Higgs Physics

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Abstract

Aspects of electroweak symmetry breaking are discussed and the phenomenology of the standard Higgs boson is reviewed.

1. Introduction

The Higgs boson lies at the heart of the unknowns of high energy physics. The three known interactions of particles are almost certainly and exclusively gauge interactions which possess universality and other virtues, although the appearance of the forces is totally different for each of the three: massless Coulomb phase for electromagnetism, confinement for the strong, and spontaneous breakdown for the weak interactions. Gauge interactions alone, however, are not sufficient to describe the world we observe. In fact, the standard model (SM) has a dark side—the Higgs and Yukawa sectors. Without them, the weak interactions would be of long range and quarks and leptons would be massless. Spontaneous breaking of the electroweak SU(2)×U(1) symmetry is caused by the Higgs sector, and fermion masses are generated by the Yukawa couplings connecting the Higgs and fermion fields.

It is therefore of utmost importance to study the properties of the Higgs sector, which is the origin of all masses in the world. Although nobody has detected these new non-gauge forces predicted by the standard model, namely, the Yukawa and Higgs (ϕ^4) interactions, we already have a piece of information on how the SU(2)×U(1) gauge symmetry is broken. The masses of W and Z tell us that the vacuum is likely to have the property $\Delta I = \frac{1}{2}$ (I is the weak isospin). In fact, the parameter $\rho \equiv m_W^2/m_Z^2 \cos^2\theta_W$ is very close to unity, being consistent with Higgs doublets. Moreover, the fermion masses can be generated only from Higgs fields with isospin $\frac{1}{2}$. Apart from this, we know almost nothing about the Higgs sector.

2. Is W a gauge boson?

We have no experimental evidence against the intermediate vector bosons being gauge bosons. Universality of the charged current interactions, observed structure of the neutral current, production and decay properties of the W and Z, everything is consistent with gauge theory. However, a direct evidence that the intermediate bosons are the quanta of Yang-Mills gauge fields is still lacking.

In particular, the Yang-Mills Lagrangian fixes the interaction among three or four gauge bosons. In our case, this includes the WWZ and $WW\gamma$ couplings. The gauge theory principle is much more restrictive than the conditions on the interaction imposed by Lorentz invariance and $\mathcal{P}, \mathcal{C}, \mathcal{T}$. (The latter discrete symmetries may not hold for these interactions, as we already know that the weak interactions violate them except for the product $\mathcal{CPT}$, for which we have a very good reason to respect.) Lorentz invariance alone allows for seven independent three-boson couplings,[1] all of which are fixed in gauge theories. These three-boson couplings will be tested only when we can produce W pairs.

On the other hand, we do have some indication that W and Z are pointlike up to the scale 100 GeV–1 TeV. If these intermediate bosons are not gauge bosons, they must be composite particles and should have some spacial extent. This can be seen as form factor effects of the couplings $f\bar{f}W$ and $f\bar{f}Z$.

If we make a simplifying assumption that W couples to left-handed fermions only, the possible form of the $f\bar{f}W$ vertex is

$$\Gamma_\mu = \frac{g}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)F(q^2) \tag{1}$$

with the normalization $F(0) = 1$. (q is the W momentum.) If W has a size comparable to its compton wavelength $1/m_W$, the form factor $F(m_W^2)$ would substantially deviate from unity. The observed W production cross section[2] in $p\bar{p}$ interactions at CERN SPS and Tevatron colliders is in a good agreement with the QCD-based prediction, showing that this is not the case.

A more restrictive result can be obtained[3] for the $f\bar{f}Z$ coupling from the precise Z width measurements[4–8] recently done at SLC and LEP. The good agreement of Γ_Z (both total and partial) with the standard model predictions imposes a limit on the "size" of Z. If we parametrize $F(q^2) = 1 \pm q^2/\Lambda_\pm^2$, the present data give a limit $\Lambda_\pm \gtrsim 1\,\mathrm{TeV}$.

We might thus conclude that the intermediate bosons are pointlike up to about 1 TeV, so they can be described as "almost" gauge bosons at least.

3. Higgs mechanism

If we accept the notion that the intermediate bosons are gauge particles, the finite range of the weak interactions means that the Higgs mechanism should be operational. Let me briefly review how the Higgs mechanism works.

The propagator of a (massless) gauge boson in the lowest order is

$$\begin{aligned} D^{(0)}_{\mu\nu} &= \frac{i}{k^2}\left[-g_{\mu\nu} + (1-\alpha)\frac{k_\mu k_\nu}{k^2}\right] \\ &= \frac{i}{k^2}\left[\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) - \alpha\frac{k_\mu k_\nu}{k^2}\right], \end{aligned} \tag{2}$$

where α is a gauge parameter. In the second line, the (4-dim.) transverse and longitudinal parts are explicitly separated. The gauge dependence is solely in the unphysical longitudinal part.

Vacuum polarization modifies the propagator. Gauge invariance restricts its form:

$$\Pi_{\mu\nu} = i(-k^2 g_{\mu\nu} + k_\mu k_\nu)\Pi(k^2)\,. \tag{3}$$

Applying this correction to the propagator (2), we find that only the transverse part of the propagator receives modification. The full propagator is

$$\begin{aligned} D_{\mu\nu} &= D^{(0)}_{\mu\nu} + D^{(0)}_{\mu\rho}\Pi^{\rho\sigma}D^{(0)}_{\sigma\nu} + \cdots \\ &= \frac{i}{k^2}\frac{1}{1+\Pi(k^2)}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) - \alpha\frac{i}{k^2}\frac{k_\mu k_\nu}{k^2}\,. \end{aligned} \tag{4}$$

If $\Pi_{\mu\nu}$ is regular at $k^2 = 0$, $\Pi(0)$ must be finite and therefore the position of the pole at $k^2 = 0$ does not change. This is how gauge invariance assures the masslessness of the photon even after the inclusion of full quantum corrections.

