A Unified Model for inelastic e-N and n eutrino-N cross sections at all $Q^2$

2009 Updates to Bodek-Yang Model

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Modeling neutrino cross sections

- Describe DIS, resonance, even photo-production ($Q^2=0$) in terms of quark-parton model. With PDFS, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section.
- Challenge:
  - Understanding of high x PDFs at very low $Q^2$?
  - Understanding of resonance scattering in terms of quark-parton model?
  - What happens near $Q^2=0$?

- NNLO QCD + Target Mass approach
  Accounts for non-pert. QCD effects at low $Q^2$ but blows up at $Q^2=0$
- Simpler to implement effective LO approach (pseudo NNLO: for MC)
  Use effective LO PDFs with a new scaling variable, $\xi_w$ to absorb target mass, higher twist, missing higher orders

- \[ W = \frac{Q^2 + m_t^2 + 0(m_t^2 - m_f^2) + A}{M_N (1 + (1 + Q^2/\nu^2)^{1/2} + B)} \]
- $\xi_w = Q^2 / 2 M_N$
- $X_{bj} = Q^2 / 2 M_N$
1. Start with GRV98 LO \((Q^2_{\text{min}}=0.80 \text{ GeV}^2)\) - dashed line- describe F2 data at high \(Q^2\).
2. Replace the Xbj with a new scaling, \(\xi_w\).
3. Multiply all PDFs by K factors for photo prod. limit and higher twist.
   \[
   \sigma(\gamma) = \frac{4\pi\alpha}{Q^2} \times F_2(x, Q^2)
   \]
   \[
   K_{\text{sea}} = \frac{Q^2}{[Q^2+C_{\text{sea}}]}
   \]
   \[
   K_{\text{val}} = [1 - G_D^2(Q^2)] \times \frac{[Q^2+C_{2V}]}{[Q^2+C_{1V}]} \text{ motivated by Adler Sum rule}
   \]
   where \(G_D^2(Q^2) = \frac{1}{1+Q^2/0.71}\).
4. Freeze the evolution at \(Q^2 = Q^2_{\text{min}}\).
   \(- F_2(x, Q^2 < 0.8) = K(Q^2) \times F_2(X_w, Q^2=0.8)\)

- Fit to all DIS F2 P/D (with low \(x\) HERA data)
  \[A=0.418, B=0.222\]

\[
\begin{align*}
C_{\text{sea}} &= 0.381, C_{1V} = 0.604, C_{2V} = 0.485 \\
\chi^2/\text{DOF} &= 1268/1200 \text{ Solid Line}
\end{align*}
\]

2004 update: Separate K factors for \(uv, dv, us, ds\)

\(A\) : initial binding/TM effect+ higher order
\(B\) : final state mass \(m_f^2, \Delta m^2\).

K Factor: Photo-prod limit \((Q^2 = 0)\), Adler sum rule

\[
\xi = \frac{Q^2 + m_f^2 + O(m_f^2-m_i^2) + A}{M\nu (1+(1+Q^2/\nu^2)^{1/2} + B}
\]

\(X_{\text{bj}} = Q^2/2 \ M\nu\)
GRV98 + B-Y 2004 Fit results

separate K factors
for uv, dv, us, ds

Separate K factors for uv, dv, us, ds provided additional parameters. They provide separate tuning for H and D data, but are not important for Heavy nuclei.
Fit results GRV98 + B-Y 2004 (SLAC, BCDMS, NMC) H + D

F2 proton

Proton experiment data fit

Deuteron experiment data fit

F2 deuterium

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Photo-production (Proton)

- Data
- GRV98 LO (no charm)
- GRV98 LO

Photo-production (Deuteron)

