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$\mathrm{B_d}$ and $\mathrm{B_s}$ mixing: mass and width differences and CP violation

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Outline

1. $\mathrm{B}-\overline{\mathrm{B}}$ mixing basics
2. Mass difference Δm
3. Width difference $\Delta\Gamma$
4. CP violation in mixing
5. Summary and Outlook

1. $\mathrm{B}-\overline{\mathrm{B}}$ mixing basics

Schrödinger equation:

$$i\frac{d}{d\,t}\begin{pmatrix}|\,B(t)\,\rangle\\ |\,\overline{B}(t)\,\rangle\end{pmatrix} = \left(M - i\,\frac{\Gamma}{2}\right)\begin{pmatrix}|\,B(t)\,\rangle\\ |\,\overline{B}(t)\,\rangle\end{pmatrix}$$

3 physical quantities in $\mathrm{B}-\overline{\mathrm{B}}$ mixing:

$$|M_{12}|\,,\qquad |\Gamma_{12}|\,,\qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Two mass eigenstates:

Lighter eigenstate: $|\,B_L\,\rangle = p|\,B^0\,\rangle + q|\,\overline{B}^0\,\rangle.$

Heavier eigenstate: $|\,B_H\,\rangle = p|\,B^0\,\rangle - q|\,\overline{B}^0\,\rangle$ with $|p|^2 + |q|^2 = 1.$

with masses $M_{L,H}$ and widths $\Gamma_{L,H}$.

Here B represents either B_d or B_s.

To determine $|M_{12}|$, $|\Gamma_{12}|$ and ϕ measure

$$\Delta m = M_H - M_L \simeq 2|M_{12}|,$$
$$\Delta\Gamma = \Gamma_L - \Gamma_H \simeq -\Delta m \,\mathrm{Re}\, \frac{\Gamma_{12}}{M_{12}} = 2|\Gamma_{12}| \cos\phi$$

and

$$a_{\mathrm{fs}} = \mathrm{Im}\, \frac{\Gamma_{12}}{M_{12}} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin\phi.$$

a_{fs} is the CP asymmetry in flavour-specific B decays (semileptonic CP asymmetry). a_{fs} measures CP violation in mixing.

Define the average rate $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$.

Standard Model expectations:

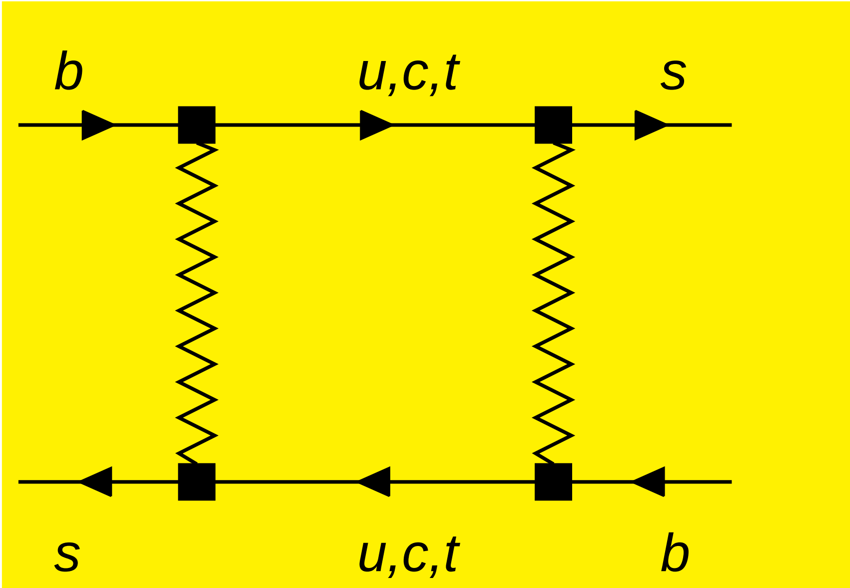
	B_d system	B_s system
$\Delta m \;=$	$0.5\,\text{ps}^{-1}$	$20\,\text{ps}^{-1}$
$\Delta\Gamma \;=$	$3\cdot 10^{-3}\,\text{ps}^{-1}$	$0.10\,\text{ps}^{-1}$
$\dfrac{\Delta\Gamma}{\Gamma} \;=$	$4\cdot 10^{-3}$	0.15
$\dfrac{\Delta\Gamma}{\Delta m} = \left\lvert\dfrac{\Gamma_{12}}{M_{12}}\right\rvert \cos\phi \;=$	$5\cdot 10^{-3} = \mathcal{O}\left(\dfrac{m_b^2}{M_W^2}\right)$	
$a_{\text{fs}} = \left\lvert\dfrac{\Gamma_{12}}{M_{12}}\right\rvert \sin\phi \;=$	$-5\cdot 10^{-4}$	$2\cdot 10^{-5}$
$\phi \;=$	$-0.9 = -5^\circ = \mathcal{O}\left(\dfrac{m_c^2}{m_b^2}\right)$	$4\cdot 10^{-3} = 0.2^\circ = \mathcal{O}\left(\lvert V_{us}\rvert^2 \dfrac{m_c^2}{m_b^2}\right)$

