α from $B \to \pi\pi, \rho\rho, \rho\pi$

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Precision determination of $\alpha(\gamma)$

- precision β determination in $b \to c\bar{c}s$ $(B \to \psi K_S)$:

$$\beta = (21.7^{+1.3}_{-1.2})^\circ$$

  with ultimate theory error at (below) percent level

  Ciuchini, Pierini, Silvestrini, 2005
  Boos, Mannel, Reuter, 2004

- Sets high standards for α, γ determinations
- what theory errors on determination of α ?
  - isospin
  - SU(3)
  - $1/m_b$

see talk by M. Beneke later in the afternoon

α in $B \to \pi^+\pi^-$

- two amplitudes with different weak phases

$$A(B^0 \to \pi^+\pi^-) = Te^{i\gamma} + Pe^{i\delta}, \qquad r = P/T$$

- time dependent decay width

$$\Gamma(B^0(t) \to \pi^+\pi^-) \propto \Gamma_{\pi^+\pi^-}\,[1 + C_{\pi^+\pi^-}\cos\Delta m t - S_{\pi^+\pi^-}\sin\Delta m t]$$

- expanding in r

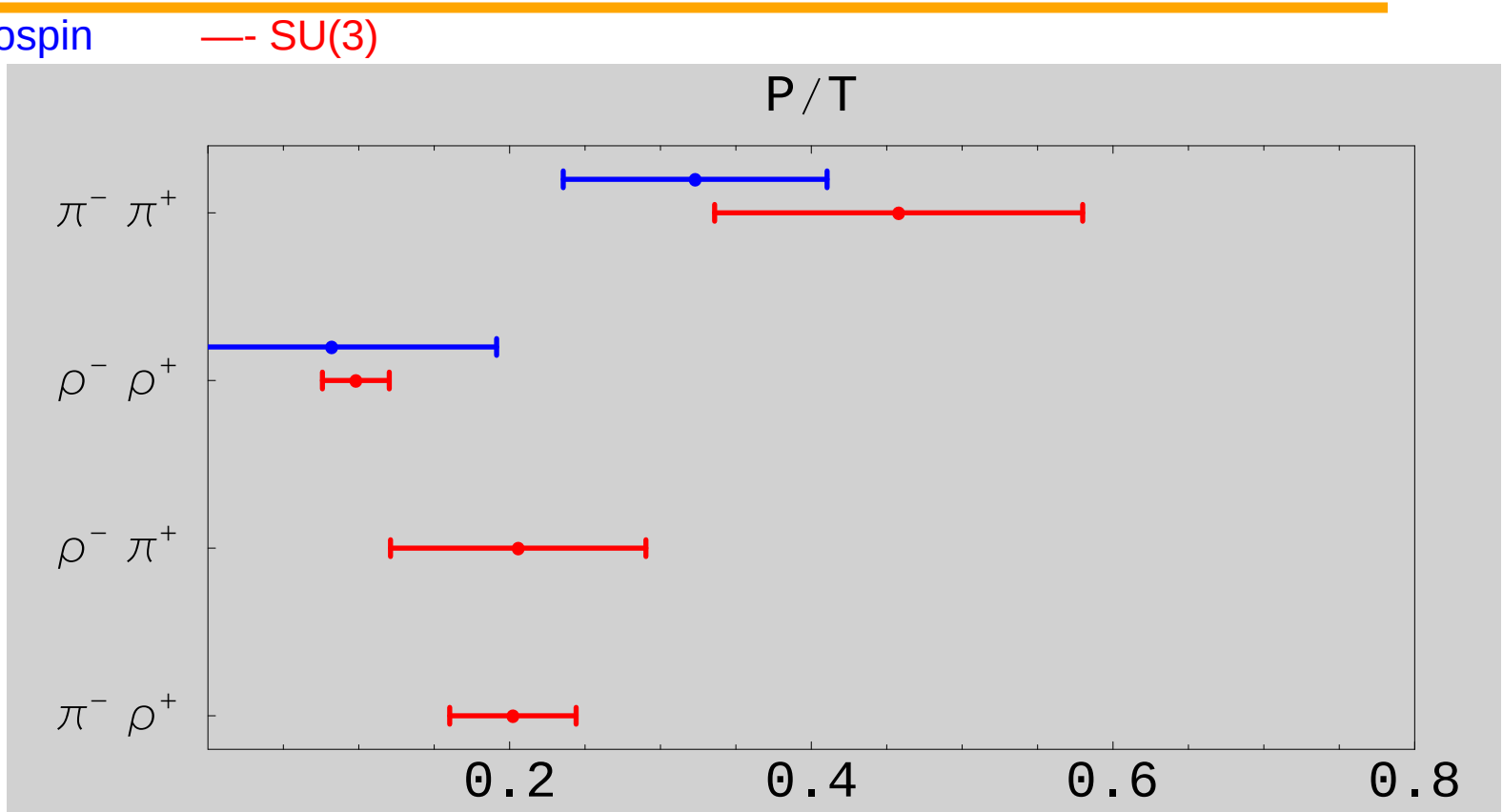
$$S_{\pi^+\pi^-} = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha + O(r^2)$$

