Verification of the Quantum Electroweak Effects through the Weak Boson Mass Relation

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ABSTRACT

Recent studies of the $M_W - M_Z$ relation using the new experimental data on the weak boson masses are reported. It is shown that those data clearly indicate the existence of the quantum effects of the electroweak interaction, and also give an upper bound on the top-quark mass.

The standard $SU(2) \times U(1)$ gauge theory of the electroweak interaction (the electroweak theory hereafter) has been so far very successful for describing a lot of weak interaction phenomena. It has been even crucial recently to take account of the radiative corrections (R.C.) in these analyses [1]. This fact gives a strong support to the validity of the electroweak theory beyond tree approximation.

Those analyses, however, have fully used the value of $\sin^2 \theta_W$ extracted from the deep inelastic neutrino-nucleon scatterings. There, actual experimental conditions have to be incorporated, and also we are forced to use the parton model to describe the quarks in the target nucleons. Although these points must have been carefully taken into account, other precision tests independent of the $\nu$ experiments are therefore strongly desired. Based on my recent analysis [2], I wish to show here that new experimental data on the weak boson masses make actually such a "new-type"
analysis possible for the first time.\footnote{In this report, I wish to show calculations using the data of LEP (ALEPH, DELPHI, L3 and OPAL collaborations) on the Z mass \cite{3} in addition to the data of MARK II, CDF and UA2 \cite{4-6} although those LEP data were not available when this meeting was held.}

Necessary theoretical tool for this purpose is only the $M_W-M_Z$ relation, which is the weak boson mass relation derived from the muon decay amplitude through the following relation \cite{7}:

$$G_F(\alpha, M_W, M_Z, m_f, m_\phi) = G_F^{exp}.$$  \hspace{1cm} (1)

Here the left-hand-side is the Fermi coupling constant calculated within the electroweak theory, and $G_F^{exp} = 1.16637 \times 10^{-5}$ GeV$^{-2}$ is the corresponding experimental data. This $G_F$ is a function of the five parameters of the theory as is expressed, but its behavior is mainly controlled by $\alpha$, $M_W$ and $M_Z$. Therefore, Eq.(1) gives a relation between $M_W$ and $M_Z$ which depends also on $m_f$ and $m_\phi$ to a certain extent once we use $\alpha = \alpha^{exp} = 1/137.036$. This is what I call "the $M_W-M_Z$ relation".

This relation is usually expressed as a value of $M_W$ calculated from $\alpha$, $G_F$, $M_Z$, $m_f$ and $m_\phi$:

$$M_W = M_Z \left[ \frac{1}{2} \left(1 + \sqrt{1 - \frac{2\sqrt{2}\pi\alpha^{exp}}{M_Z^2 G_F^{exp} (1 - \Delta r)}} \right) \right]^{1/2},$$  \hspace{1cm} (2)

where $\Delta r$ represents the higher order corrections in the $\mu$ decay amplitude (Sirlin’s notation \cite{8}), so a function of $\alpha$, $G_F$, $M_Z$, $m_f$ and $m_\phi$. This formula includes the full $O(\alpha)$ corrections plus the leading log terms to all orders in perturbation. If we assume that $m_t$ is less than a hundred GeV and $m_\phi$ less than several hundred GeV, we obtain $|M_W - M_W^{(0)}| \sim 1$ GeV where $M_W$ and $M_W^{(0)}$ are those calculated with and without the higher order corrections (i.e., $\Delta r$) respectively. This relation is therefore quite useful for a clean test of the electroweak theory at quantum correction level, which was pointed out already early in the eighties \cite{7}.

Now we are ready for numerical calculations. I use the following value for $M_Z$:

$$M_Z^{exp} = 91.11 \pm 0.05 \text{ GeV},$$  \hspace{1cm} (3)

which has been derived by combining the data of SLC: $M_Z^{exp} = 91.17 \pm 0.18$ GeV.
First, without the radiative corrections, $W$ mass is calculated by using the above $M_Z^{\text{exp}}$ as

$$M_W^{(0)} = 80.84 \pm 0.06 \ \text{GeV}. \quad (4)$$

This value is obviously inconsistent with

$$M_W^{\text{exp}} = 80.0 \pm 0.56 \ \text{GeV}, \quad (5)$$

which is obtained from $M_W^{\text{exp}} = 80.0 \pm 0.62 \ \text{GeV}$ (CDF) [5] and $M_W^{\text{exp}} = 80.0 \pm 1.33 \ \text{GeV}$ (UA2) [6].