There is an exception for this general conclusion. If $\Pi(k^2)$ has a pole at $k^2 = 0$, the position of the pole of $D_{\mu\nu}$ is shifted. This is realized if there is a massless scalar boson coupled to the current with a derivative coupling, gfk_μ. The contribution of such a Goldstone boson gives $\Pi(k^2) \simeq -g^2f^2/k^2$ near $k^2 = 0$ and

$$\frac{1}{k^2[1+\Pi(k^2)]} \simeq \frac{1}{k^2 - g^2f^2}\,, \tag{5}$$

which shows that the gauge boson receives a mass $m = gf$. This is nothing but the Higgs mechanism. What is essential for this mechanism to work is that (1) the scalar boson is massless and that (2) the scalar boson couples to the current (gauge boson). Both are true for Goldstone bosons that appear when gauge symmetry is spontaneously broken.

The necessity of the Higgs mechanism can be understood just by counting the number of physical degrees of freedom. In general, any observable quantum mechanical state should form a representation of Poincaré (inhomogeneous Lorentz) group. Representations for massive ($P^2 > 0$) and massless ($P^2 = 0$) states are different. In the massive case, a representation forms an SO(3) multiplet in the rest frame. For the spin-1 representation, three polarization states $J_z = \pm 1$, 0 should be present, which mix among themselves by rotation. In the massless case, each helicity state does not mix with others and form a representation by itself. $\mathcal{CPT}$ requires that the states should form a pair of helicity $\pm\lambda$. The so-called massless spin-1 states thus have two polarization states with helicity ± 1. In order for massless spin-1 states to become massive, an extra degree of freedom for the helicity-0 state is called for. A massless scalar can supply this degree of freedom. So, intuitively speaking, the Higgs mechanism is the algebra $2 + 1 = 3$.

The Higgs mechanism itself does not depend on the detailed nature of the Goldstone boson. But what actually are the Goldstone bosons of electroweak symmetry breaking? Obviously, this question is central to our understanding of the mechanism of SU(2)×U(1) breaking. The standard model has elementary scalar particles serving as the Goldstone bosons. So do supersymmetric models. In composite scenarios, bound states play the role as the Goldstone bosons. For instance, technipions (massless bound states of techniquarks and antitechniquarks) substitute the Higgs fields in technicolor models. In the recent scenarios[9–12] of top-induced SU(2)×U(1) breaking, tightly bound $t\bar{t}$ and $t\bar{b}$ states become the helicity-0 component of the intermediate bosons.

4. Physical Higgs boson: Should it exist?

We have seen that the intermediate vector bosons as massive gauge particles imply the Higgs mechanism and (unphysical) Higgs bosons. Existence of a physical scalar particle, on the other hand, is a separate issue, not required by the Higgs mechanism. One can show, however, that at least one physical scalar particle should exist in any theory with elementary Higgs fields. In fact, because one wants to break both the SU(2) and the hypercharge U(1), there should be at least one nontrivial SU(2) representation and at least one complex (to have a U(1) charge) representation of Higgs fields. It is easy to see that one needs more than three degrees of freedom to satisfy these conditions, whereas only three can be absorbed by the gauge fields. This proof does not directly apply if the gauge group is larger, but as long as we confine ourselves to linear field theories, there must be the "radial" mode of the vacuum expectation value, which should appear as a physical Higgs particle.

Guaranteed discovery in $J = 0$ sector

The above considerations may not apply if the Goldstone bosons are composite particles. In that case, one can expect that some other effects should show up in the scalar sector. This may be seen using the following thought experiment. The argument indicates that the known particles and their known interactions are incomplete. Something new is guaranteed to be found in the $J = 0$ sector.

Consider the process $t\bar{t} \to ZZ$. The diagrams for this process are t- and u-channel t quark exchange as depicted in Fig. 1. The vertices in these graphs are known gauge interactions. Surprisingly, these innocent-looking graphs contain a peculiarity. The calculated cross section at high energies ($\sqrt{s} \gg m_t$) is

$$\frac{d\sigma}{d\Omega} \sim \frac{\alpha^2 m_t^2}{m_Z^4} + \mathcal{O}\Big(\frac{1}{s}\Big) . \tag{6}$$

The angular distribution of the leading term is flat, indicating that it comes entirely from the $J = 0$ partial wave. This obviously *violates* unitarity, which requires $\sigma_{J=0} \lesssim 1/s$, at a c.m. energy of $\sim$TeV. Thus, something has to occur below this scale to restore unitarity. (The same argument applies to $e^+e^- \to ZZ$, which *can* be measured, but unitarity is violated only at an extremely high energy.[13])

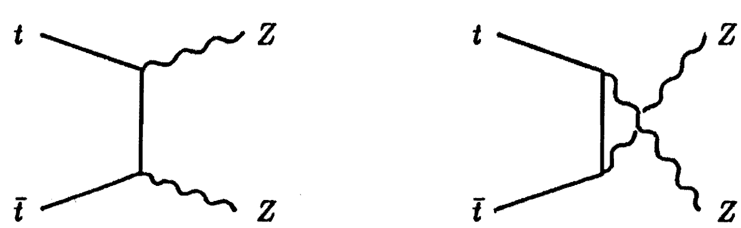

Fig. 1. Conventional diagrams for $t\bar{t} \to ZZ$.

In the standard model, Higgs is the cure for the problem. There is an additional graph (Fig. 2) with s-channel Higgs exchange. This graph, being $J = 0$, exactly cancels the bad behavior of the first two graphs. Moreover, it can be seen that the Higgs-quark coupling should be proportional to the quark mass for the cancellation to work. This is in fact the case for the standard model. Of course, several Higgs bosons instead of one can do

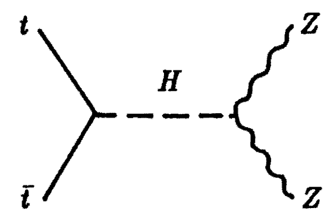

Fig. 2. Higgs exchange diagram for $t\bar{t} \to ZZ$.

the same job, provided that the couplings are correctly chosen (which is automatic in any consistent gauge model).

Another possibility is that there is no elementary Higgs particle. In this case, the top-Z (and also W) sector becomes strongly interacting at TeV energies and one may expect various phenomena such as the appearance of resonances.

