Center for Particle Cosmology at the University of Pennsylvania

Gravity on the Largest Scales and Cosmic Acceleration

Justin Khoury (UPenn)

K. Hinterbichler & J. Khoury, Phys. Rev. Lett. 104, 231301 (2010) [arXiv:1001.4525 [hep-th]]

J. Khoury & A. Weltman, Phys. Rev. Lett. 93, 171104 (2004); Phys. Rev. D 69, 044026 (2004)

22nd Rencontres de Blois July 20, 2010





⁸th Rencontres (1996)



8th Rencontres (1996)



11th Rencontres (1999), F. Bouchet



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22nd Rencontres (2010)

The Era of Precise Unknowns



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 $\Omega_{\rm c}h^2 = 0.1123 \pm 0.0035;$ $n_s = 0.963 \pm 0.012;$ $\Delta_{\mathcal{R}}^2 = (2.441 \pm 0.090) \times 10^{-9}$

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$\Omega_{\rm c} h^2 = 0.1123 \pm 0.0035 \,;$





The end of cosmology?

IS COSMOLOGY SOLVED? An Astrophysical Cosmologist's Viewpoint

P. J. E. Peebles

Joseph Henry Laboratories, Princeton University, and Princeton Institute for Advanced Study

ABSTRACT

We have fossil evidence from the thermal background radiation that our universe expanded from a considerably hotter denser state. We have a well defined, testable, and so far quite successful theoretical description of the expansion: the relativistic Friedmann-

"Does ACDM signify completion of the fundamental physics that will be needed in the analysis of ... future generations of observational cosmology? Or might we only have arrived at the simplest approximation we can get away with at the present level of evidence?"



- Prof. P. J. E. Peebles

30 Oct 1998

Already a surprise: Dark energy is coming to dominate the energy budget much more quickly than anticipated... Already a surprise: Dark energy is coming to dominate the energy budget much more quickly than anticipated...

WMAP 1st-year $\Omega_{\Lambda}=0.71$

WMAP 3-year $\Omega_{\Lambda}=0.716$

WMAP 5-year

 $\Omega_{\Lambda} = 0.742$

 \bullet

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 \bigcirc

A Richer Dark Sector

Dark energy candidates: Λ , quintessence...

> Ratra & Peebles (1988); Wetterich (1988); Caldwell, Dave & Steinhardt (1998)



Tantalizing prospect: quintessence (or any other light field) couples to both dark and baryonic matter.



 \implies ruled out?

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Not so fast. Scalar fields can "hide" themselves from local experiments through screening mechanisms

$$\rho_{\rm here} \sim 10^{30} \rho_{\rm cosmos}$$

 $\nabla^2 \phi + m^2 \phi = -\frac{g}{M_{\rm Pl}} T^{\mu}_{\mu}$

$\nabla^2 \phi + m^2 \phi = \frac{g}{M_{\rm Pl}} \rho$

$\nabla^2 \phi + M^2(\rho) \phi = \frac{g}{M_{\rm Pl}} \rho$

chameleon

$K(\rho)\nabla^2\phi + m^2\phi = \frac{g}{M_{\rm Pl}}\rho$

Vainshtein

$\nabla^2 \phi + m^2 \phi = \frac{g(\rho)}{M_{\rm Pl}} \rho$

symmetron

Chameleon Mechanism J. Khoury & Weltman, Phys. Rev. Lett. (2004); Gubser & J. Khoury, (2004) (At play in f(R) theories. Carroll, Duvvuri, Trodden & Turner (2004))

Consider scalar field ϕ with potential $V(\phi)$ and conformally-coupled to matter:

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) + g \frac{\phi}{M_{\rm Pl}} T^{\mu}_{\ \mu}$$

where $T^{\mu}_{\ \mu}$ is stress tensor of all matter (Baryonic and Dark)

