Beauty 2006:

Precision Determination *of* $|V_{ub}|$

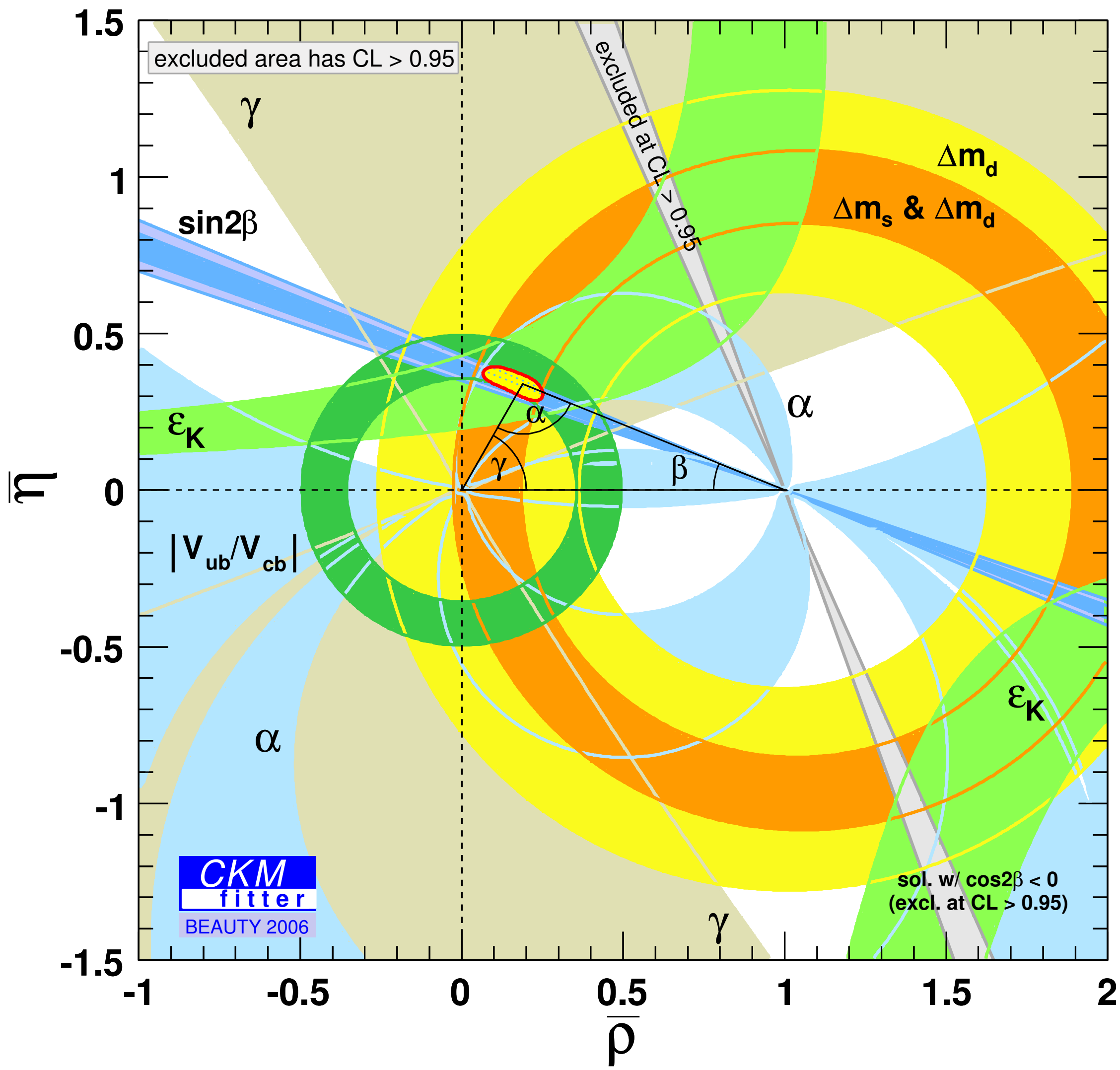
Gil Paz

Institute *for* Advanced Study, Princeton

Outline

- Introduction
- Inclusive $|V_{ub}|$ (BLNP)
- Other approaches to inclusive $|V_{ub}|$
- Improved measurement of $|V_{ub}|$ - Today!
- Improved measurement of $|V_{ub}|$ - Future
- Exclusive measurement of $|V_{ub}|$
- Lessons form leptonic B decays
- Summary