New physics

Standard Model:

M_{12} from dispersive part of box,
only internal t relevant;

Γ_{12} from absorptive part of box,
only internal u, c contribute.



New physics can barely affect Γ_{12}, which stems from tree-level decays.

M_{12} is very sensitive to virtual effects of new heavy particles.

$\Rightarrow$ $\Delta m \simeq 2|M_{12}|$ can change.

and in $\phi \simeq \arg(-M_{12}/\Gamma_{12})$ the GIM suppression of ϕ can be lifted.

$\Rightarrow$ $|\Delta\Gamma| = \Delta\Gamma_{\rm SM}\,|\cos\phi|$ is depleted

and $|a_{\rm fs}|$ is enhanced, by up to a factor of 200 in the B_s system.

To identify or constrain new physics one wants to measure both the magnitude and phase of M_{12}.

$$\rightarrow \qquad \Delta m = 2|M_{12}|$$

Information on $\arg M_{12}$ can be gained from mixing-induced CP asymmetries, in particular $a_{\rm mix}(B_s \to J/\psi\phi)$. This requires tagging, which is difficult at hadron colliders.

Three untagged measurements are sensitive to $\arg M_{12}$:

1. $|\Delta\Gamma| = \Delta\Gamma_{\rm SM}\,|\cos\phi|$
2. $a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi$ and
3. the angular distribution of $\overset{(-)}{B}_s \to VV'$, where V, V' are vector bosons.

2. Mass difference Δm

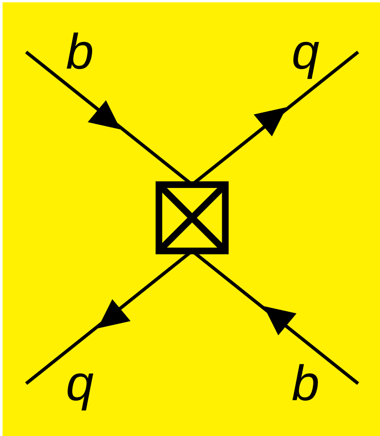
Δm is measured from the $\mathrm{B}\!-\!\overline{\mathrm{B}}$ oscillations, which are governed by $\cos(\Delta m t)$:

$$\mathcal{A}_0(t) = \frac{\Gamma(B(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(B(t) \to \overline{f})} = \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)}$$

Eq. (1.81) from *B physics at the Tevatron*

Local four-quark operator:

$$Q = \overline{q}_L \gamma_\nu b_L\, \overline{q}_L \gamma^\nu b_L$$



Theoretical uncertainty dominated by matrix element:

$$\langle\, \mathrm{B}^0\, |\mathrm{Q}|\, \overline{\mathrm{B}}^0\, \rangle = \frac{2}{3} m_B^2\, f_B^2\, B$$

Careful: Two definitions of B:

$\overline{\mathrm{MS}}$ scheme at scale $\mu = m_b$: $\quad B \approx 0.85$

scheme invariant definition: $\quad \widehat{B} = 1.5 B \approx 1.3$

I use the first definition in this talk.

