The Quantum Origin of Cosmic Structure

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Overview

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- Introduction and motivation
- Generating primordial density perturbations
  - Inflation
  - Some cosmological perturbation theory . . .
- Evolution: conserved quantities
- Observational signatures
  - Calculating observational consequences
  - Higher order observables
- Current and future observational constraints: 21cm anisotropies
- Conclusions

The evolution of the Universe

What underlying theory or theories govern the evolution of the Universe?

- on small scales:
  Quantum Field Theory, necessary to set initial conditions
- on large scale:
  Einstein's General Relativity, necessary to calculate evolution

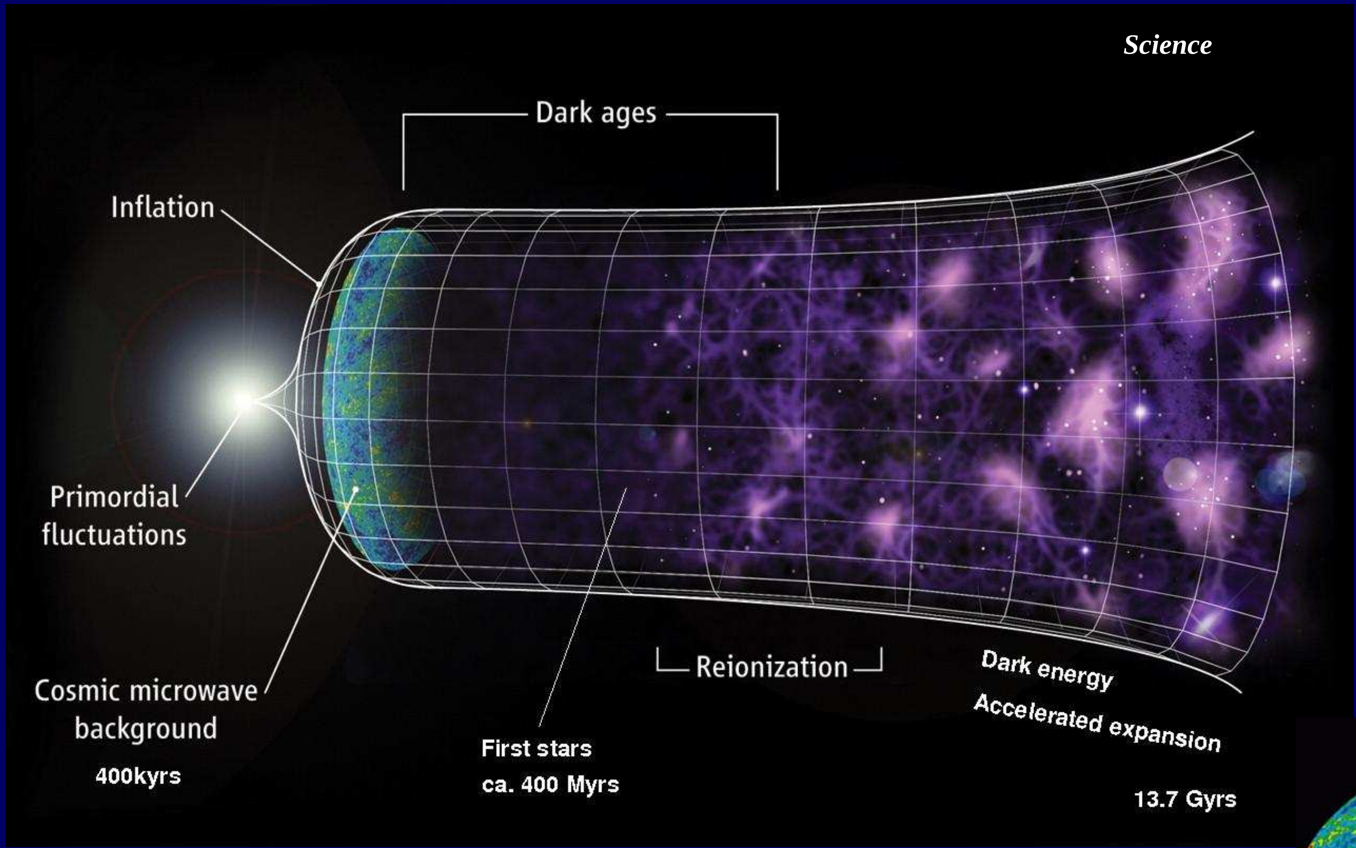
$\Rightarrow$ can work with two separate, well understood theories, instead of waiting for final theory . . .