- 3 measurables vs. 4 unknowns: T, r, δ, γ

to fix r additional input required

- larger $r \Rightarrow$ larger difference $\sin 2\alpha - \sin 2\alpha_{\rm eff}$

Size of P/T



- hierarchy: $r(\pi^+\pi^-) > r(\rho^+\pi^-) \sim r(\pi^+\rho^-) > r(\rho^+\rho^-)$
- in SU(3) (and $1/m_b$) approaches: expect the same hierarchy in the theory errors

$$B(t) \to \pi\pi$$

$B \to \pi\pi$ and isospin

Gronau, London (1990)

- completely general isospin decomposition

$$A_{+-} = \langle \pi^+\pi^- | H | B^0 \rangle = -A_{1/2} + \frac{1}{\sqrt{2}} A_{3/2} - \frac{1}{\sqrt{2}} A_{5/2}$$

$$A_{00} = \langle \pi^0\pi^0 | H | B^0 \rangle = \frac{1}{\sqrt{2}} A_{1/2} + A_{3/2} - A_{5/2}$$

$$A_{+0} = \langle \pi^+\pi^0 | H | B^+ \rangle = \frac{3}{2} A_{3/2} + A_{5/2}$$

- neglecting $A_{5/2} \sim \alpha A_{1/2}$ (i.e. $\sim 1\%$ correction)

$$A_{+-} + \sqrt{2} A_{00} = \sqrt{2} A_{+0}$$
$$\bar{A}_{+-} + \sqrt{2} \bar{A}_{00} = \sqrt{2} \bar{A}_{+0}$$

- neglecting EWP $\Rightarrow$ A_{+0} only tree contribs.