- Data
- GRV98 LO (no charm)
- GRV98 LO

Fit results GRV98 + B-Y 2004

H1

- $x = 0.00013$ [F2+7.2]
- $x = 0.0002$ [F2+6.9]
- $x = 0.00033$ [F2+6.3]
- $x = 0.0005$ [F2+5.8]
- $x = 0.0008$ [F2+5.4]
- $x = 0.0013$ [F2+5.1]
- $x = 0.002$ [F2+4.5]
- $x = 0.0032$ [F2+3.9]
- $x = 0.005$ [F2+3]
- $x = 0.008$ [F2+2.4]
- $x = 0.013$ [F2+2.1]
- $x = 0.02$ [F2+1.8]
- $x = 0.032$ [F2+1.5]
- $x = 0.05$ [F2+1.2]
- $x = 0.08$ [F2+0.9]
- $x = 0.13$ [F2+0.6]
- $x = 0.2$ [F2+0.3]
- $x = 0.32$ [F2+0]
Fit results GRV98 + B-Y 2004 muon scattering

NMC [Proton target]

NMC [Deuteron target]
Resonance F2 proton

Resonance F2 deuterium

Fit works on resonance region - Resonance data are not included in the fit!!!
2xF₁ data

- All DIS e/μ F₂ data are well described
- Photo-production data (Q²=0) also work: thus included in the latest fit
- 2xF1 data (Jlab/SLAC) also work: using F₂(ξ_w)+R1998

GRV98 + B-Y 2004
How model uses only H and D data.
For lepton/muon cross sections on nuclear targets - need to correct for Nuclear Effects measured in e/muon expt. Use also for neutrino expt.
( Note nuclear effects can be different for neutrinos)

Figure 5. The ratio of $F_2$ data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments (SLAC, NMC, E665). The band show the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].
Comparison with CCFR neutrino data (Fe) (assume $V=A$) apply nuclear corrections

- Apply nuclear corrections using e/m scattering data.
- Calculate $F_2$ and $xF_3$ from the modified PDFs with $\xi_w$.
- Use $R=R_{world}$ fit to get $2xF_1$ from $F_2$.
- Implement charm mass effect through $\xi_w$ slow rescaling algorithm, for $F_2$, $2xF_1$, and $xF_3$.

Our model describes CCFR diff. cross sect. ($E\nu=30-300$ GeV) well.
Note that no neutrino data was included in fit. (However, Let's look in more detail).

--- $\xi_w$ PDFs GRV98 modified (red line)
--- GRV98 ($x,Q^2$) unmodified (black)
Left: CCFR neutrino data -55 Gev
Right: CCFR anti-neutrino data , -55 Gev (NuFact03 version)
Note: GRV98 + B-Y 2004 is for free nucleons (H+D). Electron and muon data are corrected for radiative corrections. In addition, GRV98 has no charm sea.

Published neutrino differential cross sections:
(1) Have no radiative corrections
(2) Are on nuclear targets
(3) Have contributions from XF3 and include both axial and vector contributions.
(4) Some are at very high energy which include a contribution from the charm sea.

In order to compare to neutrino data:
(1) We need to account for difference in the scaling violations in XF3 and F2 (2009 update 1)
(2) We need to make duality work in the resonance region at very low Q2 if we want to match to the resonance region, (2009 update 2)
(3) We need to account for difference in axial and vector structure functions at low Q2 (2009 update 3)
(4) We apply and X dependent nuclear correction.
(5) However, nuclear effects may be different for muons and neutrinos, different for axial versus vector, different for F2, XF3 (will be studied in MINERva)
(6) We should to add radiation to GRV98 + B-Y 2004 (or radiatively correct the neutrino data) - not done
(7) We should add charm sea contribution at very large energy (not done)
Comparison with updated model (assume $V=A$)

CCFR Fe data/ (GRV98 + B-Y 2004)

Model underestimates neutrino data at lowest $x$ bin. At high energy, some may be from missing radiative corrections and $c$-$c\bar{c}$ contribution.
Comparison with CDHSW neutrino data (Fe)