Similar conclusion that either a scalar particle or a new strong interaction ought to undress itself can be drawn from the consideration of the process $WW \to WW$, for which the sum of the tree diagrams without Higgs exchange gives an amplitude proportional to s. WW scattering in the absence of a low mass ($\lesssim 1\,\text{TeV}$) scalar particle becomes strong at TeV energies.

5. The ρ parameter

The symmetry property of the broken vacuum is reflected in an observable: the ρ parameter defined by

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \,. \tag{7}$$

Here the meaning of the masses is obvious but the weak mixing angle may need explanation: It is defined by the ratio of the gauge couplings ($\tan\theta_W = g'/g$) and appears in the structure of the weak currents

$$\mathcal{L} = -\frac{e}{\sqrt{2}\sin\theta_W}\left(J_\mu^\dagger W^\mu + J_\mu W^{\mu\dagger}\right) - \frac{e}{\sin\theta_W \cos\theta_W} J_\mu^{(z)} Z^\mu \,, \tag{8}$$

$$J_\mu = J_\mu^1 + iJ_\mu^2 \,, \tag{9a}$$

$$J_\mu^{(z)} = J_\mu^3 - J_\mu^{(em)} \sin^2\theta_W \,, \tag{9b}$$

where J^i_μ is the i-th component of the weak isospin current and $J^{(\mathrm{em})}_\mu$ is the electromagnetic current (without the factor e). At low energies, ρ is related to the ratio of the charged- to neutral-current effective couplings

$$\rho = \frac{G_{\mathrm{NC}}}{G_F} , \tag{10}$$

with

$$\mathcal{L}_{\mathrm{eff}} = 2\sqrt{2}\left(G_F J^\dagger_\mu J^\mu + G_{\mathrm{NC}} J^{(Z)}_\mu J^{(Z)\mu}\right) . \tag{11}$$

Although we find $\rho = 1$ in the minimal standard model with a Higgs doublet, the same is not true for a general Higgs structure. If the SU(2)×U(1) symmetry is broken by a Higgs field with isospin T and hypercharge Y (such that the electromagnetic charge $Q = T_3 + Y$ is unbroken), we have[14]

$$m_W^2 = \tfrac{1}{2} g^2 \left[T(T+1) - T_3^2\right] \langle\varphi\rangle^2 , \tag{12a}$$

$$m_Z^2 = (g^2 + g'^2) T_3^2 \langle\varphi\rangle^2 , \tag{12b}$$

so that

$$\rho = \frac{1}{2}\left[\frac{T(T+1)}{T_3^2} - 1\right] , \tag{13}$$

where $\langle\varphi\rangle$ is the vacuum expectation value of the Higgs field of the component $T_3 = -Y$. Only few Higgs representations can give $\rho = 1$. Apart from $T = T_3 = \pm\frac{1}{2}$, one finds ($T = 3$, $T_3 = \pm 2$), ($T = \frac{25}{2}$, $T_3 = \pm\frac{15}{2}$), *etc.* When there are more than one Higgs fields, (13) is modified to

$$\rho = \frac{\sum_i \left[T^{(i)}(T^{(i)} + 1) - T_3^{(i)2}\right]\langle\varphi^{(i)}\rangle^2}{2\sum_i T_3^{(i)2}\langle\varphi^{(i)}\rangle^2} , \tag{14}$$

where the vacuum expectation values are constrained by

$$\sum_i \left[T^{(i)}(T^{(i)} + 1) - T_3^{(i)2}\right]\langle\varphi^{(i)}\rangle^2 = (2\sqrt{2} G_F)^{-1} . \tag{15}$$

The experimental fact $\rho = 1$ (up to about 1%) suggests that the vacuum expectation value is dominantly doublet, which should exist anyway to generate fermion masses. Any number of doublets give $\rho = 1$. Vacuum expectation values of possible non-doublet Higgs fields should be much smaller than $G_F^{-1/2}$. However, it is a logical possibility that several Higgs representations conspire to give $\rho = 1$. A certain combination of triplets with different hypercharge is shown[15] to preserve $\rho = 1$. This model has a custodial SU(2) symmetry.

So far our consideration has been at the classical level. Quantum corrections can change these results. A relatively large correction appears if the top quark is heavy. The ρ parameter in the minimal doublet model is modified at one loop as[16]

$$\rho \simeq 1 + \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \,. \tag{16}$$

In models with dynamical electroweak symmetry breaking, the value of ρ depends on the dynamics. In the simplest technicolor model with a left-handed doublet Q_L plus right-handed singlets U_R, D_R, the techniquark condensate $\langle \bar{Q}Q \rangle$ transforms as a doublet of $\mathrm{SU}(2)_L$, which gives $\rho = 1$.

6. Minimal Higgs boson

The simplest Higgs structure is that in the minimal standard model with one doublet. We find only one physical Higgs particle, which is exactly the "radial" mode of the broken vacuum. The Higgs boson thus has the same quantum numbers as the vacuum: $J^{PC} = 0^{++}$, $Q = 0$, *etc.*

The Higgs interactions are obtained by replacing the vacuum expectation value v by $v + H$ everywhere. Since every mass in the standard model comes from the Higgs vacuum expectation value, this corresponds to replacing each mass parameter m in the Lagrangian with $m(1 + H/v)$. (The Higgs self-coupling is exceptional.) The minimal standard model has a unique property of "second" universality, namely, mass–coupling proportionality. The constant of proportionality is the vacuum expectation value which is known from the muon decay constant: $v = (\sqrt{2}G_F)^{-1/2}$. This prediction, if confirmed, would give a strong evidence for the minimal Higgs structure. The property that Higgs prefers to interact with heavy particles has an important implication on the Higgs phenomenology.

7. Higgs boson mass

In the standard model, the mass of the Higgs boson cannot be predicted, just like those of quarks and leptons. The reason we *did* have predictions for the W and Z masses is that we had known the gauge couplings (from fine structure constant and $\sin^2\theta_W$) as well as the vacuum expectation value (from G_F). Since we have never perceived any Yukawa or Higgs interaction, the fermion and Higgs masses have to be determined by experiments.

