# The Equivalent Photon Approximation vs. Exact Calculation for the Single Weak Boson Production in $e^+e^-$ Collision


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The equivalent photon approximation (EPA) for the process $e^+e^- \rightarrow e\nu W$ is discussed in comparison with the exact calculation, mainly concerning the electroweak gauge cancellation. The EPA is shown to be fairly good approximation for this electroweak process.

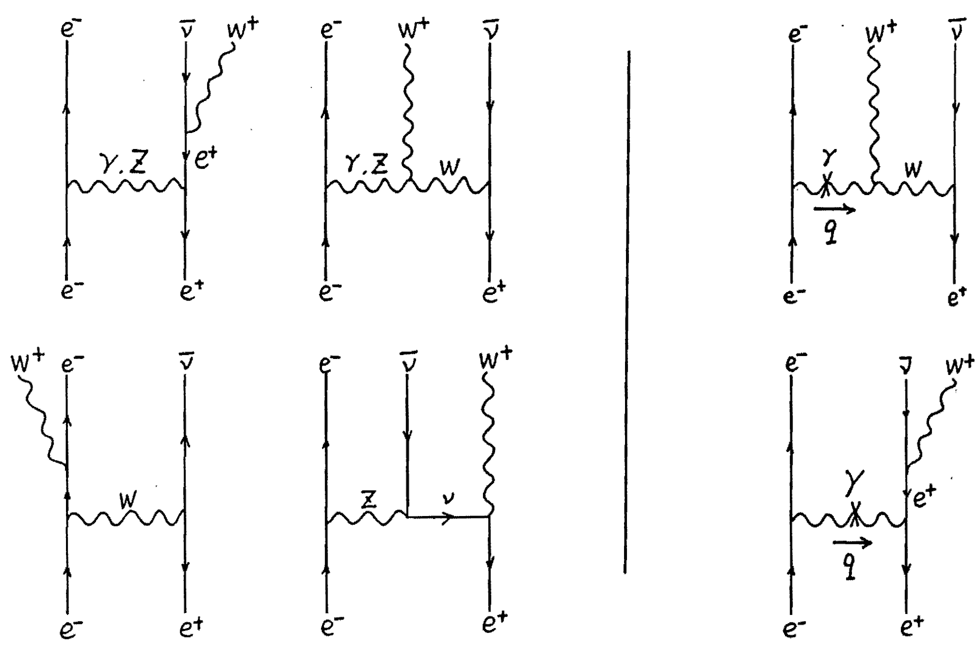

## 1. Introduction

As the energy frontier explores into the Fermi scale world, various new production processes take place and force us the evaluation of about 10–20 Feynman diagrams even in the Born approximation. One way to evaluate the matrix element for many diagrams is the numerical helicity amplitude evaluation, and another way is the automatic generation of computer codes, and, in some situations, the approximate evaluation is used. Approximations are useful for quick references and often well-describe the essential features of the processes.

For the pure QED processes, the equivalent photon approximation (EPA) is almost always valid, although for some processes it fails, mainly due to the threshold and/or cut-off effect of the subprocesses. But in the case of the process $e^+e^- \rightarrow e\nu W$, the problem is not so simple because of the W and Z boson exchanges involved.

The aim of this talk is to clarify, about an example of the process $e^+e^- \rightarrow e\nu W$, the relation between the exact matrix element evaluation[1,2] taking into account all the contribution from the diagrams in Fig.1a and the EPA[3] corresponding to Fig.1b in which the t-channel photon is assumed to be onshell.

In the followings, the EPA and its intuitive meaning, the exact evaluation and some features relevant to the EPA, and concluding remarks are given.



(a) (b)

Fig.1 Feynman diagrams for $e^+e^- \to e^-\nu W^+$. a) all the diagrams, b) the diagrams needed for EPA.

## 2. EPA

The EPA deals with the processes in which t-channel photon exchanges yield the dominant contribution. Schimatically, this approximation implies the following procedure: First one has to neglect the diagrams that do not involve any t-channel photon exchanges. Then, the photon is assumed to be emitted onshell from $e^-$ into the beam direction collinearly, with energy $xE_0$ ($E_0$: beam energy, $x$: energy fraction of photon) and with probability density $P_{\gamma/e}(x)$, and the $e^-$ is assumed to go away into the beam pipe, while rigorous kinematics allows only the space-like virtual photons, which is emitted maily into very small scattering angles. (Throughout the rest of the talk we consider only about the final state $e^-\bar{\nu}_e W^+$ just for convenience.) The rest subprocess, $\gamma e^+ \to W\bar{\nu}$, is usually treated without approximation.

