{k \partial \varphi}$

Damour, Esposito-Farèse, 92

$$V_N = - \frac{G_N \, m_f^2}{r} (1 + \beta_f^2)$$

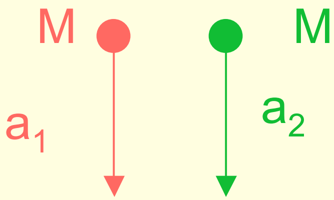


Acceleration in the Earth gravitational field :

$$a_f = \frac{G_N \, M_E}{r^2} \left[1 + \frac{m_{Pl}^2}{k^2} \left(\frac{\partial \ln M_E}{\partial \varphi} \right) \left(\frac{\partial \ln m_f}{\partial \varphi} \right) \right]$$

M_E Earth mass

For two test bodies with same mass M but different composition



$$M = N_i\, m_n + Z_i\, m_H + B_i\, \varepsilon$$

$$m_H = m_p + m_e$$
$$B_i = N_i + Z_i$$

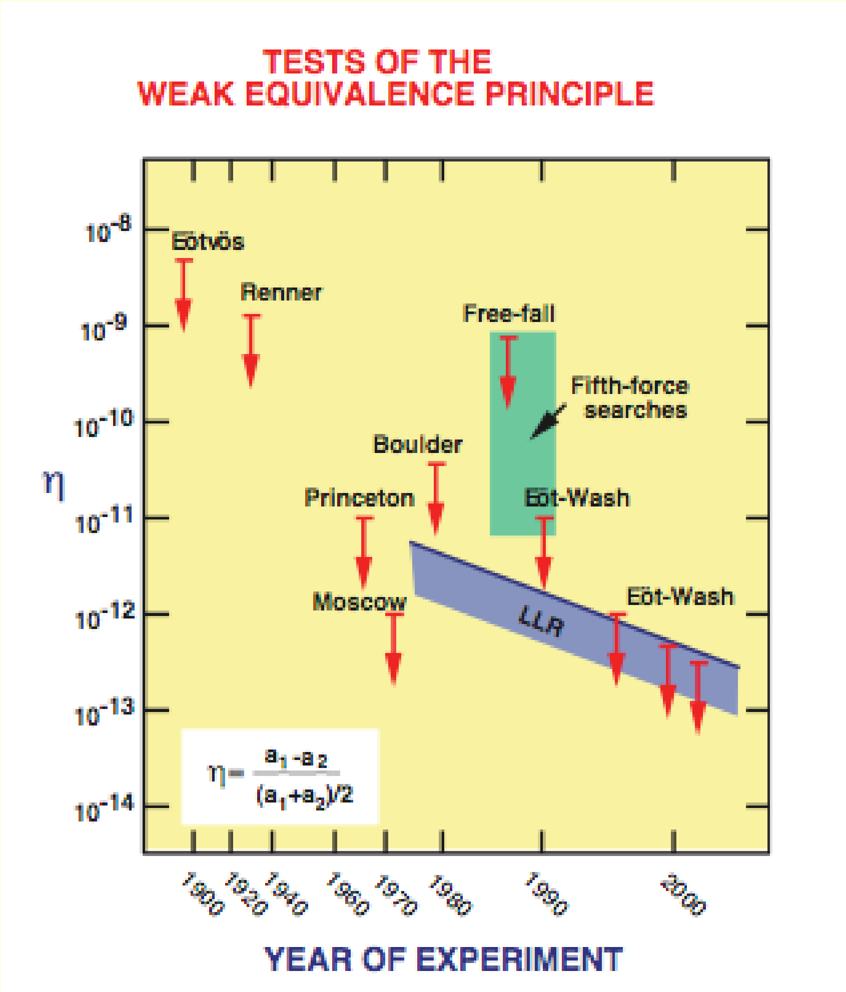
$$\Delta N = N_1 - N_2, \ldots$$

$$\eta \equiv \frac{2|a_1 - a_2|}{a_1 + a_2} = \frac{m_{Pl}^2}{k^2} \frac{\partial \ln M_E}{\partial \varphi} \left(\Delta N \frac{\partial m_n}{\partial \varphi} + \Delta Z \frac{\partial m_H}{\partial \varphi} + \Delta B \frac{\partial \varepsilon}{\partial \varphi} \right)$$

$$\Delta N\, m_n + \Delta Z\, m_H + \Delta B\, \varepsilon = 0$$

$$\eta = \frac{m_{Pl}^2}{k^2} \frac{\partial \ln M_E}{\partial \varphi} \left(\Delta Z \frac{m_H}{M} \frac{\partial \ln(m_H/m_n)}{\partial \varphi} + \Delta B \frac{\varepsilon}{M} \frac{\partial \ln(\varepsilon / m_n)}{\partial \varphi} \right)$$

$$\sim \frac{\partial \ln M_E/B_E}{\partial \varphi} \sim \frac{\partial \ln m_n/m_{Pl}}{\partial \varphi}$$



C. Will, Living Rev. Relativity, 2006

Fundamental tests probe the most crucial part of dark energy models :
the coupling of dark energy to any form of matter

Why is it so important?