Although the Higgs mass is an arbitrary parameter, some limits are available from consistency of the model. The starting point to understand these limits is the basic relation

$$m_H^2 = 2\lambda v^2 \,. \tag{17}$$

If we vary the coupling λ from 0 to ∞, the Higgs mass also moves from 0 to ∞. This observation is much too naive, however. Eq. (17) is just the lowest order relation which is subject to higher order corrections.

Lower end

If λ is very small, perturbation expansion in λ should behave well. On the other hand, the interactions of the Higgs with gauge bosons and fermions give rise to higher order corrections of order g^4 and f^4 (g and f denote gauge and Yukawa couplings, respectively), which can be numerically larger than the tree contribution of order λ. We thus expect that when $\lambda \lesssim g^4$ or f^4, the prediction (17) receives substantial corrections.

It was realized[17] that this kind of corrections can even change the pattern of symmetry breaking. The true vacuum (ground state) of the model as well as the Higgs mass can be derived from the effective potential, which is equal to the classical Higgs potential at the tree order. When $\lambda \lesssim g^4$ or f^4, the one-loop contribution to the effective potential becomes important. One may expect that the relation (17) is then modified to

$$m_H^2 \sim (g^4 \text{ or } f^4) v^2 \,. \tag{18}$$

In fact, there is a lower limit[18] on the Higgs mass of this order if there is no heavy fermions (which we now know is *not* the case):

$$m_H^2 \geq \frac{3\alpha^2}{16\sqrt{2}G_F}\,\frac{1}{\sin^4\theta_W}\left(2 + \frac{1}{\cos^4\theta_W}\right) \sim (6.5\,\mathrm{GeV})^2 \,. \tag{19}$$

A heavy fermion such as the top quark (the present bound[19] is $m_t > 89\,\mathrm{GeV}$) gives a negative contribution to m_H^2 and weakens the above bound. When $m_t \gtrsim m_W$, the bound disappears, but then m_H^2 starts to become negative, showing that the desired vacuum is unstable.[20] To prevent this, λ should increase to overcome the negative f^4 contribution, hence giving a lower limit on m_H^2 again: Very roughly, $m_H \gtrsim m_t$ is required for large m_t for the effective potential to be bounded from below. Note that these considerations apply only to the minimal standard model. If we have extra scalar particles or fermions (*e.g.*, in models with an extended Higgs sector or supersymmetry), there may be a scalar particle lighter than the above bounds.

Upper end

The relation (17) becomes untrustworthy for large λ also. For $\lambda \gg 1$, the strong coupling limit, (17) cannot escape from a huge correction and loses its significance. In the strong coupling regime, the physical "Higgs" boson, if such a particle exists, deviates from a simple elementary excitation of the Higgs field in the Lagrangian and rather resembles a composite particle. The meaning of λ and m_H^2 becomes ambiguous. We do not know what really happens for large λ, but we do know something has to occur in the scalar sector, as discussed earlier.

A benchmark Higgs mass indicating the border of the strong coupling region can be obtained[21] from partial wave unitarity. One expects that $\lambda \gtrsim 1$ corresponds to strong coupling, but a more appropriate expansion parameter may be $\lambda/4\pi^2$. Moreover, there is an ambiguity of a numerical factor in defining λ (especially the combinatorial factor). Consideration of a physical process is free from these uncertainties and can give a convention-independent value. Imposing the partial wave unitarity bound $|T^J| \leq 2$ on the $J = 0$ $W_L^+ W_L^- \to W_L^+ W_L^-$ tree amplitude at high energy limit (W_L denotes helicity-0 W) gives a bound $m_H^2 \leq 4\sqrt{2}G_F$. A slight improvement is made by treating the $W_L^+ W_L^-$, $Z_L Z_L$, and HH channels simultaneously:

$$m_H^2 \leq \frac{8\sqrt{2}\pi}{3G_F} = (1.01\,\mathrm{TeV})^2 .$$

There have been attempts to derive a nonperturbative upper limit on m_H in lattice-regularized models, either by Monte Carlo simulations[22] or analytic methods.[23] These studies yield upper bounds around 600–700 GeV. These results depend, however, on various approximations (such as the finite size of the lattice used in Monte Carlo) and rely on the regularization by a lattice to start with.

Prejudice-dependent limits

Applicability of perturbation theory is essential if we believe in grand unification. Grand unified theories, especially supersymmetric ones, gives a prediction for $\sin^2\theta_W$ which is in good agreement with experiments. This prediction, obtained using the first few terms of the β functions, would not hold if perturbation theory breaks down.

Much stronger limits on the Higgs mass have to be observed[24] in this grand unification scenario. The Higgs coupling λ is not asymptotically free and the lowest order renormalization group equation gives the solution (neglecting all other couplings)

$$\lambda(\mu) = \frac{\lambda}{1 - \frac{3\lambda}{4\pi^2}\log\frac{\mu^2}{m_H^2}} = \frac{\frac{1}{\sqrt{2}}G_F m_H^2}{1 - \frac{3G_F m_H^2}{4\sqrt{2}\pi^2}\log\frac{\mu^2}{m_H^2}} \,. \tag{20}$$

This diverges at

$$\mu_\infty = m_H \exp\left(\frac{2\sqrt{2}\pi^2}{3G_F m_H^2}\right) \,. \tag{21}$$

If you demand that $\lambda(\mu)$ should be finite (or perturbatively controllable, which gives a similar result anyway) up to a certain scale M, Eq. (20) gives you a "prejudice-dependent" upper bound on m_H. For example, taking $M = 10^{15}$ GeV or Planck mass, we find $m_H <$ 160 GeV or 140 GeV.

These bounds get modified somewhat when higher order effects and other couplings, especially the top Yukawa coupling, are taken into account. When m_t increases, the bound changes slightly but remained roughly the same until one hits the vacuum stability bound mentioned earlier. Thus, minimal grand unification implies the limits $\frac{5}{3}(m_t - 80) \lesssim m_H \lesssim$ 200 GeV. For a very detailed discussion of these bounds, see Ref. 25. The lower bound does not hold for nonminimal models, but the upper limit is rather general in theories with "elementary" Higgs fields, supposed to be applicable up to GUT/Planck scale.