Standard Model prediction:

New top mass: $m_t^{\mathrm{pole}} = 171.4 \pm 2.1$ GeV (Tevatron 2006)

$\Rightarrow \overline{\mathrm{MS}}$ mass $\overline{m}_t = 163.8 \pm 2.0$ GeV.

Effect of the downward shift of $\overline{m}_t$ by $3\,$GeV: $\quad \Delta m$ decreases by 2.7%.

$$\Delta m_d = (0.51 \pm 0.02)\,\mathrm{ps}^{-1} \left(\frac{|V_{td}|}{0.0092} \right)^2 \left(\frac{f_{B_d}}{200\,\mathrm{MeV}} \right)^2 \frac{B}{0.85}$$

$|V_{cb}| = 0.0415 \pm 0.0010 \quad \Rightarrow \quad |V_{ts}| = 0.0405 \pm 0.0010.$

$$\Delta m_s = (19.3 \pm 0.6)\,\mathrm{ps}^{-1} \left(\frac{|V_{ts}|}{0.0405} \right)^2 \left(\frac{f_{B_s}}{240\,\mathrm{MeV}} \right)^2 \frac{B}{0.85}$$

Sum rule and lattice predictions for f_{B_s} cover a wide range between $200\,$MeV and $290\,$MeV. Lattice predictions of f_{B_d} are even harder because of the chiral extrapolation to the small down quark mass.

Lattice results (Okamoto, Lattice 2005):

$$f_{B_s} = (260 \pm 36)\ \text{MeV} \qquad \text{HPQCD, } n_f = 2+1$$
$$B = 0.84 \pm 0.07,$$

Hence:

$$\Delta m_s = 22 \pm 8\ \text{ps}^{-1}$$

New $n_f = 2+1$ result (Lattice 2006)

$$f_{B_s}\sqrt{B} = 227 \pm 17\,\text{MeV} \qquad \text{Shigemitsu (HPQCD)}$$

which implies

$$\Delta m_s = 20.3 \pm 3.4\ \text{ps}^{-1}$$

If one assumes that there is no new physics in $\mathrm{B}-\overline{\mathrm{B}}$ mixing, one can determine $|V_{td}/V_{ts}|$, which determines one side of the unitarity triangle, from $\Delta m_d/\Delta m_s$.

$\longrightarrow$ see talk be t'Jampens

My focus: probing the hypothesis of new physics in $b \to s$ transitions. How much is Δm_s then constrained from Δm_d?

$$\Delta m = 20.0\, \xi^2 \frac{\Delta m_{B_d}}{R_t^2}$$

where

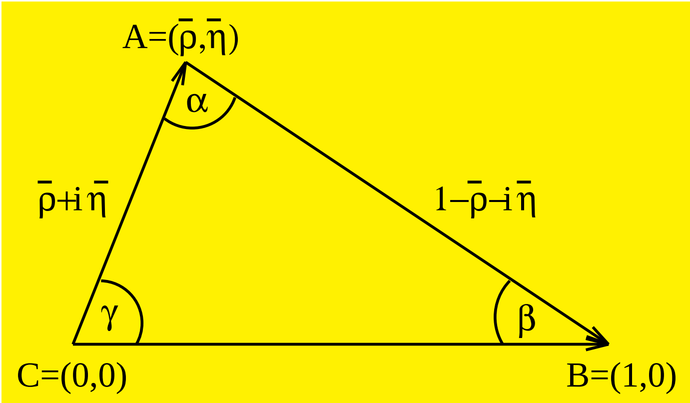
$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.21^{+0.05}_{-0.04}$$

HPQCD,JLQCD

and $B_{B_d} \simeq B_{B_s}$.