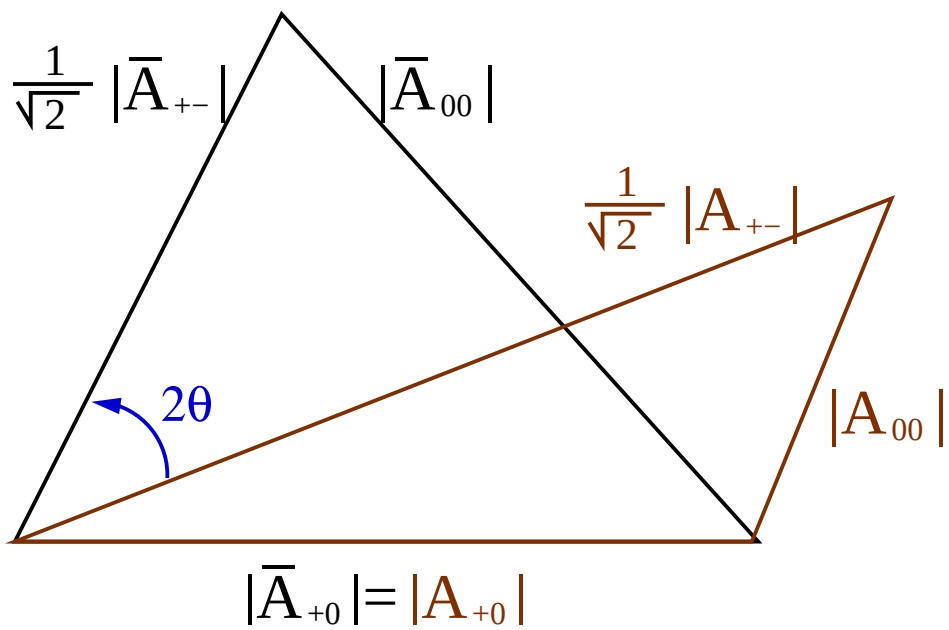
$$e^{i\gamma} A_{+0} = e^{-i\gamma} \bar{A}_{+0} \quad \Rightarrow \quad |A_{+0}| = |\bar{A}_{+0}|$$

Gronau-London triangle

- $\sin 2\alpha$ from
  $\Gamma(B^0(t) \to \pi^+\pi^-) \propto [1 + C_{\pi\pi} \cos \Delta m t - S_{\pi\pi} \sin \Delta m t]$

$$\sin(2\alpha_{\rm eff}) = S_{\pi\pi}/\sqrt{1 - C_{\pi\pi}^2}$$

$$2\alpha = 2\alpha_{\rm eff} - 2\theta$$



Isospin breaking

- sources of isospin breaking
  - d and u charges different
  - $m_u \neq m_d$

Isospin breaking

- sources of isospin breaking
  - d and u charges different
  - $m_u \neq m_d$
- extends the basis of operators to EWP $Q_{7,\ldots,10}$
- mass eigenstates do not coincide with isospin eigenstates: $\pi - \eta - \eta'$ and $\rho - \omega$ mixing
- may induce $\Delta I = 5/2$ operators not present in H_W

Electroweak penguins

- separate triangle relations still hold
- neglecting $Q_{7,8}$

Neubert, Rosner; Gronau, Pirjol, Yan; Buras, Fleischer (1999)

$$H^{\Delta I=3/2}_{\rm eff,EWP} = -\frac{3}{2}\frac{C_9+C_{10}}{C_1+C_2}\frac{V^*_{tb}V_{td}}{V^*_{ub}V_{ud}}H^{\Delta I=3/2}_{\rm eff,c-c}$$

$\Rightarrow e^{i\gamma}A_{+0} = e^{-i(\gamma+2\delta)}\bar{A}_{+0}$, but still $|A_{+0}| = |\bar{A}_{+0}|$

$\Rightarrow$ $\boxed{\alpha = \alpha_{\rm eff} - \theta - \delta}$ with $\boxed{\delta = (1.5 \pm 0.3 \pm 0.3)^\circ}$

conservatively $\sim 2(|c_7| + |c_8|)/(|c_9|) < 0.2$

- the same relation

$$e^{i\gamma}T = e^{-i(\gamma+2\delta)}\bar{T}$$

holds for $\Delta I = 3/2$ (tree) amplitudes in $\rho\rho$ and $\rho\pi$ system

$\pi^0 - \eta - \eta'$ mixing

- π^0 w.f. has η, η' admixtures

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle$$

where $\epsilon = 0.017 \pm 0.003,\ \epsilon' = 0.004 \pm 0.001$ Kroll (2004)

- GL triangle relations no longer hold

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0} \neq 0$$
$$\bar{A}_{+-} + \sqrt{2}\bar{A}_{00} - \sqrt{2}\bar{A}_{+0} \neq 0$$

