Verification of the Quantum Electroweak Effects through the Weak Boson Mass Relation

Zenrō HIOKI

Department of Physics, College of General Education
University of Tokushima
Tokushima 770, JAPAN
(BITNET: A52071 @ JPNKUDPC)

ABSTRACT

Recent studies of the M_W-M_Z relation using the new experimental data on the weak boson masses are reported. It is shown that those data clearly indicate the existence of the quantum effects of the electroweak interaction, and also give an upper bound on the top-quark mass.

The standard $SU(2) \times U(1)$ gauge theory of the electroweak interaction (the electroweak theory hereafter) has been so far very successful for describing a lot of weak interaction phenomena. It has been even crucial recently to take account of the radiative corrections (R.C.) in these analyses [1]. This fact gives a strong support to the validity of the electroweak theory beyond tree approximation.

Those analyses, however, have fully used the value of $\sin^2\theta_W$ extracted from the deep inelastic neutrino-nucleon scatterings. There, actual experimental conditions have to be incorporated, and also we are forced to use the parton model to describe the quarks in the target nucleons. Although these points must have been carefully taken into account, other precision tests independent of the ν experiments are therefore strongly desired. Based on my recent analysis [2], I wish to show here that new experimental data on the weak boson masses make actually such a "new-type" analysis possible for the first time.[♮1]

Necessary theoretical tool for this purpose is only the M_W-M_Z relation, which is the weak boson mass relation derived from the muon decay amplitude through the following relation [7] :

$$G_F(\alpha, M_W, M_Z, m_f, m_\phi) = G_F^{exp}. \tag{1}$$

Here the left-hand-side is the Fermi coupling constant calculated within the electroweak theory, and $G_F^{exp}(= 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2})$ is the corresponding experimental data. This G_F is a function of the five parameters of the theory as is expressed, but its behavior is mainly controlled by α, M_W and M_Z. Therefore, Eq.(1) gives a relation between M_W and M_Z which depends also on m_f and m_ϕ to a certain extent once we use $\alpha = \alpha^{exp}(= 1/137.036)$. This is what I call "the M_W-M_Z relation".

This relation is usually expressed as a value of M_W calculated from α, G_F, M_Z, m_f and m_ϕ:

$$M_W = M_Z \left[\frac{1}{2}\left(1 + \sqrt{1 - \frac{2\sqrt{2}\pi\alpha^{exp}}{M_Z^2 G_F^{exp}(1-\Delta r)}}\,\right) \right]^{1/2}, \tag{2}$$

where Δr represents the higher order corrections in the μ decay amplitude (Sirlin's notation [8]), so a function of α, G_F, M_Z, m_f and m_ϕ. This formula includes the full $O(\alpha)$ corrections plus the leading log terms to all orders in perturbation. If we assume that m_t is less than a hundred GeV and m_ϕ less than several hundred GeV, we obtain $|M_W - M_W^{(0)}| \sim 1$ GeV where M_W and $M_W^{(0)}$ are those calculated with and without the higher order corrections (i.e., Δr) respectively. This relation is therefore quite useful for a clean test of the electroweak theory at quantum correction level, which was pointed out already early in the eighties [7].

Now we are ready for numerical calculations. I use the following value for M_Z:

$$M_Z^{exp} = 91.11 \pm 0.05 \ \ \mathrm{GeV}, \tag{3}$$

which has been derived by combining the data of SLC: $M_Z^{exp} = 91.17 \pm 0.18$ GeV

♮1 In this report, I wish to show calculations using the data of LEP (ALEPH, DELPHI, L3 and OPAL collaborations) on the Z mass [3] in addition to the data of MARK II, CDF and UA2 [4-6] although those LEP data were not available when this meeting was held.

[4] and LEP: $M_Z^{exp} = 91.10 \pm 0.05$ GeV [3].

First, without the radiative corrections, W mass is calculated by using the above M_Z^{exp} as

$$M_W^{(0)} = 80.84 \pm 0.06 \text{ GeV}. \tag{4}$$

This value is obviously inconsistent with

$$M_W^{exp} = 80.0 \pm 0.56 \text{ GeV}, \tag{5}$$

which is obtained from $M_W^{exp} = 80.0 \pm 0.62$ GeV (CDF) [5] and $M_W^{exp} = 80.0 \pm 1.33$ GeV (UA2) [6].