- crucial tests of the most « realistic » models of dark energy
- often connected to the « Why now? » question

Some examples…

Hung; Gu, Wang, Zhang, Fardon, Nelson, Weiner, Amendola, Baldi, Wetterich;…

Mass varying neutrino scenarios

Consider a neutrino with mass depending on scalar field ϕ: $m_\nu(\phi)$

Effective potential : $V_{eff}(\phi) = V(\phi) + n_\nu\, m_\nu(\phi)$

Dark energy is the coupled fluid neutrino-scalar: $\rho_{DE} = \rho_\phi + \rho_\nu(\phi)$

But neutrinos have a tendancy to cluster (extra force due to ϕ exchange)!

Coupled dark energy

Anderson, Carroll; Casas, Garcia-Bellido, Carroll; Farrar, Peebles; Amendola; Comelli, Pietroni, Riotto; …

φ-dependent mass for the dark matter particle χ: $M_\chi(\varphi) = M_0 \exp(-\lambda\varphi)$

If the scalar potential is $V(\varphi) = V_0 \exp(\beta\varphi)$, there is an attractor corresponding to

$$\rho_\varphi \sim \rho_\chi \sim M_\chi(\varphi)\, n_\chi \sim a^{-3(1+W)} \qquad \text{with } W = -\lambda/(\lambda+\beta)$$

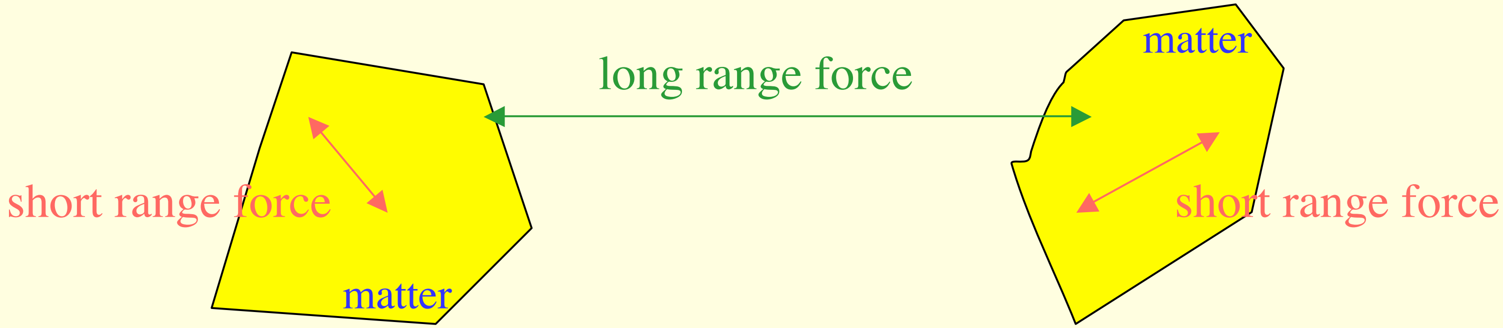
$n_\chi \sim a^{-3}$

Chameleon dark energy

Khoury, Weltmann; Brax, van de Bruck, Davis, Khoury, Weltman;…

$$V_{eff}(\phi) = V(\phi) + f(\phi)\,\rho_m$$

Then, possible to have a heavy enough scalar field ($m_\phi > 10^{-3}$ eV) in matter where constraints on the fifth force or equivalence principle apply, whereas it can be ultralight outside matter.



Thin shell effect : a tiny fraction of large objects (e.g. planets) is sensitive to the long range force. Not so for smaller objects: hence tests with satellites bring new constraints.

Galileon

Nicolis, Rattazzi, Trincherini, Deffayet, Tsujikawa,, Trodden…….

Inspired by the DGP model

Uses the same field π

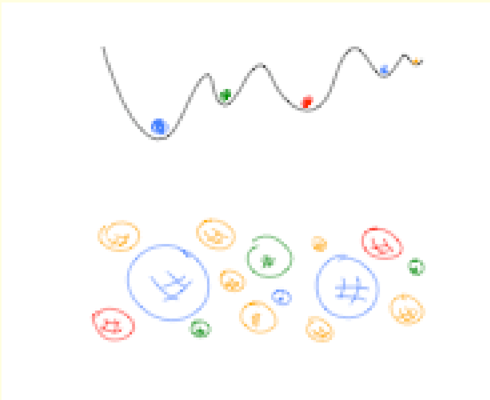
$$\mathcal{L} = f^2\, \partial\pi\partial\pi\, F(\partial\partial\pi/H_0^{\,2}) - \frac{1}{f}\pi T^{\mu}{}_{\mu}$$

In the Vainshtein region, the scalar field decouples from matter thanks to the important derivative self-interactions.