This picture gives the expression for the cross section $\sigma(s)$ of $e^+e^-$ collision expressed with respect to the subprocess cross section $\hat{\sigma}(\hat{s})$ as,

$$\sigma(s\,;e^+e^- \to e\nu\mathrm{W}) \simeq \int_{x_{min}}^{x_{max}} dx\, P_{\gamma/e}(x) \int_{q^2_{min}}^{q^2_{max}} \frac{dq^2}{q^2}\, \hat{\sigma}(\,\hat{s}\,,q^2=0\,;\gamma e \to W\nu),$$

where $\hat{s}$ denotes the c.m. energy of $\gamma e^+$, and $q$ the photon momentum.

Since we, setting $q^2 = 0$ to evaluate the subprocess cross section $\hat{\sigma}(\hat{s})$, assumed that $q^2$ dependence can be represented solely by the $q^2$-integral part, the EPA yields a constant $\ln q^2$ -distribution; $d\sigma/d(\ln q^2)$ does not depend on $\ln q^2$. This is the most remarkable aspect of EPA, and another important point of EPA resides in the treatment of the integration regions, *i.e.*, the treatment of $x_{max}$, $x_{min}$, $q^2_{max}$, and $q^2_{min}$.

### 3. Comparison of EPA with the exact calculation

To illustrate which features of the actual process $e^+e^- \to e\nu\mathrm{W}$ are extracted in the EPA calculation, several quantities are compared between the EPA and exact calculation[2] .

#### a. Total cross section

As shown in Fig.2, the contribution of t-channel photon exchange diagrams significantly over-estimates at the c.m. energy of TeV region; the deviation of it from the exact value corresponds to the electroweak gauge cancellation among the diagrams.

The EPA, though based on these t-channel photon exchanges, gives a fairly good estimation, with errors within a few percent. Since the EPA differs from the 't-channel photon only' by their photon virtualities in the hard collision subprocess, the gauge cancellation turns out to be caused by the photons with large virtualities. The counterpart of the cancellation is, of course, the other diagrams in Fig.1a.

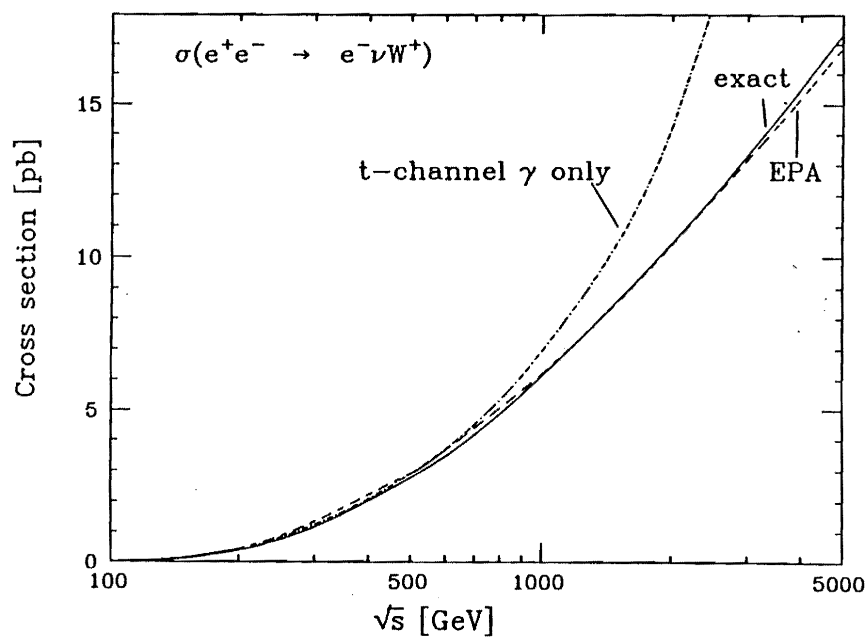



Fig.2 The total cross section for $e^+e^- \to e^-\nu W^+$. The given EPA curve was evaluated with the prescription, $x_{max} = 1$, $x_{min} = m_W^2/s$, $q^2_{max} = s$ and $q^2_{min} = m_e^2$.

**b. $\ln q^2$-distribution**

As mentioned before, the EPA *a priori* assumes a constant $\ln q^2$ -distribution. Therefore, if the process does not actually have such a distribution, the EPA does not necessarily work well. In our example, Fig.3 illustrates the situation:

The Fig.3 indicates that the constant $\ln q^2$ -distribution of the EPA gives a qualitatively good picture. But its a-few-percent accuracy seems accidental, because the actual distributions do not become constant around the minimum and maximum of the kinematically allowed region of $\ln q^2$. Indeed, $q^2_{max}$ and $q^2_{min}$ depend on the photon energy, namely on $x$ in the previous equation, this causes the smearing of the $\ln q^2$ -distribution by the integration over $x$. To take into account this dependence might lead to better approximations.