On the other hand, if we take account of the radiative corrections with m_t = 112 GeV and m_ϕ = 100 GeV (note that data of UA1, UA2 and CDF all indicate $m_t \gtrsim$ 60-70 GeV [9]), we can reproduce the above M_W^{exp}:

$$M_W = 80.00 \pm 0.06 \text{ GeV}. \tag{6}$$

Here for the light quark masses, I have used m_u=m_d=0.040 GeV and m_s=0.10 GeV. They are derived by fitting the free quark calculations of the renormalized photon self-energy to the recent numerical estimate which uses a dispersion relation and experimental data of $\sigma(e^+e^- \to hadrons)$ [10].

That is all I want to show, but discussions in terms of Δr may be easier to understand for some readers. $M_Z^{exp} = 91.11 \pm 0.05$ GeV and $M_W^{exp} = 80.0 \pm 0.56$ GeV give $\Delta r^{exp} = 0.05174 \pm 0.03161$ through Eq.(2), which rejects the tree relation (Δr=0). On the other hand, the corrected prediction from α^{exp}, G_F^{exp} and M_Z^{exp} (m_t=112 GeV and m_ϕ=100 GeV) is quite successful: $\Delta r = 0.05157 \pm 0.00002$.

Concerning the above arguments, some people may claim: "Large part of $|M_W - M_W^{(0)}|$ is given by the leading log effects which is usually expressed by the replacement $\alpha \to \alpha(M_W)$. Therefore, this is nothing but a test of the QED corrections." Or "We must not conclude yet that the quantum effects have been verified since they cannot be predicted correctly until m_t and m_ϕ are directly measured."

With the first claim I do not agree (although it is true that the leading log terms contribute mainly to the corrections), but I wish to show those people here that a much more precise and stricter test becomes possible in the near future. That is, W mass including the effect of $\alpha \rightarrow \alpha(M_W)$ only, $M_W^{l.l.}$, becomes

$$M_W^{l.l.} = 79.71 \pm 0.06 \text{ GeV}, \tag{7}$$

which is distinguishable from the one with the full corrections (Eq.(6)) once M_W is determined within $\sim$0.1 GeV at LEP II. That will be the best test of the quantum electroweak effects.

On the second claim, we can make another calculation assuming that the electroweak theory is correct at quantum correction level. That is, we can derive some constraint on the top-quark mass from $\Delta r \geq 0.02013 (= 0.05174 - 0.03161)$ since larger m_t makes Δr decrease [11]. The result is

$$m_t \lesssim 185 \text{ GeV} \quad (\text{ for } m_\phi = 100 \text{ GeV }), \tag{8a}$$

$$\lesssim 205 \text{ GeV} \quad (\text{ for } m_\phi = 1 \text{ TeV }), \tag{8b}$$

which is in good agreement with the recent analysis by Langacker [12].

I should mention here the size of other ambiguities in these calculations. Possible origins of them are the light quark masses, the non-leading higher order terms and QCD effects in heavy quark loops. They have been studied by several authors [13], and estimated to be altogether at most 0.06-0.07 GeV in M_W. That is, they are negligible in the present analysis.

Finally let us compare briefly the present results with those given by Amaldi et al. in [1]. They obtained the following value of $\sin^2\theta_W (\equiv 1 - M_W^2/M_Z^2)$ from the deep inelastic νN scatterings:

$$\sin^2\theta_W^{(\nu N)} = \begin{cases} 0.242 \pm 0.006 & (\text{ without R.C. }), \\ 0.233 \pm 0.006 & (\text{ with R.C. }). \end{cases} \tag{9}$$

Strictly speaking, we must not compare them directly with ours since the above value with R.C. is for $m_t = 45$ GeV and $m_\phi = 100$ GeV. However, it is known that

$\sin^2\theta_W^{(\nu N)}$ depends rather weakly on m_t [14], so I use them here. Eqs.(4) and (6) give

$$\sin^2\theta_W = \begin{cases} 0.2126 \pm 0.0003 & (\text{ without R.C. }), \\ 0.2290 \pm 0.0004 & (\text{ with R.C. }), \end{cases} \tag{10}$$

which clearly shows the necessity of the radiative corrections.