R_t is one side of the unitarity triangle:

$$R_t = \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2} \simeq \frac{|V_{td}|}{|V_{ts}V_{cd}|}$$



Without using $\Delta m_d/\Delta m_s$ in the UT fit, R_t is poorly known. The 2σ CL range is

$$R_t = 0.863^{+0.109}_{-0.107} \qquad \text{CKMFitter winter 2005/2006}$$

implying

$$\Delta m_s = 20^{+8}_{-6}\,\text{ps}^{-1}$$

Since there is a complex phase between the contributions from Standard Model and new particles, the precise CDF measurement

$$\Delta m^{\text{exp}} = 2|M_{12}^{\text{SM}} + M_{12}^{\text{new}}| = 17.31^{+0.33}_{-0.18}\,\text{stat} \pm 0.07\,(\text{syst})\,\text{ps}^{-1}$$

still allows

$$|M_{12}^{\text{new}}| \sim |M_{12}^{\text{SM}}|$$

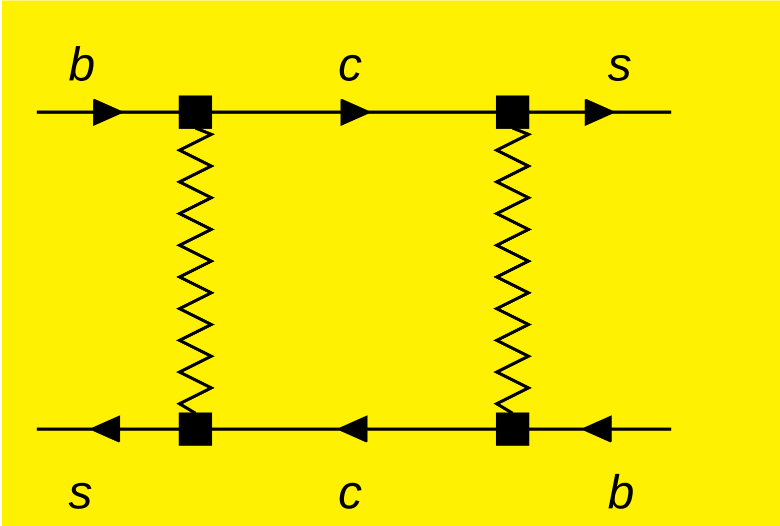
$\Rightarrow$ experimental information on $\arg M_{12}$ desirable

3. Width difference $\Delta\Gamma$

Consider $\Delta\Gamma_{B_s}$ in the Standard Model:

$B_s \sim \bar{b}s, \qquad \overline{B}_s \sim b\bar{s}$

Effect from internal up quarks negligible.



Standard Model: Identify mass eigenstates with CP eigenstates:

$$|\,B_{L,H}\,\rangle = \frac{1}{\sqrt{2}}\left[|\,B_s\,\rangle \mp |\,\overline{B}_s\,\rangle\right], \qquad B_L \text{ is CP-even and } B_H \text{ is CP-odd}$$

$$\Delta\Gamma_{B_s} \equiv \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|$$

$\Delta\Gamma_s$ stems from decays into final states which are common to B_s and $\overline{B}_s$ and contain a $(c,\bar{c})$ pair. CP-even final states like $D_s^+ D_s^-$ contribute positively to $\Delta\Gamma_s$, while decays into CP-odd states diminish $\Delta\Gamma_s$.

$\Delta\Gamma$ has a different structure than Δm:

- $\Delta\Gamma$ involves an operator product expansion at the scale m_b.
  $\Rightarrow$ expansion in Λ_{QCD}/m_b and $\alpha_s(m_b)$.
- There are two operators already at leading order of Λ_{QCD}/m_b.

Calculations:

Λ_{QCD}/m_b corrections: Beneke, Buchalla, Dunietz, Phys. Rev. **D54**, 4419 (1996)

α_s corrections: Beneke, Buchalla, Greub, Lenz, UN, Phys. Lett. B **459** (1999) 631.
Ciuchini, Franco, Lubicz, Mescia, Tarantino, JHEP **0308** (2003) 031.