Motivation



excluded area has CL > 0.95
excluded at CL > 0.95
γ
sin2β
Δm_d
Δm_s & Δm_d
ε_K
α
β
$|V_{ub}/V_{cb}|$
sol. w/ cos2β < 0
(excl. at CL > 0.95)
CKM fitter
BEAUTY 2006
$\bar{\eta}$
$\bar{\rho}$

$|V_{ub}|$

- $|V_{ub}|$: fundamental parameter of the SM
- Side of UT opposite $\beta \sim |V_{ub}|/|V_{cb}|$
  Error on inclusive $|V_{cb}| \sim 2\% \Rightarrow$ improve $|V_{ub}|$
- Measure $|V_{ub}|$ through:
  - Exclusive charmless decays: e.g. $\bar{B} \to \pi^+ \, l^- \bar{\nu}$
    Form factor uncertainty!
  - Inclusive charmless decays: $\bar{B} \to X_u \, l^- \bar{\nu}$
    $|V_{cb}| >> |V_{ub}| \Rightarrow$ Large background from $\bar{B} \to X_c \, l^- \bar{\nu}$!
    Look at regions of phase space where charm cannot be produced

Kinetmatics

- Hadronic tensor $W^{\mu\nu} \Rightarrow d^3\Gamma$
  1) Choose a basis: (a, b)
  2) Decompose $W_{\mu\nu} = \cdots W_1 + \cdots$
  3) $d^3\Gamma \sim W_i$
- Best choice (v, n) basis $\big($Lange, Neubert, GP [PRD 72, 073006 (2005)]$\big)$:
  $v = (1,0,0,0) \quad n = (1,0,0,1) \quad [\bar{n} = 2v - n = (1,0,0,-1)]$
- Motivates:
  $P_l = M_B - 2E_l, \quad \bar{n}\cdot P = P_- = E_X + |\vec{P}_X|, \quad n \cdot P = P_+ = E_X - |\vec{P}_X|$
- Exact triple rate: $y = (P_- - P_+)/(M_B - P_+)$

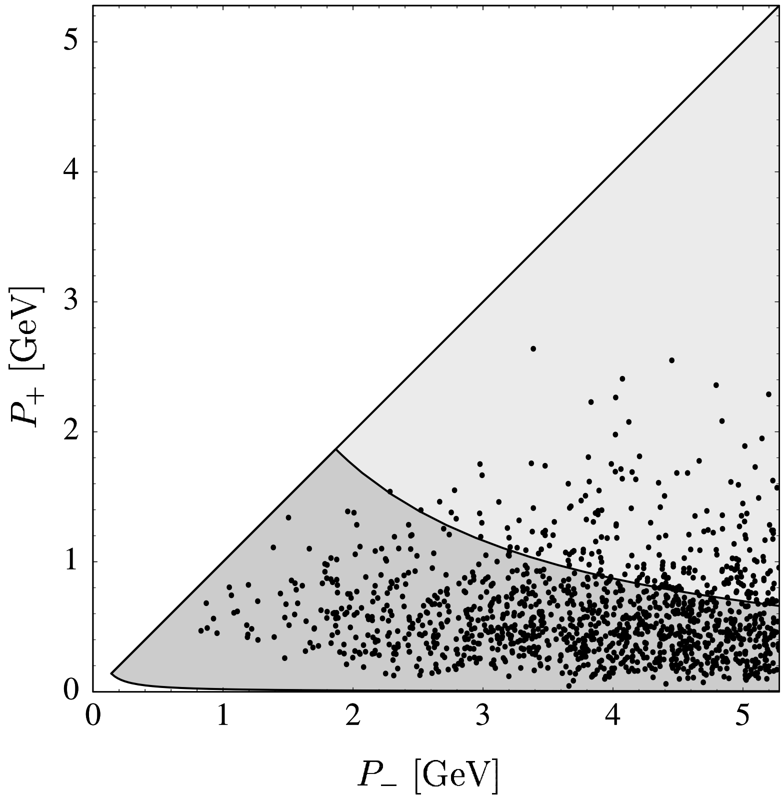
$$\frac{d^3\Gamma_u}{dP_+\, dP_-\, dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} (M_B - P_+) \Big[(P_- - P_l)(M_B - P_- + P_l - P_+)\, \tilde{W}_1$$