- varying the phases of $A_{0\eta^{(\prime)}}$, $\bar{A}_{0\eta^{(\prime)}}$ gives bound

$$|\Delta\alpha_{\pi-\eta-\eta}| \leq \sqrt{2\frac{\tau_+}{\tau_0}}\left(\epsilon\sqrt{\frac{\mathcal{B}_{0\eta}}{\mathcal{B}_{+0}}} + \epsilon'\sqrt{\frac{\mathcal{B}_{0\eta'}}{\mathcal{B}_{+0}}}\right)$$

leading to M. Gronau, J.Z. (2005), S. Gardner (2005)

$$\boxed{|\Delta\alpha_{\pi-\eta-\eta'}| < 1.6^\circ}$$

Summary $B \to \pi\pi$ isospin breaking

- not all isospin breaking effects can be calculated/constrained at present
- the ones that can be are of expected size

$$\sim (m_u - m_d)/\Lambda_{QCD} \sim \alpha_0 \sim 1\%$$

$$B(t) \to \rho\rho$$

Isospin

Isospin analysis in the same spirit as for $B \to \pi\pi$

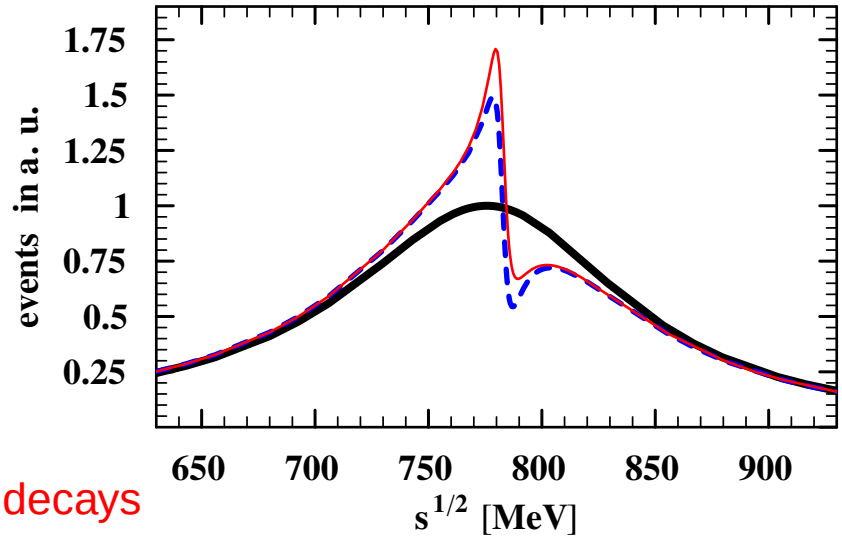
- 3 separate isospin relations (for each polarization)
- almost completely longitudinally polarized
- since $\Gamma_\rho \neq 0 \Rightarrow I = 1$ contributions possible

  Falk, Ligeti, Nir, Quinn (2003)

  - $O(\Gamma_\rho^2/m_\rho^2)$ effect
  - possible to constrain experimentally

Isospin breaking

- shift due to EWP exactly the same as in $\pi\pi$
- $\rho - \omega$ mixing, large effect expected because $|A(B^+ \to \rho^+\omega)/A(B^+ \to \rho^+\rho^0)| = 0.69 \pm 0.14$
- an estimate: neglect isos-ingl. contribs. & $P/T = 0.2$ & same strong phase for P, T



B^- decays

B^+ decays

- integrated effect of ω resonance is $< 2\%$
- other: $g_I \equiv g(\rho_I \to \pi^+\pi^-) \neq g_c \equiv g(\rho^+ \to \pi^+\pi_3)$
  [PDG: $g_c/g_I - 1 = (0.5 \pm 1.0)\%$]

$B \to \rho\rho$ and SU(3)