New operator:

$$Q_S = \overline{b}_R s_L \, \overline{b}_R s_L$$

$$\langle B_s | Q_S | \overline{B}_s \rangle = -\frac{5}{12} M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2} f_{B_s}^2 \, B_S$$

Our 1998 prediction including corrections of order α_s and Λ_{QCD}/m_b:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 [\,0.006\,B \;+\; 0.172\,B_S \;-\; 0.063]$$

Pathological situation: Both the $1/m_b$ and α_s corrections are large and decrease $\Delta\Gamma$, leading to large uncertainties. Moreover B_S dominates over B, so that $\Delta\Gamma/\Delta m$ depends on B_S/B.

In the leading order of the $1/m_b$ expansion one first encounters a third operator:

$$\widetilde{Q}_S = \overline{b}^i_R s^j_L \, \overline{b}^j_R s^i_L,$$

where i, j are colour indices.
Then $\widetilde{Q}_S$ is eliminated in favour of

$$R_0 = \widetilde{Q}_S + Q_S + \frac{Q}{2} = \mathcal{O}(\frac{1}{m_b}, \alpha_s(m_b)),$$

which is one out of five operators appearing at $\mathcal{O}(1/m_b)$. The matrix element of $\widetilde{Q}_S$ is almost five times smaller than that of Q_S:

$$\langle B_s | \widetilde{Q}_S | \overline{B}_s \rangle = \frac{1}{12} M^2_{B_s} \frac{M^2_{B_s}}{(\bar{m}_b + \bar{m}_s)^2} f^2_{B_s} \widetilde{B}_S$$

Becirevic et al. (2001) find $\widetilde{B}_S = 0.91 \pm 0.09$.
New: Shigemitsu (HPQCD), Lattice 2006: $f_{B_s} \sqrt{\widetilde{B}_S} = 245 \pm 19$ MeV.

Eliminate Q_S in favour of $\widetilde{Q}_S$ to find:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{240\,\text{MeV}}\right)^2 \left[0.161\,B \;+\; 0.057\,\widetilde{B}_S \;-\; 0.042\right]$$
$$= 0.15 \pm 0.05 \qquad \text{for } f_{B_s} = 240 \pm 40\,\text{MeV}.$$

A. Lenz, U.N.

$\Rightarrow$ $\Delta\Gamma$ is now dominated by the term proportional to B and the $1/m_b$ corrections are smaller.

The size of the $1/m_b$ corrections has decreased from 33% to 19% of the leading order (in both $1/m_b$ and α_s) result. Also the size of the α_s corrections and the scale dependence have decreased.

Better quote:

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240\,\text{MeV}}\right)^2 \left[0.11\,B \;+\; 0.037\,\widetilde{B}_S \;-\; 0.027\right]$$
$$= 0.10 \pm 0.03\,\text{ps}^{-1}.$$