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 $V_{
m eff}(\phi)$

where $T^{\mu}_{\ \mu}$ is stress tensor of all matter (Baryonic and Dark) For non-relativistic matter, $T^{\mu}_{\ \mu}\approx -\rho$, hence

$$\nabla^2 \phi = V_{,\phi} + \frac{g}{M_{\rm Pl}}\rho$$

$$V_{\rm eff}(\phi) = V(\phi) + g \frac{\phi}{M_{\rm Pl}} \phi$$

Density-dependent mass

$$V_{\rm eff}(\phi) = V(\phi) + g \frac{\phi}{M_{\rm Pl}} \rho$$

e.g.
$$V(\phi) = \frac{M^5}{\phi}$$



Thus $m=m(\rho)$ increases with increasing density Laboratory tests => set $m^{-1}(\rho_{\rm local}) \lesssim {
m mm}$

Generally implies: $m^{-1}(\rho_{\rm cosmos}) \lesssim {
m Mpc}$

Nevertheless, $m^{-1}(\rho_{\text{solar system}}) \lesssim 10 - 10^4 \text{ AU}$ \implies ruled out by post-Newtonian tests?

 $\rho = \rho_{\rm out}$

 $= \rho_{\rm in}$ $R_{\rm c}$

 $\rho = \rho_{\rm out}$



 $\rho = \rho_{\rm out}$







$$\rho = \rho_{\text{out}} \qquad \qquad \rho = \rho_{\text{in}}$$

$$\Rightarrow \qquad \rho = \rho$$

Chameleon Searches

Eot-Wash

Adelberger et al., Phys. Rev. Lett. (2008)





GammeV, Fermilab

Chou et al., Phys. Rev. Lett. (2008)



ADMX

P. Sikivie & co., arXiv:1004.5160



Vainshtein Mechanism

Vainshtein (1972); Arkani-Hamed, Georgi, Schwartz (2003) Deffayet, Dvali, Gabadadze & Vainshtein (2002); Luty, Porrati & Rattazzi (2003); Nicolis & Rattazzi (2004)

4d effective theory in DGP:
$$\mathcal{L}_{\pi} = 3(\partial \pi)^2 \left(1 + \frac{\nabla^2 \pi}{3\Lambda^3}\right) + \frac{\pi}{M_{\rm Pl}}\rho$$

which enjoys Galilean symmetry: $\partial_{\mu}\pi \to \partial_{\mu}\pi + c_{\mu}$

$$3\nabla^2 \pi + \frac{1}{\Lambda^3} \left[(\nabla^2 \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \right] = \frac{\rho}{2M_{\rm Pl}}$$

Solution around point source of mass M:

Vainshtein radius:

$$R_V \equiv \frac{1}{\Lambda} \left(\frac{M}{M_{Pl}}\right)^{1/3}$$

5th force on a test particle, relative to gravity:

 $\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2} \sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases}$

$$\frac{F_{\pi}}{F_{\text{Newton}}} = \frac{\pi'(r)/M_{Pl}}{M/(M_{Pl}^2 r^2)} = \begin{cases} \sim \left(\frac{r}{R_V}\right)^{3/2} & R \ll R_V \\ \sim 1 & R \gg R_V \end{cases}$$

Field generated on a background below Vainshtein radius of large object: $\pi = \pi_0 + \varphi$, $T = T_0 + \delta T$

$$\mathcal{L} = -3(\partial\varphi)^{2} + \frac{2}{\Lambda^{3}} \left(\partial_{\mu}\partial_{\nu}\pi_{0} - \eta_{\mu\nu}\Box\pi_{0}\right) \partial^{\mu}\varphi\partial^{\nu}\varphi$$
$$- \frac{1}{\Lambda^{3}} (\partial\varphi)^{2}\Box\varphi + \frac{1}{M_{\mathrm{Pl}}}\varphi \,\delta T \qquad \qquad \sim \left(\frac{R_{\mathrm{V}}}{r}\right)^{3/2} \gg 1$$

Kinetic term is enhanced, which means that, after canonical normalization, coupling to δT is suppressed. The non-linear coupling scale is also raised.

Other examples:

Generalized Galileons
 Nicolis, Rattazzi and Trincherini (2009)

k-Mouflage Babichev, Deffayet and Ziour (2009)

Symmetron Fields

K. Hinterbichler and J. Khoury, Phys. Rev. Lett. (2010) See also Olive & Pospelov (2008); Pietroni (2005)

Instead of $m(\rho)$, here it is the coupling to matter that depends on density.

