Scaling FFAG lattices for muon acceleration

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Motivations

Use the large transverse acceptance of scaling FFAG lattices while using constant RF frequency acceleration to reach high accelerating gradient.
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Use the large **transverse acceptance** of scaling FFAG lattices while using constant RF frequency acceleration to reach **high accelerating gradient**.

Possible with **harmonic number jump** acceleration!
Outline

I. Reminder on harmonic number jump acceleration.

II. Harmonic number jump with RF cavities all around the ring.
   1- Each cavity has to work at a different frequency: need for a double beam lattice.
   2- Lattice example and tracking results.
   3- Issue of the excursion: need dispersion suppressor!

III. Scaling FFAG lattice with reduced excursion areas.
   1- Example of a FFAG dispersion suppressor.
   2- Lattice example Lattice details and tracking results.
Reminder on harmonic number jump acceleration

To jump one harmonic every turn: \( T_{i+1} - T_i = \frac{1}{f_{RF}} \)
Reminder on harmonic number jump acceleration

To jump one harmonic every turn: \[ T_{i+1} - T_i = \frac{1}{f_{RF}} \]

Energy gain per turn must follow: \[ \Delta E_i = \frac{1}{f_{RF} \cdot \left[ \frac{\Delta T}{\Delta E} \right] E_i} \]

Figure 1 - Revolution time as a function of particle energy in the case of a 3 to 10 GeV scaling FFAG ring, with \( k = 145 \) and average radius = 120 m.
HNJ with cavities distributed around the ring

Figure 2 - N cavities homogeneously distributed around the ring.

Assuming that the initial number of harmonic $h_0$ is large we get\(^(*)\):

$$f_k \approx f_0 \left(1 - \frac{1}{h_0} \cdot \frac{k}{N}\right)$$

Every cavity working at a constant frequency $f_k$ but the frequency has to be tuned to a slightly different value!

\(^(*)\)look at the proceedings of PAC’09 for all details.

$\mu^+$ and $\mu^-$ beams cannot be accelerated simultaneously if they circulated in opposite directions...
Need for a double beam lattice

A solution to circulate a particle and its antiparticle in the same direction in a scaling FFAG ring is to use a FD-symmetric lattice:

Figure 3 - Double beam FFAG lattice (k = 145). Closed orbits of $\mu^+$ and $\mu^-$ circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.
**3 to 10 GeV muon double beam FFAG**

<table>
<thead>
<tr>
<th>Table 1 - ring parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
</tr>
<tr>
<td>Number of cells</td>
</tr>
<tr>
<td>Field index $k$</td>
</tr>
<tr>
<td>Packing factor</td>
</tr>
<tr>
<td>$B_{max}$ (at 10 GeV)</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
</tr>
<tr>
<td>Verti. phase adv. per cell</td>
</tr>
<tr>
<td>Mean RF frequency</td>
</tr>
<tr>
<td>RF peak voltage</td>
</tr>
<tr>
<td>Number of RF cavities</td>
</tr>
</tbody>
</table>

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring.
1st example: 3 to 10 GeV muon double beam FFAG

4D tracking - 8 turns acceleration cycle with a constant RF peak voltage
= 1.6 GV/turn:

Figure 5 - 8 turns acceleration cycle plotted in the longitudinal phase space, at
the location of the first cavity. Initial beam emittance is
0.21 eV.s x 10 000 π mm.mrad (normalized).
Issue of the excursion: need for dispersion suppressor insertions!

Harmonic jump condition: \( T_{i+1} - T_i = \frac{1}{f_{RF}} \)

In the same time: \( \frac{\Delta C_i}{\beta c} = T_{i+1} - T_i \)

In case of highly relativistic particles: \( \Delta R_i \approx \frac{c}{2\pi f_{RF}} = \frac{\lambda_{RF}}{2\pi} \)

**average excursion** = \( \lambda_{RF} \cdot \frac{N_{\text{turns}}}{2\pi} \) \( \rightarrow \) Need for excursion reduced areas!
Dispersion suppressor with FFAG magnets

\[
\frac{2}{k_2 + 1} = \frac{1}{k_1 + 1} + \frac{1}{k_3 + 1}
\]
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Figure 4 (slide #10)- Schematic view of a 3 to 10 GeV double beam muon FFAG ring.