Triviality: Higgs sector as an effective theory

The well-known triviality of ϕ^4 theories[26] may be cast in a physical language in this context. The mathematical statement of triviality is that if we keep the bare ϕ^4 coupling finite, the renormalized coupling is zero, *i.e.*, there is no interaction. What is crucial here is to understand the meaning of a finite bare coupling. In the framework of renormalization group, the bare coupling corresponds to the running coupling at infinite momentum scale (or zero distance). We have seen that the solution of the (lowest-order) renormalization group equation diverges at a finite μ (see Eq. (20)). If one wants $\lambda(\mu)$ to be finite at any μ, the only possibility is to choose $\lambda = 0$. The mathematical proof of triviality (I am not concerned here how rigorous the proof is) puts this perturbation-based observation on a more solid basis. (Triviality of a gauge-Higgs system is not proven, though, which may or may not mean the system is trivial.)

The moral of triviality of the Higgs sector is just that we cannot regard a ϕ^4 theory as an ultimate theory. It has to have a limit of applicability ($< \mu_\infty$). Of course, nobody would believe that the standard model is the final theory. Apart from many aesthetic problems, it does not contain gravitation. One thus expects that the elementary Higgs sector provides a good approximation to the real world *at most* up to Planck scale. (The scale may be as low as the Higgs mass itself, if the composite scenarios are correct.) The most restrictive limit from this consideration is $m_H \lesssim 140\,\text{GeV}$ as derived in the last section.

Three Categories

To summarize, we may classify possible Higgs masses into three regions:

	m_H (GeV)	Category
(1)	$\lesssim 10$	Radiative
(2)	10–200	Elementary
(3)	$\gtrsim 200$	Composite

Category (1) is quite special and would suggest that the Higgs mass comes from radiative or quantum effects with indeterminate tree-order potential. Recent limits from LEP excludes a standard Higgs with mass $\lesssim 25\,\text{GeV}$, however (see the following section).

Category (2) corresponds to the mass region expected in models with elementary Higgs fields, such as grand unified theories and supersymmetric models. If the Higgs mass is larger, the self-coupling blows up before the unification scale is reached. In particular, in the minimal supersymmetric model, the mass of the lighter scalar Higgs boson is less than m_Z. This is because the quartic coupling in this model arises entirely from the D term, whose size is fixed by the gauge couplings. Generically speaking, models with perturbative unification should have at least one scalar particle in this category.

Category (3) is the region where one would expect the Higgs as a composite state. Even in the standard model, strong interactions make the physical Higgs particle rather composite-like (strongly varying form factors, *etc.*).

An interesting case lies on the border between the categories (2) and (3). This is where the self-coupling becomes large *exactly* at the unification or Planck scale. This situation is realized in the recent scenarios of top-originated Higgs sector. For example, Bardeen *et al.*[11] predicts $m_H \sim 240\,\text{GeV}$ and $m_t \sim 220\,\text{GeV}$ for $\Lambda \sim 10^{19}\,\text{GeV}$. In these models, the Higgs behaves as an elementary (pointlike) particle below the scale Λ.

8. Present experimental limits for standard Higgs boson

There has been a great forward leap in the search for a relatively light Higgs boson in 1989. The number of relevant experimental articles is more than doubled. In the following I discuss main results on the Higgs boson mass limit. A fuller compilation may be found in Ref. 27.

A limit for a very light Higgs ($m_H \lesssim 2m_e$) comes from the measurement[28] of hyperfine splitting of muonic atoms. The splittings agree with QED predictions at the level of 10^{-6}. A Higgs boson with $m_H \lesssim 1\,\text{MeV}$ is excluded at more than three standard deviations, even if a conservative estimate of the Higgs-nucleon coupling‡ is used. The 68%CL limit is 10 MeV if $\eta_{HNN} = 2/9$ is used.

The search for a rare kaon decay $K^+ \to \pi^+ H$, where H is not detected, is also relevant to a very light H. A BNL experiment[31] gives upper limits for the branching ratio of less than 10^{-8} (90%CL) for $m_H \lesssim 10\,\text{MeV}$, $< 10^{-7}$ for $m_H \lesssim 20\,\text{MeV}$. The decay amplitude for this process contains a "hard" part from $s \to dH$, which has been calculated by many groups,[32] and a "soft" long-distance part. Chiral perturbation theory is employed to get estimates[33–35] for the amplitude, which indicates that the two contributions can interfere destructively. The natural order of magnitude for the branching ratio is at the 10^{-4} level, but accidental cancellation is possible, if unlikely, because of our ignorance of the soft contribution. For the charged kaon decay $K^+ \to \pi^+ H$, the imaginary part of the hard amplitude gives a lower limit on the branching ratio,[34] which however depends on m_t and the Kobayashi-Maskawa parameter η. If we take a rather conservative estimate $\eta \gtrsim 0.1$[36] and $m_t \gtrsim 80\,\text{GeV}$, we find $\mathrm{B}(K^+ \to \pi^+ H) \gtrsim 1 \times 10^{-6}\bar{\beta}$, where $\bar{\beta}$ is the S-wave phase space factor. The experimental result thus excludes $m_H \lesssim 20\,\text{MeV}$.

‡ If one parametrize the Higgs-nucleon coupling as $g_{HNN} = \eta_{HNN}(\sqrt{2}G_F)^{1/2}m_N$, the estimate by Shifman *et al.*[29] assuming three heavy flavors gives $\eta_{HNN} = 2/9$, which I regard conservative. More recent estimates[30] with nonzero strange-quark matrix element of proton give $\eta_{HNN} \simeq 0.56$.

Nuclear 0^+–0^+ transitions by Higgs emission followed by $H \to e^+e^-$ were searched for with negative results.[37] This can exclude the mass range $2m_e$–10 MeV, the upper end again depending on the Higgs-nucleon coupling.

An electron beam dump experiment at Orsay[38] excludes the region 1.2–52 MeV at 90%CL. The result depends only on the Higgs-electron coupling, which is unambiguously predicted in the standard model.

The rare pion decay $\pi^+ \to e^+\nu H$ followed by $H \to e^+e^-$ was searched for[39] by SINDRUM Collaboration at SIN (now PSI), The limit for the branching ratio is between 10^{-9} and 10^{-11} for m_H = 10–100 MeV. The theoretical prediction[40,34] is quite robust and the mass range 10–100 MeV is excluded at 90%CL.