$\Rightarrow$ can calculate how quantum fluctuation evolve into large scale structure (LSS)

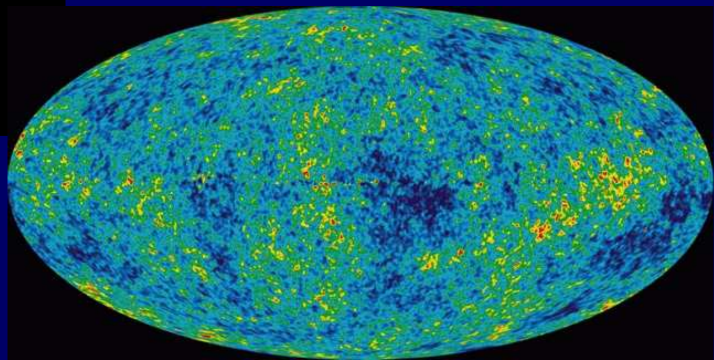
Note: in the following *Cosmological standard model* (nothing too strange . . .)

The evolution of the Universe

The cosmological standard model



the CMB as seen by WMAP



Inflation

Dynamics controlled by Einstein equations $G_{\mu\nu} = 8\pi\, G\, T_{\mu\nu}$

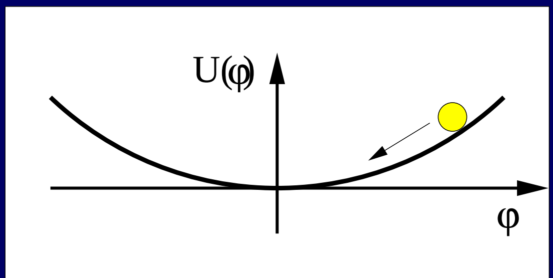
Cosmological inflation:

Starobinsky (1980), Guth (1981)

- period of accelerated expansion in the very early universe
- “easily” achieved using scalar field φ with potential U:

$$P = \frac{1}{2a^2}\varphi'^2 - U(\varphi)$$

$\Rightarrow$ negative P during period of potential domination (field is “slowly rolling”)



Background dynamics (using conformal time η):

$$\varphi'' + 2\mathcal{H}\varphi' + a^2 U_{,\varphi} = 0$$
$$\mathcal{H}^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\varphi'^2 + a^2 U\right)$$

Vacuum fluctuations

Fluctuations in the scalar field (assuming slow roll, in Fourier space, wavenumber k) governed by Klein-Gordon equation

$$\delta\varphi'' + 2\mathcal{H}\delta{\varphi_1}' + k^2\delta\varphi + a^2 U_{,\varphi\varphi}\delta\varphi = 0$$

can be solved in terms of Hankel functions

- Initial conditions: *making contact with Quantum Field Theory* on small scales ($|k\eta| \gg 1$)

$$\delta\varphi \sim \frac{e^{-ik\eta}}{a\sqrt{2k}}$$

- power spectrum $\mathcal{P}_{\delta\varphi}(k) \equiv \left(\frac{k^3}{2\pi^2}\right)\left|\delta\varphi\right|^2$ and we get "iconic" result for fluctuation amplitude at horizon crossing (i.e. $k = aH$)

$$\delta\varphi\big|_{k=aH} = \frac{H}{2\pi}$$

Starobinsky; Hawking; Guth and Pi (1982); Mukhanov and Chibisov (1981)