Model underestimates neutrino data at lowest x bin. At high energy, some may be from missing radiative corrections and c-cbar contribution.
Comparison with CDHSW anti-neutrino data (Fe)

Model underestimates antineutrino data at lowest x bin - also lowest Q2. At high energy, some may be from missing radiative corrections and c-cbar contribution.
Comparison with CHORUS data (Pb)

E=15 GeV

E=90 GeV
How should the model be used

- Duality is not expected to work for quasielastic or the delta. This is because these cross sections have definite isospin final states. Therefore PDFs will not give the correct ratio of neutrino vs. antineutrino and proton versus neutron scattering for quasielastic and delta production.
- Duality should work in the region of higher resonances since these regions include several resonances with different isospins.
- MINOS has used the 2004 Bodek-Yang model above W=1.8.
- They used other models for quasielastic, the delta, and the 1520 resonance region and matched them to Bodek-Yang in the W=1.8 region.
Total cross sections
Bodek-Yang 2004 used above $W=1.8$ AND matched to resonance and quaselastic models.
Find that predicted total neutrino and antineutrino cross sections are lower than high energy measurements (5%). The antineutrino to neutrino ratio is also low.
Some may come from the need to apply radiative corrections and include the $c$-$c$ bar sea at very high energy (no $c$-$c$ bar sea in GRV98). Some may be differences in nuclear effects between electrons and neutrinos—But is this all?
2009 Update 1: \( H(x) = \text{NLO Correction to } xF_3 \)

- Scaling variable, \( \xi_w \) absorbs higher order QCD and higher twist in \( F_2 \), but \( xF_3 \) may be different.
  (\( F_2 \) data was used in the fitting our corrections to leading order PDF)
- 1st Update: Use double ratio correction \( H(x) \) from QCD

\[
\frac{x F_3(\text{NLO})}{x F_3(\text{LO})} \cdot \frac{F_2(\text{NLO})}{F_2(\text{LO})}
\]

=> not 1 but indep. of \( Q^2 \)
Effect of $x F_3$ NLO correction $H(x)$

- Parameterized $x F_3$ correction as a function of $x = H(x)$
- Neutrino cross section down by 1%
- Anti-neutrino cross section up by 3%
2009 update 2: Axial contribution at low Q2

\[ K_i^{\text{vector}}(Q^2) = \frac{Q^2}{Q^2 + C}, \quad K_i^{\text{axial}}(Q^2) = \frac{Q^2 + C_1}{Q^2 + C_2}, \]

\[ F_2^\nu(x, Q^2) = \sum_i \left[ K_i^{\text{vector}}(Q^2) + K_i^{\text{axial}}(Q^2) \right] \times \xi_w \left[ q_i(\xi_w, Q^2) + \bar{q}_i(\xi_w, Q^2) \right] \]

\[ xF_3^\nu(x, Q^2) = 2 \sum_i \left[ \sqrt{K_i^{\text{vector}}(Q^2)K_i^{\text{axial}}(Q^2)} \right] \times H(x, Q^2) \left[ xq_i(\xi_w, Q^2) - x\bar{q}_i(\xi_w, Q^2) \right], \]

- In our neutrino previous cross section model we assumed \( K^{\text{axial}} = K^{\text{vector}} \).
  This is only true for free quarks (which is a correct assumption for \( Q^2 > 0.5 \text{ GeV}^2 \)).
- However: We expect that axial-vector is not suppressed at \( Q^2=0 \).
  - 2009 Update 2: \( K^{\text{axial}} = 1 \) as a first try
Axial-contribution

CCFR diff. cross at $E_\nu = 55$ GeV

$K_{axial} = \frac{Q^2}{(Q^2+C)}$

Black line GRV98; red line with B-Y 2004 And $K_{axial}=K_{vector}$, blue $K_{axial}=1$