The kaon decay $K \to \pi H$ has been searched for by several experiments. Apart from the search for $K \to \pi +$ "nothing" mentioned earlier, there are two significant results. NA31 Collaboration at CERN[41] looked for $K_L \to \pi^0 H$, $H \to e^+e^-$ and derived a bound for the product of the branching ratios of 1×10^{-7} at 90%CL for $m_H \gtrsim 15$ MeV and 2×10^{-8} for $m_H \gtrsim 50$ MeV (the limit extends to above 300 MeV). Although there is no theoretical lower bound for $\Gamma(K_L \to \pi^0 H)$ in contrast to the charged kaon case, the result is likely to exclude the mass range 15 MeV–$2m_\mu$ barring accidental cancellation.

A BNL experiment[42] searched for $K^+ \to \pi^+ H$ followed by $H \to \mu^+\mu^-$, with a limit for the product of the branching ratios of 1.5×10^{-7} at 90%CL for m_H = 220–320 MeV. Compared to the theoretical lower bound mentioned earlier, the limit is marginal especially at the higher side, because of the phase space suppression ($\bar{\beta} \simeq 0.4$ at $m_H = 320$ MeV) and the opening of $H \to \pi\pi$ decay at 270 MeV. Nevertheless, the mass range is likely to be excluded.

The importance of the B decay $B \to HX$ has been recognized recently. The decay rate[32] $\Gamma(b \to sH)$ is proportional to m_t^4 and the branching ratio exceeds 20% if one uses the CDF limit[19] for m_t. Several decay chains $B \to HX$, $H \to e^+e^-$ (Mark II at PEP[43]), $B \to HX$, $H \to \mu^+\mu^-$, and $B \to HK$, $H \to \mu^+\mu^-$, $\pi^+\pi^-$ (CLEO[44]) have been searched for and the mass range 70 MeV–3.6 GeV is excluded with high confidence.

The Υ decay $\Upsilon \to H\gamma$[45] produces a monochromatic line in the photon energy spectrum. A limit comes from CUSB,[46] excluding 210 MeV–5 GeV if $\mathcal{O}(\alpha_s)$ corrected theoretical prediction[47] is used for the rate. The $\mathcal{O}(\alpha_s)$ correction is large (reducing the rate by a factor of 2) and the limit is very sensitive to additional corrections.

Very recently, new limits have been announced by LEP groups, which looked for the decay $Z \to Z^* H$, Z^* (virtual Z) $\to \ell\bar{\ell}$, $\nu\bar{\nu}$, $(q\bar{q})$. The expected rate is not so small for light Higgs masses. ALEPH[48] excludes the region 32 MeV–24 GeV, OPAL[49] rules out 3.0–25.3 GeV, and DELPHI[50] 210 MeV–14 GeV, all at 95%CL. A more recent report from ALEPH[51] extends the excluded region fully down to zero mass, making use of the missing momentum signature.[52]

WH and ZH associated production in $p\bar{p}$ interactions at $E_{\rm cm} = 1.8\,$TeV was searched for by CDF,[53] where H decays to $\mu^+\mu^-$, $\pi^+\pi^-$, or K^+K^-. They exclude the interval $2m_\mu$–$2m_K$ (except for 818–846 MeV) at 90%CL, though the statistics (based on $4.4\,\mathrm{pb}^{-1}$) is quite marginal.

To summarize, the standard Higgs boson with mass less than 3.6 GeV is excluded by various low energy experiments, especially muonic-atom hyperfine splitting, electron beam dump, pion decay, and B decay, with overlapping excluded regions. Recent LEP results extend the limit to 25 GeV.

9. Higgs decay modes

The Higgs boson decays to the heaviest particle pair accessible, because of the mass-coupling proportionality. The decay width to a fermion pair is

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 m_H \beta_f^3}{4\sqrt{2}\pi} \times \begin{cases} 1 & \text{lepton} \\ 3 & \text{quark} \end{cases} , \tag{22}$$

where $\beta_f = (1 - 4m_f^2/m_H^2)^{1/2}$, and those to a weak boson pair are

$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3 \beta_W}{8\sqrt{2}\pi}\left(1 - \frac{4m_W^2}{m_H^2} + \frac{12m_W^4}{m_H^4}\right) , \tag{23}$$

$$\Gamma(H \to ZZ) = \frac{G_F m_H^3 \beta_Z}{16\sqrt{2}\pi}\left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12m_Z^4}{m_H^4}\right) , \tag{24}$$

where β_W and β_Z are defined similarly as β_f. Other two-body decays $H \to gg$, $\gamma\gamma$,[54] $Z\gamma$[55] do not exist at the tree level but occur through higher-order loops. The rates are generally small.

There is an important effect which is not so widely known: the QCD correction[56–60] to the decay $H \to q\bar{q}$. The correction factor is large and positive near threshold, eventually leading to the mixing of the Higgs with quarkonium states.[61,59] In the opposite extreme case $m_H \gg m_q$, the correction is again large but negative, with a logarithmic factor $\log(m_q^2/m_H^2)$. A bulk of the correction can be absorbed[56,57] if one replaces the quark mass in the tree-level formula by a running quark mass, which actually sums over leading logarithms. In any case, the decay rate to quarks is reduced by the correction to a large extent for $m_q \ll m_H$, which may be relevant for the intermediate-mass Higgs searches.

The calculated branching ratio of the standard Higgs boson is shown in Fig. 3, for the two cases $m_t = 100$ and 200 GeV. Several comments should be made:

1) Since $m_t \gtrsim m_W$, the $b\bar{b}$ mode is dominant until the W^+W^- mode takes over. The $t\bar{t}$ mode is *never* dominant, at most 20–30% which occurs at somewhat above the threshold.

2) The W^+W^- and ZZ rates in Fig. 3 include one-virtual-one-real configurations,[62] *i.e.*, WW^* and ZZ^*. It can be seen that the three-body mode $Wf\bar{f}$ already dominates $b\bar{b}$ at $m_H \sim 140$ GeV, below the real WW threshold. A detailed plot of the branching ratios in this region is shown in Fig. 4.