On looking at the enormous enhancement in the distribution for the photon exchanges, most part of which is due to the photons coupled to the longitudinal W bosons, one can understand that $W_L$ in the final state is another factor of the deviation of the 't-channel $\gamma$ only' curve in Fig.2 from the exact one at the TeV scale. Or, one may infer that $W_L$ coupled to the photon with large virtuality plays the central role in the gauge cancellation. The cancellation imposes an effective cut-off on large $q^2$, which makes $q^2_{max}$ used in EPA unpredictable.

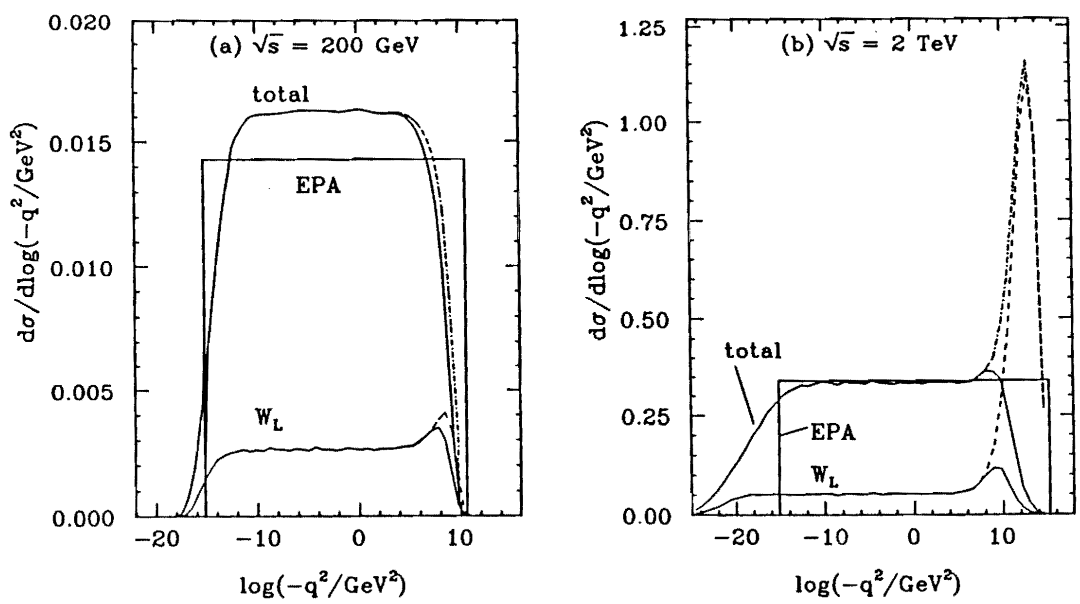



Fig.3 $\ln q^2$ -distributions for $e^+e^-$ c.m. energies 200 GeV and 2 TeV. The solid box in each figure expresses the EPA estimation, and the dashed lines the contributions only from the diagrams with t-channel photon exchanges. $W_L$ and 'total' mean the longitudinal W boson only and the sum of all the W boson helicity states, respectively.

## c. $p_T$-distribution and others

The $p_T$-distribution in the EPA is sometimes said to be a drawback of it, but, in $e^+e^- \rightarrow e\nu W$, the $p_T$ of W distributes broadly, and there is no significant difference between the real and approximated distributions. Even in the processes which have the double-logarithmic enhancement and yield very narrow $p_T$-distributions like $e^+e^- \rightarrow eeZ$, however, it is possible to improve the EPA as in the Gabrielli's method[4], so that the resulting distribution should become quite close to the exact one.

The energy distributions of any of final particles generally can be well-estimated by the EPA. The only and apparent difference of the EPA and exact calculations lies in the tagging of the final beam particle; since the EPA assumes that the initial beam particle which emitted the collinear photon runs away into the beam pipe with zero scattering angle, the EPA can never be used for the analyses of tagged events.

## 4. Concluding remarks

From the analysis of the process $e^+e^- \to e\nu\mathrm{W}$, one may induce that the EPA is still valid for the electroweak processes, even for the cases which involve large cancellation owing to the electroweak gauge symmetry. The reason why the quantitative accuracy remains within errors of a few percent is beyond the scope of the talk. (See Fig.3.) And, on using the EPA, one sometimes has to modify the prescription into more suitable forms according to the kinematical situations[2,4] .

## References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2. K. Hagiwara, H. Iwasaki and D. Zeppenfeld, in preparation.
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4. E. Gabrielli, in *Proc. of La Thuile Workshop on Physics at Future Accelerators*, La Thuile, Italy, Jan. 1987, CERN 87–07