Vacuum fluctuations

- Inflation solves many problems of Big Bang model (flatness, horizon, monopole), but was designed to do so
- Arguably the greatest success of inflation:
  vacuum fluctuations $\delta\varphi$ get (nearly) scale-invariant power spectrum $\mathcal{P}_{\delta\varphi}$
  $\Rightarrow$ seeds for structure formation

To relate primordial spectrum to spectrum of temperature fluctuations in the CMB or distribution of galaxies: need GR

Cosmological perturbation theory

- GR is nonlinear: need approximation scheme, such as perturbation theory
- perturbing metric *and* matter variables, e.g. above $\varphi = \varphi + \delta\varphi$
  - GR is covariant, splitting variables is not: spurious gauge modes get introduced $\Rightarrow$ construct gauge invariant variables
  - E.g.: a first order coordinate transformation $x^\mu \to \widetilde{x^\mu} = x^\mu + \delta x_1{}^\mu$, induces a change in metric variable, *curvature perturbation*, and energy density:

$$\widetilde{\psi}_1 = \psi_1 + \frac{a'}{a}\delta\eta_1 \,, \qquad \widetilde{\delta\rho_1} = \delta\rho_1 + \rho_0'\delta\eta_1$$

combine both $\Rightarrow$ get gauge-invariant quantity

$$-\zeta_1 = \psi_1 + \frac{\mathcal{H}}{\rho_0'}\delta\rho_1$$

curvature perturbation on uniform density hypersurfaces

Bardeen (1980)

Conserved quantities

- Variables like $\delta\varphi$ evolve $\Rightarrow$ would need evolution from end of inflation, "horizon exit" to "horizon entry"
- use instead *conserved quantities*: only need to calculate at "horizon exit"
- popular example: ζ_1
- using energy conservation, can show that on large scales for adiabatic perturbations

$$\zeta_1' = 0$$

- *Why is that useful?* Can calculate observable quantities in early universe, e.g. at end of inflation after horizon exit, then map them onto ζ $\Rightarrow$ observables won't change until they reenter horizon

To put it crudely:
$\zeta_1 \sim$ gravitational potential wells $\Rightarrow$ dark matter and other fluids "fall in" $\Rightarrow$ CMB and other anisotropies (in neutral hydrogen and LSS)