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) + \frac{\phi^2}{2M^2} T^{\mu}_{\ \mu}$$

where $T^{\mu}_{\ \mu}$ is stress tensor of all matter (Baryonic and Dark)

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Potential is of the spontaneous-symmetrybreaking form:

$$V(\phi) = -\frac{1}{2}\mu^{2}\phi^{2} + \frac{1}{4}\lambda\phi^{4}$$

Most general renormalizable potential with $\phi \to -\phi$ symmetry.

Effective Potential

$$V_{\text{eff}}(\phi) = \frac{1}{2} \left(\frac{\rho}{M^2} - \mu^2 \right) \phi^2 + \frac{1}{4} \lambda \phi^4$$

. Whether symmetry is broken or not depends on local density

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 ${\rm @}$ Outside source, $\rho=0$, symmetron acquires VEV and symmetry is spontaneously broken.

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 ${\rm \bullet}$ Outside source, $\rho=0$, symmetron acquires VEV and symmetry is spontaneously broken.

 ${\rm \bullet}$ Inside source, provided $~\rho>\mu~^2M^2,$ the symmetry is restored.

Effective Coupling Perturbations $\delta\phi$ around local background value couple as: $\mathcal{L}_{\text{coupling}} \sim \frac{\phi}{M^2} \delta \phi \rho$ Symmetron fluctors decouple in high-density regions In voids, where \mathbb{Z}_2 symmetry is broken, $\mathcal{L}_{\text{coupling}} \sim \frac{\mu}{\sqrt{\lambda}M^2} \delta \phi \rho$ $\sim \frac{\delta\phi}{M_{\rm Pl}}\,
ho$ gravitational strength Gravitational-strength, Mpc-range 5th force in voids.

Inspiration...

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Symmetron Couch (\$9500.00)

"NASA-style gravity reduction."

"Offers a unique multi-phase wave experience."



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Thin-Shell Screening Effect Behavior of solution depends on $\alpha \equiv \frac{\rho R^2}{M^2} = 6 \frac{M_{\rm Pl}^2}{M^2} \Phi_{\rm N}$

Tor sufficiently massive objects, such that $\alpha \gg 1$, $\delta \phi \sim \frac{\dot{\phi}}{M^2} \frac{\delta \mathcal{M}}{r}$ solution is suppressed by thin-shell effect:

$$\phi_{\text{exterior}}(r) \sim \frac{1}{\alpha} \frac{\mathcal{M}}{M_{\text{Pl}}^2 r} + \phi_0$$

@ For small objects, $lpha \ll 1$, we find $\phi pprox \phi_0$ everywhere

$$\Rightarrow \quad \phi_{\text{exterior}}(r) \sim \frac{\mathcal{M}}{M_{\text{Pl}}^2 r} + \phi_0$$

Parameter Constraints

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \frac{\phi^2}{2M^2}T^{\mu}_{\ \mu}$$

Necessary (and sufficient) condition is that Milky Way has thin shell:

$$\alpha_{\rm G} = 6 \frac{M_{\rm Pl}^2}{M^2} \Phi_{\rm G} \gtrsim 1$$

$$\Phi_{\rm G} \sim 10^{-6}$$



 $\lambda \sim \frac{M_{\rm Pl}^4 H_0^2}{M^6} \gtrsim 10^{-100}$

>
$$M \lesssim 10^{-3} M_{\rm Pl}$$

$$\implies \mu \sim \frac{M_{\rm Pl}}{M} H_0 \gtrsim {\rm Mpc}^{-1}$$

Predictions for Tests of Gravity

Test	Effective parameter	Current bounds
Time delay/light deflection	$ \gamma - 1 \approx 10^{-5}$	$ \gamma - 1 \approx 10^{-5}$
Nordvedt effect	$ \eta_{\rm N} \sim 10^{-4}$	$ \eta_{\rm N} \sim 10^{-4}$
Mercury perihelion shift	$ \gamma - 1 \approx 4 \cdot 10^{-4}$	$ \gamma - 1 \approx 10^{-3}$
Binary pulsars	$\omega_{\rm BD}^{\rm eff}\gtrsim 10^6$	$\omega_{\rm BD}^{\rm eff}\gtrsim 10^3$

Astrophysical signatures

Khoury and Weltman (2004) Hui, Nicolis and Stubbs (2009)

Look at dwarf galaxies in voids



 ${\, @ }$ Stars are screened ($\Phi \sim 10^{-6}$), but hydrogen gas is unscreened. (Gas itself has only $\, \Phi \sim 10^{-11}$.)