Back to the cosmological constant

A multitude of universes?

Eternal inflation



String theory

Anthropic approach

Vilenkin, Weinberg, Linde, string theorists

Consider regions (universes) with different values of t_G and t_Λ :

• when ρ_Λ starts to dominate (at t_Λ), the Universe enters a de Sitter phase of exponential expansion

• galaxy formation (at t_G) must precede this phase (otherwise no observer available)

Hence $t_G \leq t_\Lambda$

• Regions with $t_\Lambda \gg t_G$ have not undergone yet any de Sitter phase of reacceleration and are thus phase space suppressed compared with regions with $t_\Lambda \sim t_G$:

Hence $t_\Lambda \gtrsim t_G$

$\rho_\Lambda \sim \rho_M$

But

- the precise prediction for λ is larger than what is observed

- the argument does not involve $\hbar$

The holographic approach

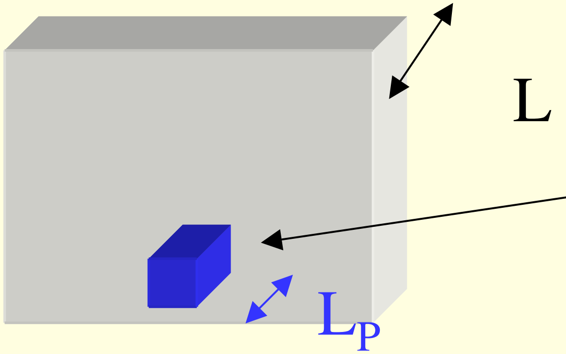
Padmanabhan, C. Hogan

Trying to understand from first principles:

$$\rho_\Lambda = \frac{1}{8\pi G \ell_\Lambda^2} = \frac{\hbar}{\ell_P^2 \ell_\Lambda^2} \equiv \frac{\hbar}{\ell_{DE}^4}$$

Consider a 3-dimensional domain of size L

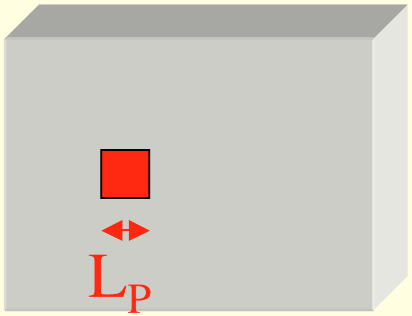
$N = (L/L_P)^3$



N elementary cells of size L_P
Poissonian fluct. in energy :
$\Delta\varepsilon \sim 1/L_P$

$$\Delta E^2 \sim N/L_P^{\,2} \Rightarrow \rho \sim \sqrt{N}/(L_P L^3) \sim \frac{1}{L_P^{5/2} L^{3/2}}$$

If the relevant degrees of freedom are only on the surface:

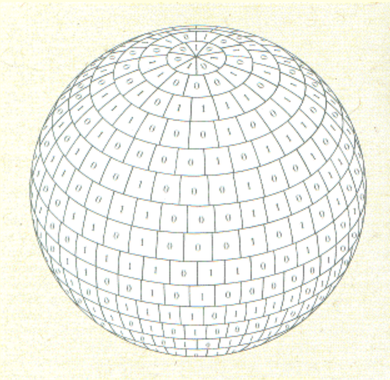


$N = (L/L_P)^2$

$$\Delta E^2 \sim N/L_P^{\,2} \Rightarrow \rho \sim \sqrt{N}/(L_P L^3) \sim \frac{1}{L_P^{2} L^{2}}$$

√

Such considerations leads Padmanabhan to write a microscopic theory of gravity with 2-dim degrees of freedom (↔horizons)



Gravity as an emergent, long wavelength phenomenon

Action invariant under the shift :

$$T_{\mu\nu} \rightarrow T_{\mu\nu} + \lambda\, g_{\mu\nu}$$

Allows to gauge away the vacuum energy associated with matter

Conclusion

+	--
A rich array of proposed models	Not so many are complete (problem of interactions of dark energy with the rest of matter)
A rich program of observations and experiments	Is dark energy a « complex physical phenomenon »? so far described basically by 2 numbers: Ω_Λ and w
The issue of dark energy will contribute to the development of large and deep surveys: expect progress for cosmology at large	Vacuum energy: do we under-stand the connection between inflation and dark energy?

The identification of dark energy does not solve the problem of the vacuum energy

THE END