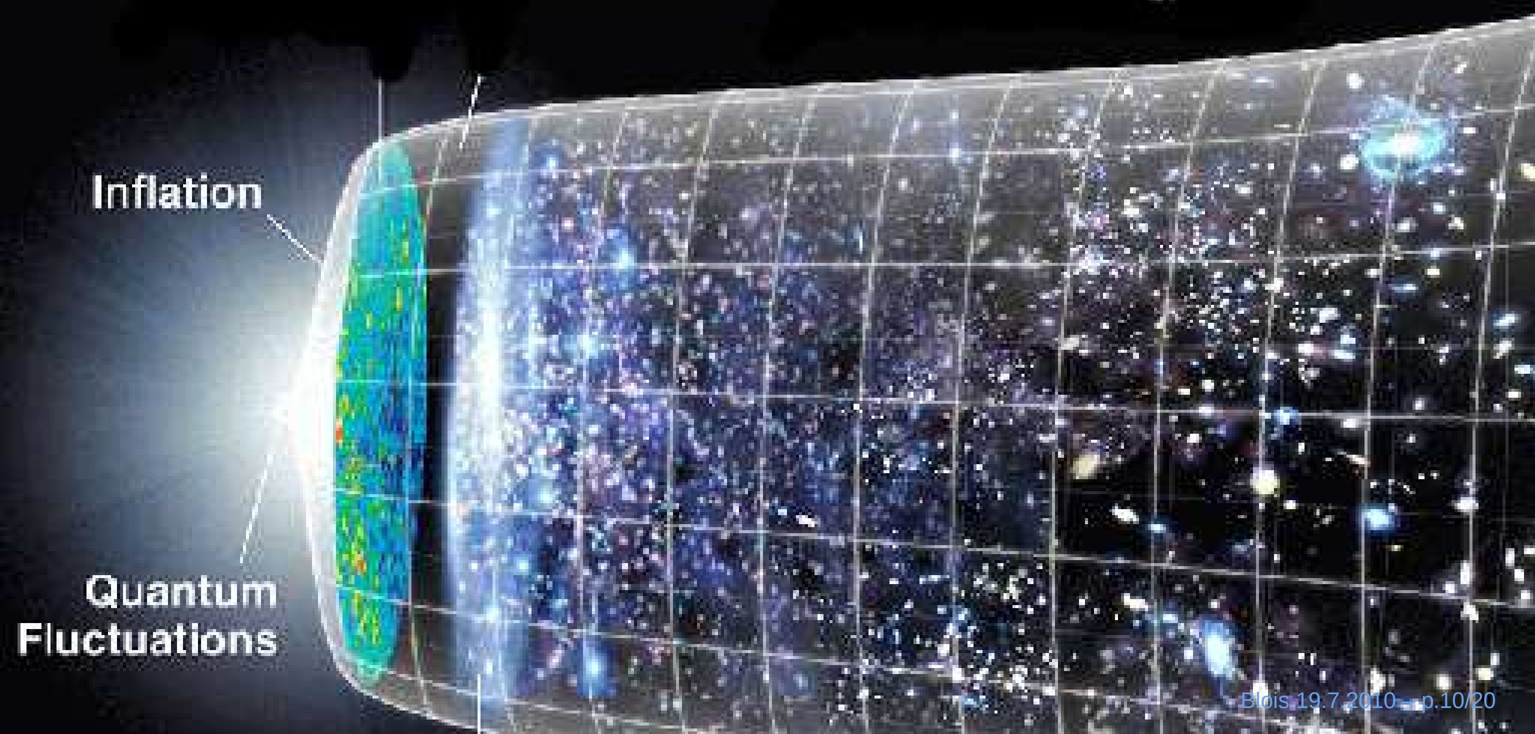
Putting it in context . . .

The cosmological standard model, again . . .

During final stages, at end of inflation: calculate the observable of choice, e.g. $\langle \delta\varphi_1 \delta\varphi_1 \rangle$

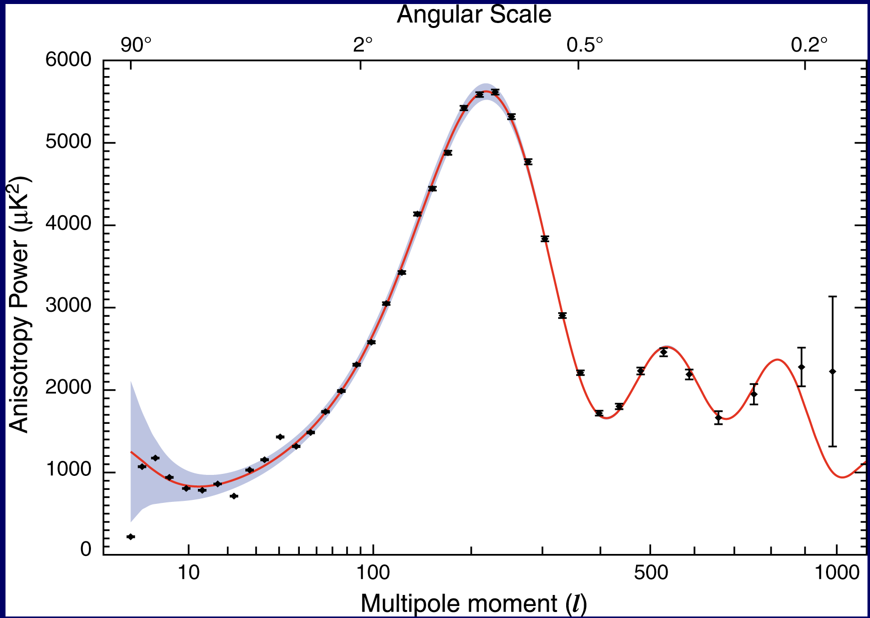
Since the observables evolve, translate into conserved quantities, say ζ