In conclusion, I have shown my recent analysis on the electroweak quantum effects. The main results are as follows: The electroweak theory has been tested beyond the tree approximation independent of the neutrino experiments for the first time by using the M_W-M_Z relation [7]. The new data are in excellent agreement with the calculations including the quantum effects, while those at tree level cannot reproduce the data, or we can obtain a strong constraint on m_t assuming that the electroweak theory is correct beyond tree approximation. Theoretical ambiguities are enough small in this analysis and do not affect our conclusion.

REFERENCES

[1] U. Amaldi, A. Böhm, L. S. Durkin, P. Langacker, A. K. Mann, W. J. Marciano, A. Sirlin and H. H. Williams, *Phys. Rev.* **D36** (1987), 1385;
G. Costa, J. Ellis, G. L. Fogli, D. V. Nanopoulos and F. Zwirner, *Nucl. Phys.* **B297** (1988), 244.

[2] Z. Hioki, *Prog. Theor. Phys.* **82** (1989), 875.

[3] ALEPH Collaboration : D. Decamp et al., Preprint CERN-EP/89-132 (October 13, 1989);
DELPHI Collaboration : P. Aarnio et al., Preprint CERN-EP/89-134 (October 16, 1989);
L3 Collaboration : B. Adeva et al., L3 Preprint ♯001 (October 11, 1989);
OPAL Collaboration : M. Z. Akrawy et al., Preprint CERN-EP/89-133 (October 13, 1989).

[4] MARK II Collaboration : G. Feldman, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989.

[5] CDF Collaboration : F. Abe et al., *Phys. Rev. Letters* **63** (1989), 720; M. Campbell, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989.

[6] UA2 Collaboration : A. Weidberg, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989.

[7] Z. Hioki, *Prog. Theor. Phys.* **68** (1982), 2134; **71** (1984), 663;
L. Maiani, in: ***Proceedings of the International Conference on Unified Theories and their Experimental Tests,*** Venice, ITALY, March 16–18, 1982 (Inst. Naz. Fis. Nucl. ITALY, 1983), p. 55;
W. J. Marciano and A. Sirlin, *Phys. Rev.* **D29** (1984), 945; **D31** (1985), 213Erratum.

[8] A. Sirlin, *Phys. Rev.* **D22** (1980), 971.

[9] CDF Collaboration : P. Sinervo, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989;
UA1 Collaboration : K. Eggert, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989;
UA2 Collaboration : L. DiLella, Talk presented at the XIV International Symposium on Lepton and Photon Interactions, Stanford, USA, August 7 - 12, 1989.

[10] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, *Zeit. für Phys.* **C43** (1989), 497.

[11] M. Consoli, S. Lo Presti and L. Maiani, *Nucl. Phys.* **B223** (1983), 474;
Z. Hioki, *Nucl. Phys.* **B229** (1983), 284;
S. Bertolini and A. Sirlin, *Nucl. Phys.* **B248** (1984), 589.

[12] P. Langacker, *Phys. Rev. Letters* **63** (1989), 1920.

[13] A. Sirlin, *Phys. Rev.* **D29** (1984), 89;
W. Hollik and H.-J. Timme, *Zeit. für Phys.* **C33** (1986), 125;
F. Jegerlehner, *Zeit. für Phys.* **C32** (1986), 425;
B. W. Lynn, D. Kennedy and C. Verzegnassi, Preprint SLAC-PUB-4127;
B. A. Kniehl, J. H. Kühn and R. G. Stuart, *Phys. Letters* **214B** (1988), 621;
M. Consoli, W. Hollik and F. Jegerlehner, *Phys. Letters* **227B**(1989), 167.

[14] J. Ellis and G. Fogli, *Phys. Letters* **213B** (1988), 526.