Should find systematic O(1) discrepancy in the mass estimates based on these two tracers.

NOTE: Effect also possible in chameleon theory but not generic. In the symmetron case, it is generic. Tantalizing Hints?Wyman & J. Khoury, astro-ph/1004.2046
Lima, Wyman & J. Khoury, in progressi) Large Scale Bulk FlowsLocal bulk flow within $50 h^{-1}$ Mpc is 407 ± 81 km/s
Watkins, Feldman & Hudson (2008)• LCDM prediction is ≈ 180 km/s

Find: $v < 240 \ {\rm km/s}$

ii) Bullet Cluster (1E0657-57) • Requires $v_{infall} \approx 3000 \text{ km/s}$ at 5Mpc separation Mastropietro & Burkett (2008) • Probability in LCDM is between 3.3×10^{-11} and 3.6×10^{-9}

Lee & Komatsu (2010)

Find: 10^4 enhancement in prob.

iii) Void phenomenon

Peebles, astro-ph/0712.2757 Nusser, Gubser & Peebles, PRD (2005)

$$V(r)=-rac{eta Gm^2}{r}e^{-r/r_s}$$
 with $eta\sim \mathcal{O}(1)\;;\;\;r_s\sim \mathrm{Mpc}$

* However, Yukawa force is tightly constrained on galactic scales: $\beta < 0.1$ Kesden & Kamionkowski, PRL (2007) (See, however, Peebles et al. (2009).)

But screening mechanism helps...



Conclusions

If new forces are associated with dark sector, then some screening mechanism is required by local tests of gravity

Chameleon and Symmetron mechanisms rely on densitydependent mass and coupling, respectively.

Rich phenomenology for laboratory, solar-system and cosmological tests of gravity

Cosmological consequences?

Peculiar velocities, high-velocity mergers, void phenomenon

Topological defects

1. Symmetron Defects Hinterbichler, Hui & Khoury, in progress In void regions larger than $\mu^{-1} \approx {
m Mpc}$, symmetron takes values $\phi=\pm\mu/\sqrt{\lambda}$



Multiple symmetrons \implies global strings, monopoles...?

2. Cosmology

Levy, Matas, Hinterbichler, Hui & Khoury, in progress

* Hubble mass:

e.c

s:

$$V(\phi) = -\frac{1}{2}\mu^{2}\phi^{2} + \frac{1}{4}\lambda\phi^{4}$$

 $m = \mu$

More general $V(\phi)$

 $m = H_0$

.
$$V(\phi) = H_0^2 M_{\rm Pl}^2 \left(e^{-\phi^2/M^2} + \frac{M}{M_{\rm Pl}} e^{\phi^2/M_{\rm Pl}^2} \right)$$

* Self-acceleration? $\tilde{g}_{\mu\nu} = \left(1 + \frac{\phi^2}{2M^2} + \mathcal{O}\left(\frac{\phi^4}{M^4}\right)\right)^2 g_{\mu\nu}$

If no acceleration in Einstein frame, then can we have acceleration in Jordan frame because $\Delta\phi\sim M$?

Fixing Ideas

$$V_{\text{eff}}(\phi) = \frac{1}{2} \left(\frac{\rho}{M^2} - \mu^2\right) \phi^2 + \frac{1}{4}\lambda\phi^4$$

Gravitational-strength symmetron-mediated force in vacuum

$$\phi_0 \equiv \frac{\mu}{\sqrt{\lambda}} \sim \frac{M^2}{M_{\rm Pl}} \ll M$$

Hence field excursion is within validity of effective theory, i.e. can consistently neglect $\,\mathcal{O}(\phi^4/M^4)$ corrections to matter coupling.

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Potential becomes tachyonic around current cosmic density

$$\mu^2 \sim \frac{H_0^2 M_{\rm Pl}^2}{M^2} \implies \lambda \sim \frac{M_{\rm Pl}^4 H_0^2}{M^6} \ll 1$$

Will see later that local tests of gravity constrain $M \lesssim 10^{-3} M_{
m Pl}$

$$\implies m_0 = \sqrt{2}\mu \sim \frac{M_{\rm Pl}}{M}H_0 \sim {\rm Mpc}^{-1}$$

Gravitational-strength, Mpc-range 5th force in voids.