3) The effect of the negative QCD correction can be seen in the comparison of $\tau^+\tau^-$ and $c\bar{c}$ rates which shows a crossover. The branching ratio for $\tau^+\tau^-$ is 6–8% when $b\bar{b}$ is dominant.

4) The loop-induced modes gg, $\gamma\gamma$, $Z\gamma$ depend on m_t for $m_H \gtrsim 100$ GeV. The branching ratio for gg is never above 10%, and those for $\gamma\gamma$ and $Z\gamma$ are at most 2×10^{-3}.

The main modes are thus as follows:

m_H (GeV)	Dominant decay
10–140	$b\bar{b}$
140–180	W^+W^- (incl. $Wf\bar{f}$)
> 180	W^+W^-, ZZ (~2:1)

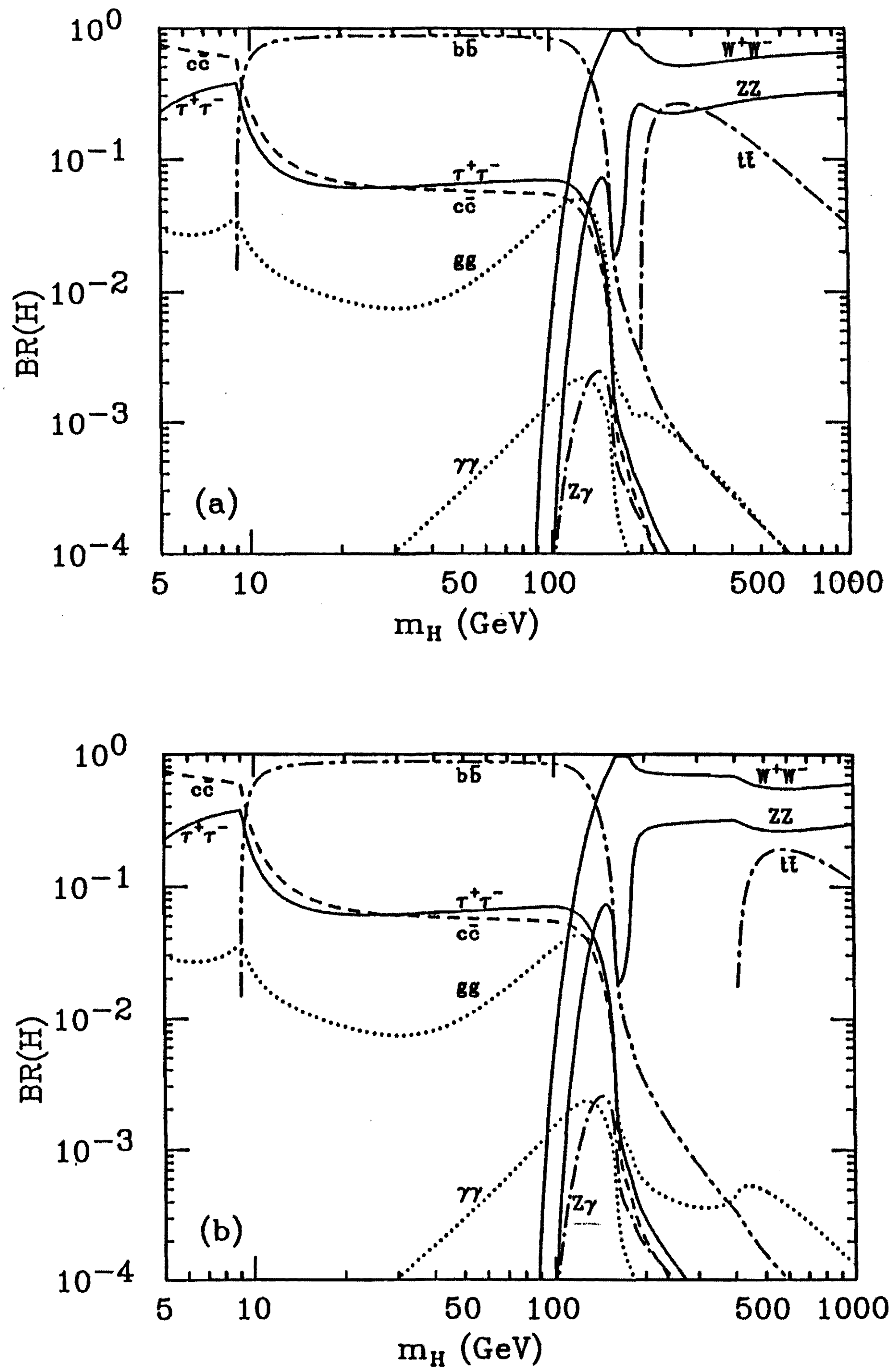

Fig. 3. Branching ratios of the standard Higgs boson for (a): $m_t = 100\,\text{GeV}$; (b): $m_t = 200\,\text{GeV}$.

10. Future Higgs searches

LEP-I: Limits from the first year's data are already available, as discussed earlier. The search utilizes the decay $Z \to Z^*H$, $Z^* \to \ell\bar{\ell}$, $\nu\bar{\nu}$, $(q\bar{q})$. The branching ratio[63] decreases rapidly as m_H increases. The reason is twofold: (1) For small m_H, the virtual Z can become closer to the pole, enhancing the rate; (2) For large m_H, the three-body decay is suppressed by the factor $(m_Z - m_H)^5$. The exploration of the larger mass region thus requires a much larger data sample and we should await for another year or two. Eventually one should be able to detect the signal up to 40 GeV or so.[64]

The decay $Z \to H\gamma$ occurs via loops. The expected rate[65] is too small to be really useful. Being a two-body decay, the branching ratio for large m_H exceeds that for $Z \to Z^*H$. The rate becomes too small, however.