$$+ (M_B - P_-)(P_- - P_+) \frac{\tilde{W}_2}{2} + (P_- - P_l)(P_l - P_+) \left(\frac{y}{4}\tilde{W}_3 + \tilde{W}_4 + \frac{1}{y}\tilde{W}_5 \right) \Big]$$

- Simplest phase space: $\frac{M_\pi^2}{P_-} \leq P_+ \leq P_l \leq P_- \leq M_B$
- No explicit dependence on m_b ! Can predict partial rates instead of fractions

(Pedestrian introduction to inclusive $|V_{ub}|$ chapter 1 of GP hep-ph/0607217)

Kinetmatics



- $P_+ P_- = M_X^2 \quad q^2 = (M_B - P_-)(M_B - P_+)$
- Experimental cuts $\Rightarrow$
  $P_+ \sim \Lambda_{\rm QCD} \sim 0.5$ GeV $\quad P_- \sim m_b \sim 5$ GeV

Dynamics - OPE region

- In order to calculate $d^3\Gamma$ we need to know $\tilde{W}_i$
- If we had no charm background...
  Integrate over P_+, P_- up to M_B, and use HQET based OPE

$$\tilde{W}_i \sim c_0 \frac{\langle O_0 \rangle}{m_b} + c_2 \frac{\langle O_2 \rangle}{m_b^2} + c_3 \frac{\langle O_3 \rangle}{m_b^3} + \cdots$$

- c_i calculable in PT:
  c_0 known at $\mathcal{O}(\alpha_s)$, c_2 and c_3 at $\mathcal{O}(\alpha_s^0)$
- $\langle O_i \rangle$ are HQ parameters, taken from experiment:
  $\langle O_0 \rangle = 1$
  $\langle O_2 \rangle \longrightarrow \mu_G^2 = [(M_B^*)^2 - (M_B)^2]/4, \quad \mu_\pi^2$
  $\langle O_3 \rangle \longrightarrow \rho_{LS}^3, \rho_D^3$
- OPE works very well for $\bar{B} \to X_c\, l^- \bar{\nu}$
  $\Rightarrow$ Error on V_{cb} is 2%, know HQ parameters
- Similar OPE for total $\bar{B} \to X_s \gamma$ rate, which we can't measure...

Dynamics - SF region

- Because of the charm background, forced into regions of phase space where HQET based OPE is not valid ("OPE breaks down")

- We do have a systematic $1/m_b$ expansion, calculated using SCET:

$$\tilde{W}_i \sim H_u \cdot J \otimes S + \frac{1}{m_b} \sum_k h_u^k \cdot j_u^k \otimes s_u^k + \cdots$$

- H - physics at scale $\mu \geq m_b$ - Calculable in PT
  J - physics at scale $\mu \sim \sqrt{m_b \Lambda_{\mathrm{QCD}}}$ - Calculable in PT
  S - physics at scale $\mu \sim \Lambda_{\mathrm{QCD}}$ - Non perturbative function

- For $\bar{B} \to X_s \gamma$ near endpoint:

$$\frac{d\Gamma}{dE} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_k h_s^k \cdot j_s^k \otimes s_s^k + \cdots$$