Macroscopic Violations of Equivalence Principle Khoury & Weltman (2003); Hui, Nicolis and Stubbs (2009)

Because of thin-shell screening, macroscopic objects fall with different acceleration in g-field

$$\vec{a} = -\vec{\nabla}\Phi + (1-\epsilon)\frac{\phi}{M^2}\vec{\nabla}\phi$$

Unscreened objects ($\epsilon = 1$) follow geodesics in Jordan frame
Screened objects ($\epsilon = 0$) do not.

To maximize effect, look for

– large (~ Mpc) void regions, so that symmetry is broken and $~\bar{\phi}/M^2 = 1/M_{\rm Pl}$

– look for unscreened objects (i.e. $\Phi < 10^{-7}$) in these voids

Distinguishable from Other Screening Mechanisms

Chameleon

Optimizable Potential is non-renormalizable, e.g. $V(\phi) = M^{4+n}/\phi^n$

Tightest constraint comes from laboratory tests of gravity, and this results in tiny signals for solar system tests <u>Khoury & Weltman (2003)</u>

Galileon

$$3\nabla^2 \pi + \frac{1}{\Lambda_s^3} \left[(\nabla^2 \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \right] = \frac{\rho}{2M_{\rm Pl}}$$

 $V_{\rm eff}(\phi)$

 $V(\phi)$

Predicts LLR signal measurable by APOLLO, but insignificant timedelay/light deflection signals. Dvali, Gruzinov and Zaldarriaga (2002)

No macroscopic violations of EP Hui, Nicolis and Stubbs (2009)

Strong coupling?

$$V(\phi) = \frac{M^5}{\phi} \qquad M = 10^{-3} \text{ eV}$$

Perturb around minimum:

$$V = \bar{V} + \ldots + \frac{\delta \phi^n}{\Lambda^{n-4}} + \ldots$$

where

$$\frac{\Lambda}{M} = \left(\frac{\bar{\phi}}{M}\right)^{\frac{n+1}{n-4}} = \left(\frac{M^2}{m^2}\right)^{\frac{n+1}{3(n-4)}} > \left(\frac{M^2}{m^2}\right)^{\frac{1}{3}}$$

 $V_{\rm eff}(\phi)$

 $V(\phi)$

Cosmologically: $m \sim Mpc^{-1} \implies \Lambda \sim 10^5 \text{ GeV}$ Locally: $m \sim 10^{-3} \text{ eV} \implies \Lambda \sim 10^{-3} \text{ eV}$

 $\begin{array}{l} \mbox{Relation to f(R) gravity} & \mbox{Carroll, Duvvuri, Trodden & Turner (2004);} \\ S = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4 x \sqrt{-\tilde{g}} f(\tilde{R}) + S_{\rm matter}[\tilde{g}_{\mu\nu}] \end{array}$

Special case of chameleon theories:

$$S = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-\tilde{g}} \left\{ f(\psi) + \frac{df}{d\psi} (\tilde{R} - \psi) \right\} + S_{\rm matter} [\tilde{g}_{\mu\nu}]$$
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$$\implies S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + S_{\rm matter} \left[g_{\mu\nu} e^{\sqrt{2/3}\phi/M_{\rm Pl}} \right]$$
$$\frac{M_{\rm Pl}^2}{M_{\rm Pl}^2} \left(g_{\mu\nu} \frac{df}{df} - f \right)$$

 $V = \frac{11 \left(\frac{d\varphi}{d\psi} - \frac{d\varphi}{d\psi}\right)}{2 \left(\frac{df}{d\psi}\right)^2}$

where

Relation to f(R) gravity $S = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-\tilde{g}} f(\tilde{R}) + S_{\rm matter}[\tilde{g}_{\mu\nu}]$ Carroll, Duvvuri, Trodden & Turner (2004); Capozziello, Carloni & Troisi (2004)

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where
$$V = \frac{M_{\rm Pl}^2 \left(\psi \frac{df}{d\psi} - f \right)}{2 \left(\frac{df}{d\psi} \right)^2}.$$