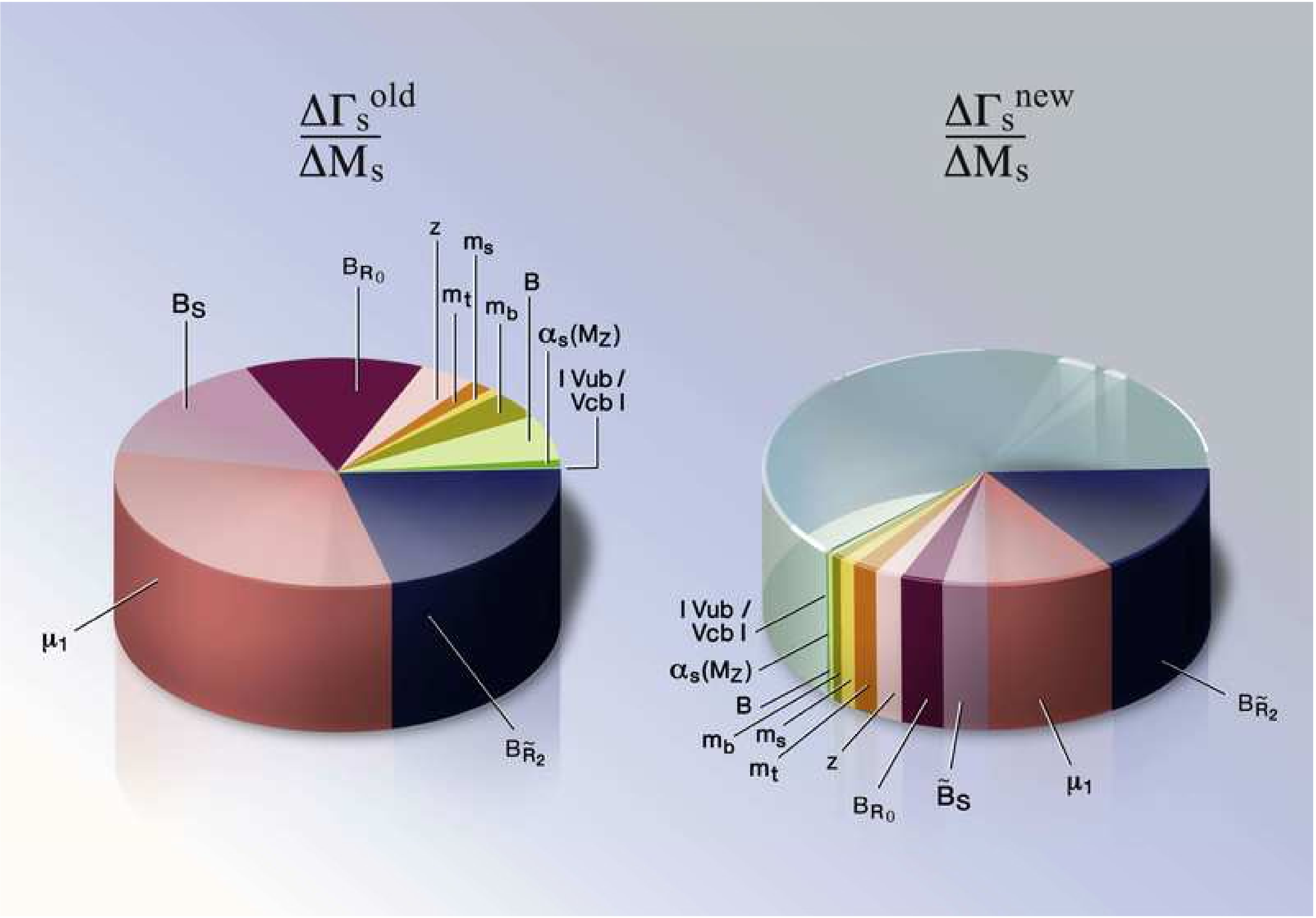
f_{B_s} drops out from $\Delta\Gamma/\Delta m$. Now include the uncertainties of the coefficients:

$$\frac{\Delta\Gamma}{\Delta m} = \left[34 \pm 6 + (16 \pm 1)\frac{\widetilde{B}_S}{B}\right] \cdot 10^{-4}$$
$$= (52 \pm 10) \cdot 10^{-4}$$

Here the matrix elements of the $1/m_b$-suppressed operators are evaluated in the vacuum insertion approximation (i.e. bag factors set to 1).

$$\Delta\Gamma_s \;=\; \frac{\Delta\Gamma}{\Delta m}\,\Delta m_s^{\text{exp}} \;=\; (0.090 \pm 0.017)\,\text{ps}^{-1}.$$

Next page: error budget



$\frac{\Delta\Gamma_s^{old}}{\Delta M_s}$
$\frac{\Delta\Gamma_s^{new}}{\Delta M_s}$
z
m_s
B_{R_0}
m_t
B
B_S
m_b
$\alpha_s(M_Z)$
$|V_{ub}/V_{cb}|$
μ_1
$B_{\tilde{R}_2}$
$|V_{ub}/V_{cb}|$
$\alpha_s(M_Z)$
B
m_b
m_s
m_t
z
B_{R_0}
$\tilde{B}_S$
μ_1
$B_{\tilde{R}_2}$

The largest uncertainties now come from ...