Dynamics - SF$\leftrightarrow$OPE

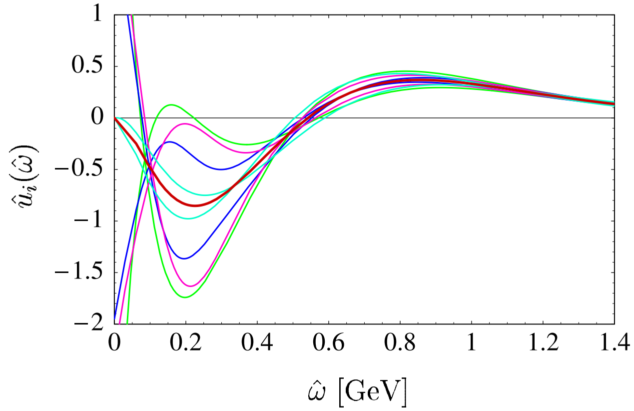
- What is the relation between the two regions?
- Moments of SFs related to HQ parameters, e.g.:
  First moment of $S \leftrightarrow m_b$ (two loop relation)
  Second moment of $S \leftrightarrow \mu_\pi^2$ (two loop relation)
- $\Rightarrow$ Good knowledge of HQ parameters, constrain the SFs
- Integrate over large enough regions of phase space, recover OPE result

Inclusive $|V_{ub}|$: BLNP approach

- BLNP approach (Lange, Neubert, GP [PRD 72, 073006 (2005)]):
  use all that we currently know about $\bar{B} \to X_u l \bar{\nu}$ and $\bar{B} \to X_s \gamma$:
  - LO in $1/m_b$: H_u, H_s, J at $\mathcal{O}(\alpha_s)$
  - $1/m_b$ subleading SFs at $\mathcal{O}(\alpha_s^0)$
  - Known $1/m_b \cdot \alpha_s$ terms from OPE
  - Known $1/m_b^2$ terms from OPE
- Extract S from $\bar{B} \to X_s \gamma$ and use as input for $\bar{B} \to X_u l \bar{\nu}$
- Model subleading SFs using moment constraints
- Error Analysis:
  - LO SF taken from experiment
  - Pertrubative error
  - Subleading SFs: 3 functions, 9 models each, scan over $9^3 = 729$ combinations

Inclusive $|V_{ub}|$: BLNP approach

- Error Analysis (continued):
  - Subleading SFs modeling:



  - WA: take as fixed % of rate

- BLNP results: $|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \cdot 10^{-3}$ with
  - 4.2% HQ error
  - 3.8% Theory error (Pertrubative + Subleading SFs)
  - 1.9% WA

Inclusive $|V_{ub}|$: $M_x - q^2$ approach

- For cut on **low** q^2 can use OPE, but with $1/m_c$ expansion
- To optimize efficiency and theoretical uncertainty, Bauer, Ligeti, and Luke (BLL) suggest to use combined $M_x - q^2$ cut [PRD 64, 113004(2001)]:
  - OPE assumed to be valid for combined cut
  - LO SF sensitivity estimated by convoluting tree level decay rate with ("tree level SF" model - δ function model)
  - Subleading SFs assumed to be small, not assessed
  - HFAG average: $|V_{ub}| = (5.02 \pm 0.26 \pm 0.37) \cdot 10^{-3}$ with 3% from SF sensitivity
- BLNP analysis doesn't find reduced SF sensitivity
- BLL should updated analysis, e.g. estimate SF sensitivity beyond tree level, estimate subleading SFs contribution etc.