LEP-II: Above the Z pole, associated production $e^+e^- \to Z^* \to ZH$ has a reasonable cross section if $\sqrt{s} > m_Z + m_H$. LEP-II is expected to cover the range $m_H \lesssim m_W$ by this process.[66]

Tevatron: Difficult if not impossible. The cross section is too small for not too light Higgs bosons, and the background is huge. There are proposals[67,59] to use the $\tau^+\tau^-$

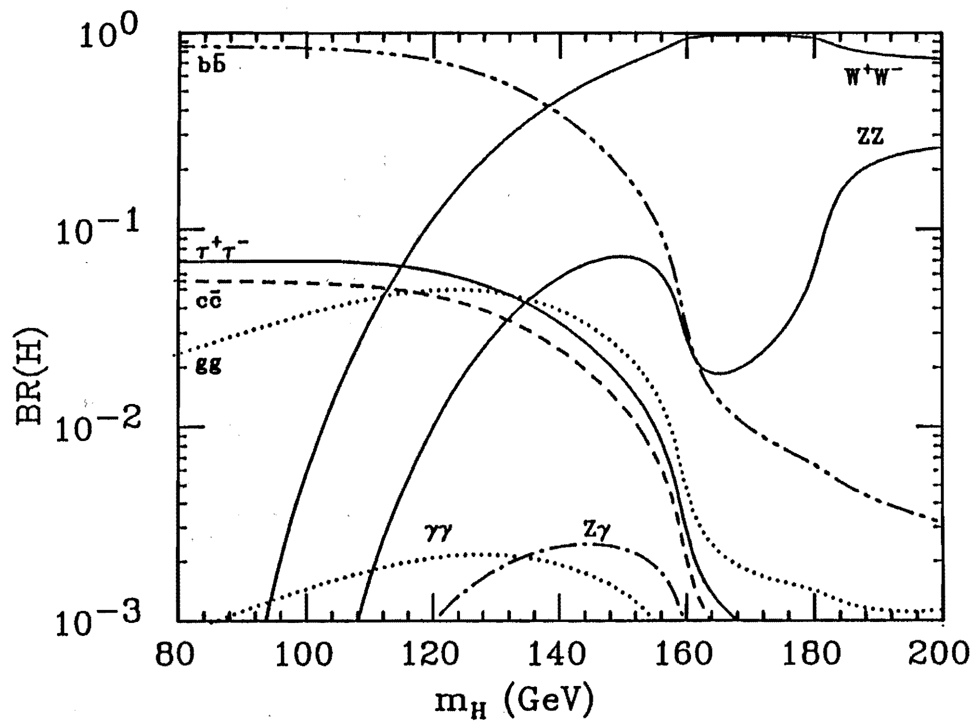

Fig. 4. Standard Higgs branching ratios in the "intermediate" region $m_H =$ 80–200 GeV. $m_t = 100\,\text{GeV}$ assumed.

mode to tag the event, but it can only be used if $m_H \lesssim 10\,\text{GeV}$, which is already excluded by LEP.

SSC/LHC: Two mechanisms—gluon fusion and W fusion—can produce the Higgs with a not-too-pessimistic cross section. If $m_H \gtrsim 200$ GeV, the decay chain $H \to ZZ \to \ell\bar{\ell}\ell\bar{\ell}$ or $\ell\bar{\ell}\nu\bar{\nu}$ is thought to be a good signature. However, including realistic estimates of the acceptances and efficiencies does not leave so many events above backgrounds and the accessible mass range has not been firmly established.[68] For the intermediate mass region $m_H \lesssim 200\,\text{GeV}$, the detection is probably impossible.

TeV e^+e^- colliders: Two processes are important: $e^+e^- \to Z^* \to ZH$ for lighter mass region and WW fusion ($e^+e^- \to \nu\bar{\nu}H$) for higher masses. ZZ fusion ($e^+e^- \to e^+e^-H$) has a cross section only 1/10 of that of the WW fusion. A collider with $E_{\text{cm}} = 1\,\text{TeV}$ can explore quite a wide range up to $m_H \sim 500\,\text{GeV}$.[69] In particular, the intermediate mass region is accessible only at e^+e^- colliders. Higgs search is thus a particularly suited subject for a TeV e^+e^- collider.

11. Summary

We already possess some circumstantial evidence that the intermediate vector bosons are massive gauge particles. This requires the Higgs mechanism at work, which in turn implies the existence of (unphysical) Goldstone bosons tied to the breaking of SU(2)×U(1). The nature of these Goldstone bosons is not known yet. They can be either elementary particles as in the standard model or supersymmetric models, or composite pion-like particles as in the technicolor models. The existence or nonexistence of a physical Higgs particle is a different matter, but we find at least one physical scalar particle in any elementary-type model as the radial mode of the vacuum. Although such a particle may not exist in composite-type models, a simple argument shows that some new physics ought to uncover itself in the $J = 0$ sector, either one or more scalar particles or strongly interacting W–Z–top system.

Whatever the cause of the SU(2)×U(1) breaking may be, the measured ratio m_W/m_Z suggests that its transformation property is $\Delta I = \frac{1}{2}$. Any number of Higgs doublets as well as usual techniquark scenarios are consistent with this observation.

The mass of the Higgs boson remains unpredictable without any information on the dynamics of the SU(2)×U(1) breaking. A Higgs boson with mass $\lesssim 10\,\text{GeV}$ represents a special case and would indicate that its mass comes from radiative effects. A borderline separating the "elementary" and "composite" regions can be drawn at about 200 GeV.

Unification ideas generally result in masses below that value. The minimal supersymmetry predicts a Higgs boson lighter than Z^0 due to its special feature of the potential. Recent scenarios of the third-generation induced breaking predict the Higgs mass just on the boundary of the two regions. Above 1 TeV, the predicting power of perturbation theory is lost and the Higgs boson becomes inevitably strongly interacting.

Present experimental bound for the standard Higgs boson is about 25 GeV from the search for the decays $Z^0 \to H^0\ell^+\ell^-$, $H^0\nu\bar{\nu}$ at LEP.

Future searches at LEP-I should extend the sensitivity to ~40 GeV, LEP-II to ~80 GeV. Super hadron colliders such as SSC may probe the mass range above 200 GeV with the decay mode $H \to ZZ$. The region between 80 and 200 GeV can only be approached by next-generation e^+e^- linear colliders such as JLC, which is also powerful for masses above 200 GeV.

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