... the matrix element of the $1/m_b$-suppressed operator

$$\widetilde{R}_2 = \frac{1}{m_b^2}\,\overline{b}_L^i \overleftarrow{D}_\rho \gamma_\mu D^\rho s_L^j \,\overline{b}_L^j \gamma^\mu s_L^i$$

$\Rightarrow$ challenging lattice calculation

... the dependence on the renormalisation scale. Its reduction requires a NNLO calculation

$\Rightarrow$ challenging loop calculation

4. CP violation in mixing

$$a_{\rm fs} = {\rm Im}\,\frac{\Gamma_{12}}{M_{12}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi.$$

$a_{\rm fs}$ is the CP asymmetry in decays $B \to f$ which are flavour-specific, i.e.

$$\overline{B} \not\to f \quad \text{and} \quad B \not\to \overline{f}.$$

Examples: $B_{d,s} \to X\ell^+\nu_\ell$ or $B_s \to D_s^-\pi^+$.

$$a_{\rm fs} = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \overline{f})}$$

The time dependence of the decay rates $\Gamma(\overline{B}(t) \to f)$ and $\Gamma(B(t) \to \overline{f})$ drops out.

$a_{\rm fs}$ measures CP violation in mixing.

No tagging is necessary: With $\Gamma[f,t] = \Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to f)$,

$$a_{\rm fs}^{\rm untagged} = \frac{\Gamma[f,t] - \Gamma[\overline{f},t]}{\Gamma[f,t] + \Gamma[\overline{f},t]} = \frac{a_{\rm fs}}{2}\left[1 - \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)}\right]$$

Even time-integrated rates are useful:

$$A_{\rm fs,unt} \equiv \frac{\int_0^\infty dt [\Gamma[f,t] - \Gamma[\overline{f},t]]}{\int_0^\infty dt [\Gamma[f,t] + \Gamma[\overline{f},t]]} = \frac{a_{\rm fs}}{2}\,\frac{x^2+y^2}{x^2+1} \simeq \frac{a_{\rm fs}}{2},$$

where $x = \Delta m/\Gamma$, $y = \Delta\Gamma/(2\Gamma)$.

$\Rightarrow$ Only need to count e.g. positive vs. negative leptons from untagged B^0 decays.

$a_{\rm fs}$ in the Standard Model

$$a_{\rm fs} = \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}$$

$$a_{\rm fs}(B_s) = (2.0 \pm 0.4) \cdot 10^{-5}$$

Negligible. $\Rightarrow$ If you see it, it's new physics.

$$a_{\rm fs}(B_d) = (-5.2 \pm 1.1) \cdot 10^{-4}$$

New physics can lift $|a_{\rm fs}(B_s)|$ and $|a_{\rm fs}(B_d)|$ to $\sim 5 \cdot 10^{-3}$.

5. Summary and Outlook

- Large effects from new physics are still possible in Δm_s.
- The large size of the $1/m_b$ corrections in $\Delta\Gamma$ is an artifact of a poor choice of operators in the leading order of the $1/m_b$ expansion.
- The choice of $\widetilde{Q}_S$ instead of Q_S further diminishes the dependence of $\Delta\Gamma/\Delta m$ on hadronic parameters.
- A more precise measurement of $\Delta\Gamma$ will constrain new physics by excluding a region in the complex M_{12} plane through $\Delta\Gamma/\Delta m$.
- $a_{\rm fs}$ will contribute to constrain M_{12}, once experimental upper bounds $|a_{\rm fs}| \leq 5 \cdot 10^{-3}$ become available.