On the other hand, if we take account of the radiative corrections with $m_t = 112 \ \text{GeV}$ and $m_b = 100 \ \text{GeV}$ (note that data of UA1, UA2 and CDF all indicate $m_t \sim 60-70 \ \text{GeV}$ [9]), we can reproduce the above $M_W^{\text{exp}}$:

$$M_W = 80.00 \pm 0.06 \ \text{GeV}. \quad (6)$$

Here for the light quark masses, I have used $m_u = m_d = 0.040 \ \text{GeV}$ and $m_s = 0.10 \ \text{GeV}$. They are derived by fitting the free quark calculations of the renormalized photon self-energy to the recent numerical estimate which uses a dispersion relation and experimental data of $\sigma(e^+e^- \rightarrow \text{hadrons})$ [10].

That is all I want to show, but discussions in terms of $\Delta r$ may be easier to understand for some readers. $M_Z^{\text{exp}} = 91.11 \pm 0.05 \ \text{GeV}$ and $M_W^{\text{exp}} = 80.0 \pm 0.56 \ \text{GeV}$ give $\Delta r^{\text{exp}} = 0.05174 \pm 0.03161$ through Eq.(2), which rejects the tree relation ($\Delta r=0$). On the other hand, the corrected prediction from $\alpha^{\text{exp}}$, $G_F^{\text{exp}}$ and $M_Z^{\text{exp}}$ ($m_t=112 \ \text{GeV}$ and $m_b=100 \ \text{GeV}$) is quite successful: $\Delta r = 0.05157 \pm 0.00002$.

Concerning the above arguments, some people may claim: "Large part of $|M_W - M_W^{(0)}|$ is given by the leading log effects which is usually expressed by the replacement $\alpha \rightarrow \alpha(M_W)$. Therefore, this is nothing but a test of the QED corrections." Or "We must not conclude yet that the quantum effects have been verified since they cannot be predicted correctly until $m_t$ and $m_b$ are directly measured."
With the first claim I do not agree (although it is true that the leading log terms contribute mainly to the corrections), but I wish to show those people here that a much more precise and stricter test becomes possible in the near future. That is, $W$ mass including the effect of $\alpha \rightarrow \alpha(M_W)$ only, $M_W^{ll}$, becomes

$$M_W^{ll} = 79.71 \pm 0.06 \text{ GeV},$$

which is distinguishable from the one with the full corrections (Eq.(6)) once $M_W$ is determined within $\sim 0.1$ GeV at LEP II. That will be the best test of the quantum electroweak effects.

On the second claim, we can make another calculation assuming that the electroweak theory is correct at quantum correction level. That is, we can derive some constraint on the top-quark mass from $\Delta r \geq 0.02013 (= 0.05174 - 0.03161)$ since larger $m_t$ makes $\Delta r$ decrease [11]. The result is

$$m_t \lesssim 185 \text{ GeV } \quad (\text{for } m_\phi = 100 \text{ GeV }),$$

$$\lesssim 205 \text{ GeV } \quad (\text{for } m_\phi = 1 \text{ TeV }),$$

which is in good agreement with the recent analysis by Langacker [12].

I should mention here the size of other ambiguities in these calculations. Possible origins of them are the light quark masses, the non-leading higher order terms and QCD effects in heavy quark loops. They have been studied by several authors [13], and estimated to be altogether at most 0.06-0.07 GeV in $M_W$. That is, they are negligible in the present analysis.

Finally let us compare briefly the present results with those given by Amaldi et al. in [1]. They obtained the following value of $\sin^2 \theta_W (\equiv 1 - M_W^2/M_Z^2)$ from the deep inelastic $\nu N$ scatterings:

$$\sin^2 \theta_W^{(\nu N)} = \begin{cases} 
0.242 \pm 0.006 & \text{(without R.C.)}, \\
0.233 \pm 0.006 & \text{(with R.C.)}. 
\end{cases}$$

Strictly speaking, we must not compare them directly with ours since the above value with R.C. is for $m_t = 45$ GeV and $m_\phi = 100$ GeV. However, it is known that
\( \sin^2 \theta_W^{(\nu N)} \) depends rather weakly on \( m_t \) [14], so I use them here. Eqs.(4) and (6) give

\[
\sin^2 \theta_W = \begin{cases} 
0.2126 \pm 0.0003 \text{ (without R.C.)}, \\
0.2290 \pm 0.0004 \text{ (with R.C.)}, 
\end{cases} 
\]  

(10)

which clearly shows the necessity of the radiative corrections.

In conclusion, I have shown my recent analysis on the electroweak quantum effects. The main results are as follows: The electroweak theory has been tested beyond the tree approximation independent of the neutrino experiments for the first time by using the \( M_W-M_Z \) relation [7]. The new data are in excellent agreement with the calculations including the quantum effects, while those at tree level cannot reproduce the data, or we can obtain a strong constraint on \( m_t \) assuming that the electroweak theory is correct beyond tree approximation. Theoretical ambiguities are enough small in this analysis and do not affect our conclusion.

REFERENCES


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