Inclusive $|V_{ub}|$: DGE approach

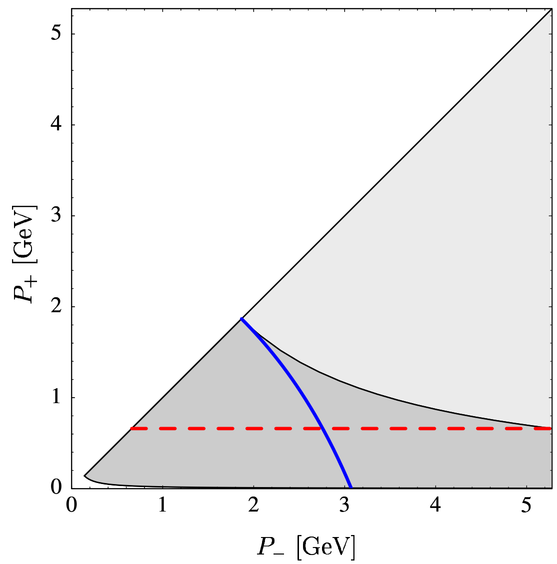
- Dressed Gluon Exponentiation (DGE) approach advocated by Andersen and Gardi [JHEP 0601:097 (2006)]
  - "All is preturbative" approach
  - Only input parameter m_b and α_s
  - HFAG average: $|V_{ub}| = (4.46 \pm 0.20 \pm 0.20) \cdot 10^{-3}$
- No power corrections are included or estimated
  "present exp. data no power correction are needed"
  [Gardi hep-ph/0606081]
- Unclear how the OPE result is recovered

Weak Annihilation (WA)

- Appears at order $1/m_b^3$ in the OPE: $\bar{b}\Gamma u\bar{u}\Gamma b$
- Effects neutral and charged B differently [Bigi, Uraltsev NPB 423, 33 (1994)]
- Appears for every cut that includes $q^2 = m_b^2$
- Currently, only estimates on its magnitude:
  - Voloshin [PLB 515, 74 (2001)]: Use $\Gamma_{sl}(D^0) - \Gamma_{sl}(D_s)$
  - Neubert, GP, Sanz-Cillero in preparation: use χPT, preliminary results in hep-ph/0609002
  - CLEO limits $(\Gamma_{\rm WA}/\Gamma_{b\to u}) < 7.4\%$ (90% confidence level) [PRL 96, 121801 (2006)]

Improved $|V_{ub}|$ Today! : WA

- Another strategy, following the queen of hearts: "**OFF WITH ITS HEAD!**"
- Lange, Neubert, GP [PRD 72, 073006 (2005)]: Cut on **high** $q^2 < q^2_{\max}$ e.g. $q^2_{\max} = (M_B - M_D)^2$, combined with M_X or P_+ cut



- Loose efficiency but also the WA error and its uncertainty, Preliminary study gives smaller error with such cut
- **Still waiting for experimental implementation!**

Weight Function

- Weight function idea: relate photon spectrum in $\bar{B} \to X_s \gamma$ **directly** to $\bar{B} \to X_u \, l^- \bar{\nu}$ spectra

- How does it work:

$$W \sim \frac{\Gamma_u}{\Gamma_s} \sim \frac{H_u \cdot J[m_b y(P_+ - \hat{\omega})] \otimes S(\hat{\omega})}{H_s \cdot J[m_b(P_+ - \hat{\omega})] \otimes S(\hat{\omega})}$$

  W calculated from theory

- Babar used calculation of weight function for M_X spectrum by Leibovich, Low, Rothstein (LLR) [PLB 486, 86 (2000)]

- Babar: $|V_{ub}| = (4.43 \pm 0.45 \pm 0.29) \cdot 10^{-3}$ [PRL 96, 221801 (2006)]

Improved $|V_{ub}|$ Today! : Weight Function

- Recent calculation: Lange, Neubert, GP [JHEP 0510 084 (2005)]: Relate the **normalized** photon spectrum in $\bar{B} \to X_s \gamma$ to $\bar{B} \to X_u \, l^- \bar{\nu}$ P_+ spectrum
- Contain **two loop** corrections **and** subleading SF corrections **and** the known $\alpha_s \cdot 1/m_b$ corrections
- Theoretical error 5%
- Similar weight function calculated for general spectra by Lange [JHEP 0601:104 (2006)]
- Has potential to be the best extraction of $|V_{ub}|$!
- **Still waiting for experimental implementation!**