Figure 8 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 2 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Figure 8 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 2 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Table 2 - Ring main cells parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>120 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>$2 \times 30$</td>
</tr>
<tr>
<td>cell opening angle</td>
<td>5 deg.</td>
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<tr>
<td>Field index $k$</td>
<td>145</td>
</tr>
<tr>
<td>Packing factor</td>
<td>0.9</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>2.3 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>92.9 deg.</td>
</tr>
<tr>
<td>Verti. phase adv. per cell</td>
<td>31.1 deg.</td>
</tr>
</tbody>
</table>

Figure 8 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 2 excursion reduced insertions.
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Figure 8 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 2 excursion reduced insertions.

Table 3 - Dispersion suppressor cells parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>120 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>4 \times 2</td>
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<tr>
<td>cell opening angle</td>
<td>4.24 deg.</td>
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<tr>
<td>Field index ( k )</td>
<td>192.4</td>
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<tr>
<td>Packing factor</td>
<td>0.9</td>
</tr>
<tr>
<td>( B_{\text{max}} )</td>
<td>2.7 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>90.0 deg.</td>
</tr>
<tr>
<td>Verti. phase adv. per cell</td>
<td>26.6 deg.</td>
</tr>
</tbody>
</table>
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Figure 8 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 2 excursion reduced insertions.

Table 4 - excursion reduced areas cells parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean radius</td>
<td>360 m</td>
</tr>
<tr>
<td>Number of cells</td>
<td>2×10</td>
</tr>
<tr>
<td>细胞 opening angle</td>
<td>1.304 deg.</td>
</tr>
<tr>
<td>Field index $k$</td>
<td>858.1</td>
</tr>
<tr>
<td>Packing factor</td>
<td>0.3</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>2.4 T</td>
</tr>
<tr>
<td>Horiz. phase adv. per cell</td>
<td>55.9 deg.</td>
</tr>
<tr>
<td>Verti. phase adv. per cell</td>
<td>14.8 deg.</td>
</tr>
</tbody>
</table>
Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 9 - Tune variation between 3 and 10 GeV in the lattice with insertions (from stepwise tracking in a soft edge field model).
Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 10 - **Horizontal** beta function at 6 GeV (half a turn is presented).
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

Figure 11 - **Vertical** beta function at 6 GeV (half a turn is presented).
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Beta function variation with energy:

Figure 12 - Horizontal beta function at 3 GeV (blue) and 6 GeV (red).
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Very large transverse acceptance: here ~ 50 000 π.mm.mrad (normalized) at 6 GeV.

Figure 13 - **Horizontal** phase space plot of 5 particles ($E_{\text{kin}} = 6$ GeV) with different initial amplitudes (over 300 turns).
Conclusion

Advantages of this scheme:
* Large transverse acceptance.
* Large longitudinal acceptance, and no emittance degradation during acceleration.
* Possible with RF frequency in the 200 MHz to 400 MHz range.
* Can accelerate $\mu^+$ and $\mu^-$ simultaneously.

To be improved:
* Assuming super-ferric type of magnets (Bmax ~ 2.5T) ring size is still large (about 850 m circumference).
* Excursion in the reduced excursion area is still about 0.5 m-, needs to be further reduced.
Thank you!
Additional material...
First example: 3 to 10 GeV muon double beam FFAG

4D tracking - 8 turns acceleration cycle with a constant RF peak voltage = 1.6 GV/turn:

Figure 5 - 8 turns acceleration cycle plotted in the **longitudinal phase space**, at the location of the first cavity. Initial beam emittance is $0.21 \text{ eV.s x 10 000 } \pi \text{ mm.mrad (normalized)}$.

Figure 6 - First turn (red squares) and last turn (green dots) of the 8 turns acceleration cycle plotted in **transverse phase space**. Initial beam emittance is $0.21 \text{ eV.s x 10 000 } \pi \text{ mm.mrad (normalized)}$. 
3 to 10 GeV muon double beam FFAG + excursion reduced areas

Beta function variation with energy:

**Horizontal** beta function at 6 GeV (red) and 3 GeV (blue).

**Horizontal** beta function at 6 GeV (red) and 10 GeV (Green).
3 to 10 GeV muon double beam FFAG + excursion reduced areas

**Vertical** beta function variation with energy:

**Vertical** beta function at 6 GeV (red) and 3 GeV (blue).

**Vertical** beta function at 6 GeV (red) and 10 GeV (Green).