- small $\mathcal{B}(\rho^0\rho^0) = (1.16 \pm 0.46) \times 10^{-6}$,
  while $\mathcal{B}(\rho^+\rho^-), \mathcal{B}(\rho^+\rho^0) = 20 - 30 \times 10^{-6}$
- no measurement of $C_{\rho^0\rho^0}$ yet
- $|\alpha - \alpha_{\rm eff}| < 22.4^\circ$ (95% CL) see talk by S. t'Jampens
- but $P/T \sim 0.1 \Rightarrow$ can SU(3) analysis compete with isospin?
- Note: in isospin analysis the theory errors not necessarily P/T suppressed
  - the $\Delta\alpha$ shift due to EWP exactly the same as in $B \to \pi\pi$

Basic idea

- in $\Delta S = 1$ decays:

$$T' \sim V_{ub}V_{us}^* \qquad\qquad P' \sim V_{cb}V_{cs}^*$$

- in $\Delta S = 0$ decays:

$$T \sim V_{ub}V_{ud}^* \qquad\qquad P \sim V_{cb}V_{cd}^*$$

- thus P'/T' is $1/\lambda^2$ enhanced over $P/T \Rightarrow$ can use it to bound P/T

Stronger bound on P/T from SU(3)

Beneke, Gronau, Rohrer, Spranger (2006)

- instead of isospin and $\mathcal{B}(\rho^0\rho^0)$ use SU(3) and $\mathcal{B}_L(K^{*0}\rho^+)$
- $\mathcal{B}(K^{*0}\rho^+) = (9.3 \pm 1.7) \cdot 10^{-6}$, $f_L = 0.48^{+0.09}_{-0.08}$, mostly P
- Long. CP-av. rate: $|A_L(K^{*0}\rho^+)|^2_{\rm CP-av} = F\left(\frac{|V_{cs}|f_{K^*}}{|V_{cd}|f_\rho}\right)P^2$
  - SU(3) breaking factorized: $F = 1$, expect corrections
  - $F \neq 1$: nonfact. SU(3)-breaking, P^c_{EW}, PA
  - conservative range: $0.3 \leq F \leq 1.5$
- $F = 1 \Rightarrow P/T = 0.10 \pm 0.02$
- include pre-ICHEP06 exp. errors on S_L, C_L and vary F:

$$\alpha = [91.2^{+9.1}_{-6.6}\ ({\rm exp})^{+1.2}_{-3.9}\ ({\rm th})]^\circ$$

- th. error from F is smaller than $|\alpha - \alpha_{\rm eff}|_{\bf isospin} < 22.4^\circ$

Lesson

- once we know $(P/T)_{\rho^+\rho^-}$ is small

  $\Downarrow$

  SU(3) breaking (or other corrections) introduce a small error in α
- could be comparable to theory errors in isospin analysis of $B \to \rho\rho$

$$B(t) \to \rho\pi$$

$B \to \pi^+\pi^-\pi^0$ Dalitz plot

- model the Dalitz plot (similarly for $A(\bar{B}^0 \to 3\pi)$)

$$A(B^0 \to \pi^+\pi^-\pi^0) = \overbrace{A(B^0 \to \rho^+\pi^-)}^{A_+} D_{\rho\rho}(s_+)\cos\theta_+ +$$
$$+ \underbrace{A(B^0 \to \rho^-\pi^+)}_{A_-} D_{\rho\rho}(s_-)\cos\theta_- + \underbrace{A(B^0 \to \rho^0\pi^0)}_{A_0} D_{\rho\rho}(s_0)\cos\theta_0$$

- other resonances need to be included in the fit
- $\rho - \omega$ mixing treated in the same way as in $\rho\rho$
- possible to determine

$$A_+,\ \bar{A}_+,\ A_-,\ \bar{A}_-,\ A_0,\ \bar{A}_0$$

up to overall phase $\Rightarrow$ 11 independent measurables

Snyder-Quinn

Snyder, Quinn (1993), Lipkin et al. (1991), Gronau (1991)

- rescale $A_i(\bar{A}_i) \to e^{i\beta} A_i(e^{-i\beta}\bar{A}_i)$
- tree and penguin defined according to CKM (T_i, P_i now complex)

$$\mathcal{A}_{\pm,0} = e^{-i\alpha} T_{\pm,0} + P_{\pm,0} \ , \qquad \bar{\mathcal{A}}_{\pm,0} = e^{+i\alpha} T_{\pm,0} + P_{\pm,0}$$