Horizon re-entry: translate conserved quantities into temperature fluctuations, use Boltzmann codes to evolve further



Calculating observables

getting information from the data:



- calculate two-point correlator or power spectrum $\langle \delta\varphi\delta\varphi \rangle$
- translate into conserved quantity sourcing CMB anisotropies, e.g. curvature perturbation on uniform density hypersurfaces, $\langle \zeta_1 \zeta_1 \rangle$

- feed into Einstein equations and Boltzmann solver
- get theoretical predictions for CMB anisotropies
- compare with observations

Comparing theory with observations

Theoretical input: formalism sketched above, and *model of inflation*, i.e. the potential $U(\varphi)$

- too many models to list, some grouping of model zoo into:
  - single field models versus multi-field models
  - large field models compared to small field models
- simplest “chaotic inflation” models still viable (single and large field): e.g.

$$U(\varphi) = \frac{1}{2} m^2 \varphi^2$$

Linde (1983)

- Biggest problem of inflation: what is the inflaton, the field that drives inflation?

WMAP7 ruled out Harrison-Zel‘dovich-Peebles spectrum (more than 3 σ)

Cosmological parameters

Some parameter values from WMAP7 cosmological interpretation paper

- take primordial power spectrum as power law, with amplitude $\Delta^2_\zeta(k_0)$ and spectral index n_s

$$\Delta^2_\zeta(k) = \Delta^2_\zeta(k_0)\left(\frac{k}{k_0}\right)^{n_\mathrm{s}}$$

  and $\Delta^2_\zeta(k_0) = 2.43 \times 10^{-9}$, $n_\mathrm{s} = 0.969$ (at 68%C.L.), pivot scale $k_0 = 0.002\ Mpc^{-1}$
- scalar to tensor ratio (contribution of gravitational waves to power spectrum) $r < 0.36$
- “running” or scale dependence of spectral index: $-0.084 < dn_\mathrm{s}/d\ln k < 0.010$
- many more not directly related to primordial perturbations, like Hubble parameter H_0, background energy densities, . . .

Higher order observables

At linear order in perturbation theory primordial perturbations from inflation are (very nearly) Gaussian distributed, at higher this is no longer the case. Note, here and in the following: classical perturbation theory, not loops!

- at linear order two point correlation function:
  $\langle\zeta\zeta\rangle \Rightarrow$ power spectrum $P(k) \sim Ak^{n_{\rm s}}$, with amplitude A , spectral index $n_{\rm s}$ (and wavenumber k)
- at second order three point correlation function:
  $\langle\zeta\zeta\zeta\rangle \Rightarrow$ bispectrum, more complicated
  However: in its simplest form can be characterised by a number, non-linearity parameter $f_{\sf NL}$, roughly:

$$f_{\sf NL} \propto \frac{\zeta_2}{\zeta_1^2}$$

where $\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \dots$

Higher order observables

- at third order four point correlation function: $\langle\zeta\zeta\zeta\zeta\rangle \Rightarrow$ trispectrum, even more complicated However: in its simplest form can be characterised by parameters $\tau_{\rm NL}$, $g_{\rm NL}$

Note:
at present $f_{\rm NL}$ treated as a constant (as is spectral index in many studies), though eventually – when sufficient data available – allow for scale and configuration dependence.