$K_{axial} = 1$ better
CCFR diff. cross at $E_\nu = 35$ GeV

$K_{\text{axial}} = K_{\text{vector}} = \frac{Q^2}{Q^2 + C}$

$K_{\text{axial}} = 1$

Black line GRV98; red line with B-Y 2004

And $K_{\text{axial}} = K_{\text{vector}}$, blue $K_{\text{axial}} = 1$
In resonance region, duality works down to $Q_2=0.5$ GeV$^2$, but breaks down at $Q_2=0$.
Not important for the Vector part, since $Q_2=0$ contributes zero to the vector part of the neutrino cross section.

$$\sigma(\gamma\text{-proton}) = 4\pi\alpha/Q^2 \ast F_2(\xi_w, Q^2)$$
where $F_2(\xi_w, Q^2) = Q^2/(Q^2 + C) \ast F_2(\xi_w)$

Update 3: Improve the model so that it is also valid in the resonance region at $Q_2=0$
We will fix it by applying a low Ehad K factor
Important for axial part
Update (3): apply a low $\nu(E_{\text{had}})$ K factor

$$K(\nu) = (\nu^2 + C_{2\nu}) / \nu^2$$

Where $C_{2\nu} = 0.20$

It makes duality work for resonance all the way to $Q^2 = 0$.

So vector part is now modeled everywhere including resonance region down to very low $\nu\text{u}$ and very low $Q^2$.

For a heavy nucleus, Fermi motion will smear all of the resonances.
Low nu K factor pushes the validity of the model for electron scattering in the resonance region down to $Q^2=0$

- Proton data
- (note that for nuclear targets the resonances will be smeared by Fermi Motion)

**Photo-production $Q^2=0$**

Black line includes Low Ehad K factor

Red line does not
Summary and Discussions

- We updated our Effective LO model with $\xi_w$ and $K(Q^2)$ factors.
- (1) Updated to include a low $\nu K$ factor to describe all charged lepton inelastic continuum as well as resonance data including photo-production data. The vector part of the neutrino cross section is now modeled very well. Note: By Gauge Invariance, the vector structure functions must go to zero at $Q^2=0$ for both resonances and inelastic continuum.
- (2) Updated to account for the difference in the higher order QCD corrections between $F_2$ and $X F_3$. This is accounted for with a $H(x)$ factor. Therefore, the axial part is also well described for $Q^2>1$ GeV$^2$, where axial and vector are expected to be the same.
- (3) Updated to use $K_{\text{axial}}(Q^2)=1$ for both the resonance and inelastic continuum region. This is expected since we know that neutrino quasielastic and resonance production form factor are not zero at $Q^2=0$.
- The lowest $Q^2$ bins in the neutrino and antineutrino measured differential cross sections favor $K_{\text{axial}}(Q^2)=1$. Needs to be studied in more detail.
- The total cross section as measured in high energy neutrino scattering favors $K_{\text{axial}}(Q^2)=1$. 

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Things left to do

- Use $K_{\text{axial}} (Q^2)=1$ for now, but it will be tuned further in the future.
- We can tune the axial vector $K$ factor by including low $Q^2$ neutrino and antineutrino differential cross sections in the fit.

However, the electron data has been radiatively corrected. A proper comparison to neutrino differential cross sections needs to include both radiative corrections and the $c$-$\bar{c}$ contribution at high energies (which are not included in the GRV98 PDFS). And what about the nuclear effects?

We plan tune $K_{\text{axial}} (Q^2)$ to get better agreement with the neutrino and antineutrino measured total cross sections (Here we need to separately add the quasielastic, delta and $c$-$\bar{c}$ contributions, (but no need to include the radiative corrections since these integrate away in the total cross section). We will have this comparison soon.

- In the future more detailed information on the axial form factor would come from MINERvA: by combining JUPITER Jlab (e-N vector) with the MINERvA (neutrino-N vector+axial) data.
- There could be different nuclear effects ($e$ vs $\nu$), $F_2$ vs $xF_3$, and for axial $F_2$ versus vector $F_2$. This will also be studied in MINERvA.