- an isospin relation only between penguins

$$P_0 + \tfrac{1}{2}(P_+ + P_-) = 0$$

(EWP and isospin breaking neglected)

- 10 unknowns: e.g. α, $|T_\pm|$, $|T_0|$, $\arg T_\pm$, $|P_\pm|$, $\arg P_\pm$ enough info to determine them

Effect of isospin breaking

- isospin breaking affects only the relation between penguins!
  - not true for the pentagon analysis
- largest contribution from EWP because they are related to tree

$$P_- + P_+ + 2P_0 = P_{EW}$$

where P_{EW} can be obtained from J. Charles, PhD thesis

$$\mathcal{A}_+ + \mathcal{A}_- + 2\mathcal{A}_0 = Te^{-i\alpha} + P_{EW}$$

and

$$\frac{P_{EW}}{T} = -\frac{3}{2}\left(\frac{c_9 + c_{10}}{c_1 + c_2}\right)\frac{|V_{tb}V_{td}|}{|V_{ub}V_{ud}|} = +0.013\frac{\sin(\beta+\alpha)}{\sin\beta}$$

Other isospin breaking

- other isospin breaking effects are $P/T \sim 0.2$ suppressed
- using similar approach of SU(3) relations as in $\pi\pi$ to estimate shift due to $\pi^0 - \eta - \eta'$ mixing

$$|\Delta\alpha_{\pi-\eta-\eta'}| = \frac{|\epsilon P_{\rho\eta} + \epsilon' P_{\rho\eta'}|}{|T|} \leq 0.024\epsilon + 0.069\epsilon' \leq 0.1^\circ$$

- this does not include all isospin breaking but shows the suppression exists

SU(3) approach

- in Snyder-Quinn crucial interf. regions in Dalitz plot
- yet ρ bands do not overlap head-on
- alternative: use just $\rho^\pm\pi^\mp$ final state without interference info + SU(3) Gronau, JZ (2004)
- to constrain $p_\pm/t_\pm$ in $A(B^0 \to \rho^\pm\pi^\mp) = e^{i\gamma}t_\pm + p_\pm$ use SU(3) related modes $B^0 \to K^{*+}\pi^-, K^+\rho^-$ and $B^+ \to K^{*0}\pi^+, K^0\rho^+$
- @90% CL: $\boxed{0.16 \leq |p_+/t_+| \leq 0.24}$ $\boxed{0.12 \leq |p_-/t_-| \leq 0.29}$
- SU(3) breaking on extracted α are small, of order $p_\pm^2/t_\pm^2$ $\Rightarrow$ MC study with up to 30% SU(3) breaking on penguins gives $\sqrt{\langle(\alpha^{\rm out} - \alpha^{\rm in})^2\rangle} \sim 2^\circ$
- similar approach possible in $B \to a_1^\pm\pi^\mp$ M. Gronau, JZ, 2005

Conclusions

- the isospin breaking effects that have been analysed are of expected size
- in certain cases they can be further suppressed: in $B \to \rho\pi$ by P/T
- in modes with small penguin amplitudes ($B \to \rho\rho, \rho\pi$) SU(3) approaches can be competitive with isospin analysis

Backup slides

t conv. vs. c conv.

- c-convention:

$$A = Te^{i\gamma} + Pe^{i\delta}, \qquad r = P/T$$

$$S = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha + O(r^2)$$

- t-convention

$$A = -Te^{i\gamma} + Pe^{i(-\beta+\delta)}, \qquad r = P/T$$

$$S = \sin 2\alpha + r\cos\delta\big(\sin(\alpha) - \sin(3\alpha)\big) + O(r^2)$$

$$\frac{2\sin(\beta+\alpha)\cos 2\alpha}{\sin(\alpha) - \sin(3\alpha)} \sim -0.9$$