Higher order observables: f_{NL}

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Use CMB data (and in future also galaxy surveys) to further constrain models

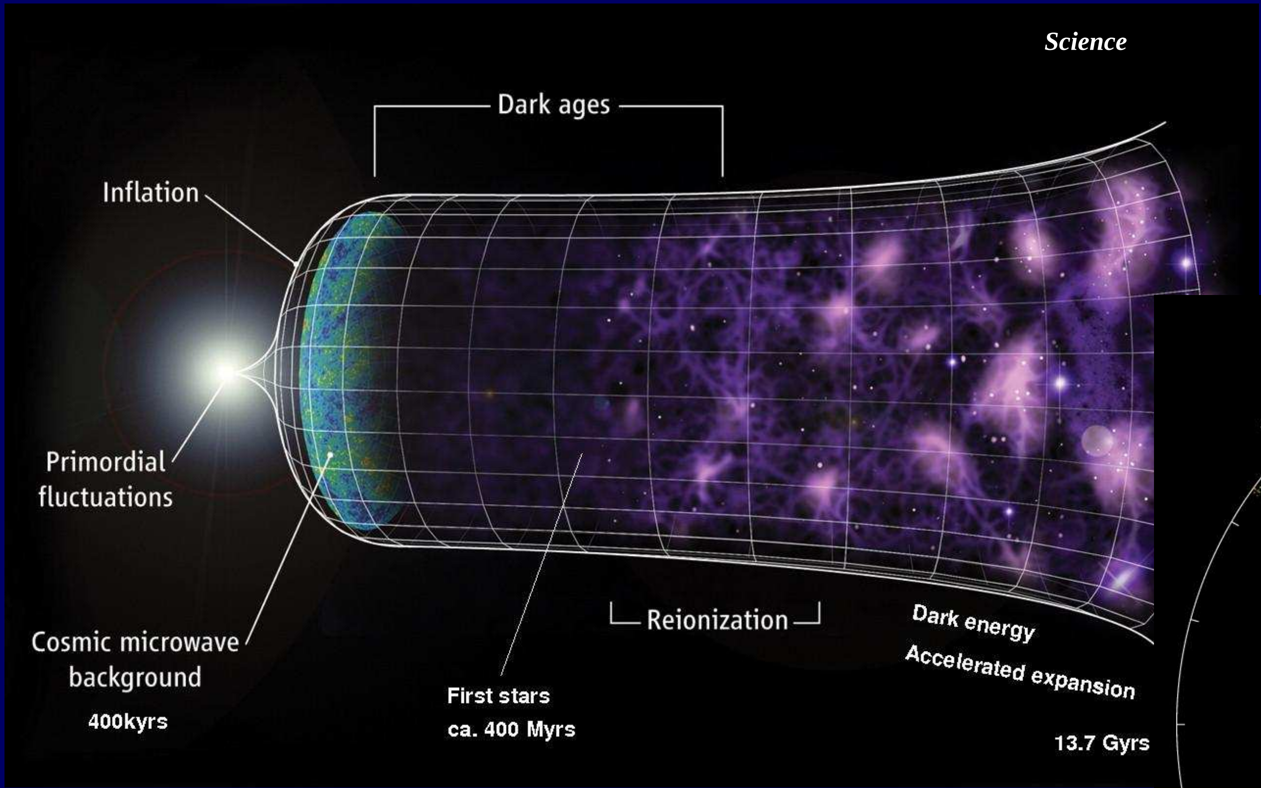
- at linear order most models pass observational tests
- non-linearity parameter f_{NL} is currently becoming a very strong model discriminator