Improved $|V_{ub}|$: Future

- O_7 for $\bar{B} \to X_s\gamma$ is known at $\mathcal{O}(\alpha_s^2)$, other ops. are being calculated
  Once they are known, want $\bar{B} \to X_u\, l^- \bar{\nu}$ at $\mathcal{O}(\alpha_s^2)$:
  "Only" need H_u at $\mathcal{O}(\alpha_s^2) \Rightarrow$ full 2 loop weight function
- Subleading SFs at order $\mathcal{O}(\alpha_s) \Leftrightarrow$ OPE at $\mathcal{O}(\alpha_s)$
- Can we find a way to extract subleading SFs from data?
- Complete subleading SF basis for $\bar{B} \to X_s\gamma$
  Lee, Neubert, GP in preparation
  hep-ph/0609224: Preliminary results and new non perturbative effects for **total** $\bar{B} \to X_s\gamma$ rate

Exclusive $|V_{ub}|$

- Exclusive $|V_{ub}|$ extracted from $B \to \pi l \bar{\nu}$ (see L. Gibbons talk)
- Need input about form factor $f_+(q^2)$
- input from (unquenched) LQCD ($q^2 >$16 GeV^2)
  - HPQCD $|V_{ub}| = (3.93 \pm 0.26 + 0.59 - 0.41) \cdot 10^{-3}$ [PRD 73, 074502(2006)]
  - FNAL $|V_{ub}| = (3.51 \pm 0.23 + 0.61 - 0.40) \cdot 10^{-3}$ [hep-lat/0409116]
- Input from Light Cone Sum Rules ($q^2 < 16$ GeV^2)
  Ball-Zwicky $|V_{ub}| = (3.38 \pm 0.12 + 0.56 - 0.37) \cdot 10^{-3}$ [PRD 71, 014015 (2005)]
- Exclusive (central) values are lower then inclusive, "Historically" always the case

Lessons from Leptonic B decays

- In April 2006 Belle reported on "Evidence of the Purely Leptonic Decay $B^- \to \tau^- \bar{\nu}_\tau$" [hep-ex/0604018]
- $f_B \cdot |V_{ub}| = (7.73^{+1.24}_{-1.02}(\text{stat})^{+0.66}_{-0.58}(\text{syst})) \times 10^{-4}$ GeV
- Using the inclusive value: $|V_{ub}| = (4.39 \pm 0.33) \cdot 10^{-3}$

  $f_B = 0.176^{+0.028}_{-0.023}(\text{stat})^{+0.020}_{-0.019}(\text{syst})$ GeV
- Comparing to unquenched lattice value: [HPQCD PRL 95, 212001 (2005)]

  $f_B = 0.216 \pm 0.022$ GeV
- IS THE INCLUSIVE VALUE TOO HIGH ???

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- Comparing to unquenched lattice value: [HPQCD PRL 95, 212001 (2005)]

  $f_B = 0.216 \pm 0.022$ GeV
- IS THE INCLUSIVE VALUE TOO HIGH ???
- But in ICHEP 2006, Belle reported it found a coding error!
- New Value: $f_B \cdot |V_{ub}| = (10.1^{+1.6}_{-1.4}{}^{+1.1}_{-1.3}) \times 10^{-4}$ GeV
- Using the **same** value of $|V_{ub}|$ they now find:

  $f_B = 0.229^{+0.036}_{-0.031}{}^{+0.030}_{-0.034}$ GeV
- IS THE EXCLUSIVE VALUE TOO LOW ???

$|V_{ub}|$: Summary

- Impressive improvement in determination of $|V_{ub}|$
  Result of hard experimental and theoretical work
- Error on inclusive V_{ub}: 18% in PDG 2004 $\Rightarrow$ 8% in PDG 2006
- Improve $|V_{ub}|$ today!
  - Cut on **high** q^2 to eliminate WA
  - Advanced two loop relations between $\bar{B} \to X_u\, l^- \bar{\nu}$ and $\bar{B} \to X_s \gamma$
- Need to compare approaches: assumptions, perturbative corrections, non perturbative corrections
- More room for theoretical improvement