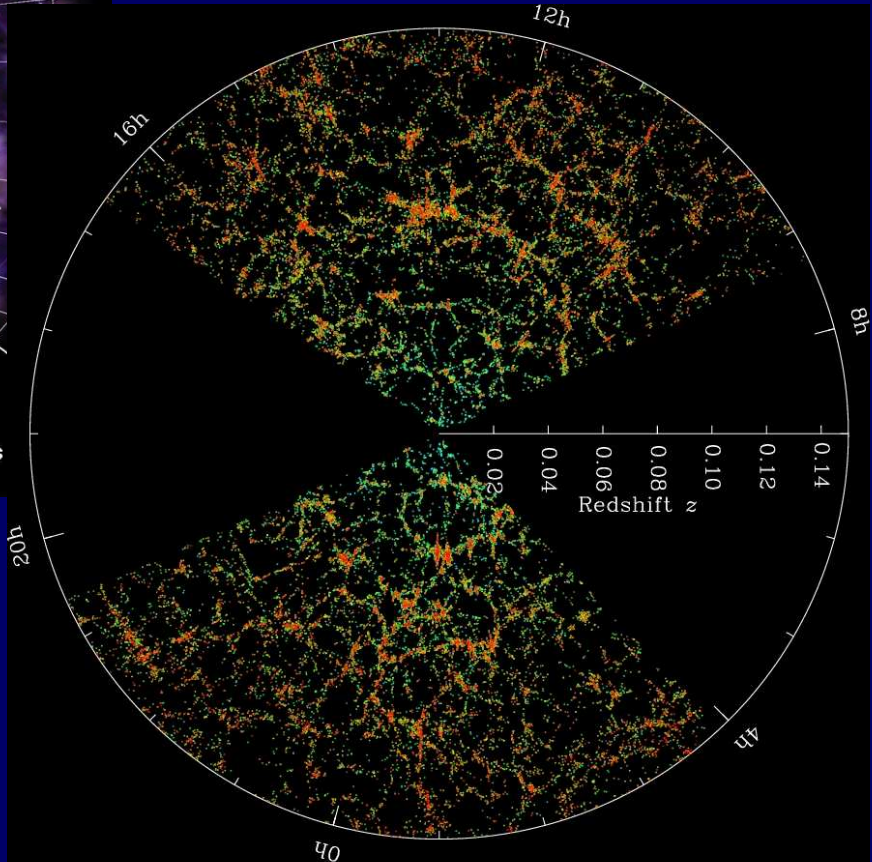
Gangui et al. (1994), Komatsu and Spergel (2001), Maldacena (2003)

- very simplest single field inflation models (“vanilla”) predict $f_{\mathrm{NL}} \sim 1$
- the 68% hint in WMAP7 data: $f_{\mathrm{NL}} = 32 \pm 21 \Rightarrow$ “vanilla” inflation model *might be* ruled out, multi-field inflation (e.g. curvaton, generic multi-field model) favoured

LSS and 21cm anisotropies

The cosmological standard model





the distribution of galaxies as observed by SDSS

21cm anisotropies

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Maps of the neutral hydrogen have the potential to provide the data we need to study non-gaussianity:

- neutral hydrogen left over from the Big Bang can be mapped using its 21cm transition
- “same” mechanism in forming 21cm anisotropy involved as in forming temperature anisotropy of CMB (primordial perturbations sourcing potential “wells”)
- signal is generated after decoupling but before galaxy formation at redshift $200 \lesssim z \lesssim 30$ (compare to formation of CMB at decoupling at $z \simeq 1100$)
- no Silk damping on small scales
- by “tuning” or shifting the observed wavelength tomography possible (slices of the universe at different redshifts) $\Rightarrow$ 21cm maps are in full 3D glory

21cm . . . continued

Implications for studies of the early universe and non-gaussianity

- Silk damping makes CMB observation impossible on small scales, no signal beyond $l \sim 3000$ or $k \lesssim 0.2 \,\mathrm{Mpc}^{-1}$
- 21cm observations possible up to $l \sim 50000$
- amount of data in 21cm compared to CMB is $\sim 10^{10}$ higher

Loeb and Zaldarriaga 2003

Using the 21cm data it should be possible to constrain the non-linearity parameter $\Delta f_{\mathrm{NL}} \lesssim 1$ (PLANCK temperature + polarisation $\Delta f_{\mathrm{NL}} \sim 6$)

Many experiments projected or underway: LOFAR, SKA, PAST . . .

Conclusion

- Cosmological standard model works!
- inflation:
  - introduced to solve problems of hot Big Bang
  - vacuum fluctuations in inflaton generate nearly scale-invariant primordial spectrum to source CMB and LSS
- new observable quantities, in particular at higher order in cosmological perturbation theory, and new and better data will allow to constrain parameter space further
- there are also problems: *what is the inflaton?*
- although no candidate for the inflaton obvious at present, at very least an excellent parametrisation.