PURELY STRINGY MODEL BUILDING WITH LOWER-RANK GAUGE GROUPS

Hikaru Sato
Department of Physics
Hyogo University of Education

§1 Introduction

§2 Noncommuting Wilson lines

§3 Gauge symmetry breaking

§4 Model building

§5 Conclusions

§1 INTRODUCTION

Compactification of 10-d heterotic string theory may provide a unified framework for all fundamental interactions in 4-d.

Compactification of 6 space dimensions

(1) Calabi-Yau manifold
  tensor product of minimal N=2 superconformal theories — Gepner

(2) Orbifold
- standard Z orbifold

$$E_8 \times E_8' \longrightarrow E_6 \times SU(3) \times E_8'$$

- orbifold with Wilson lines
  homomorphism of the space group defining the torus into $E_8 \times E_8$
  - gauge symmetry breaking
  - number of generations

  $\Big\} \Rightarrow$ realistic models

  $E_8 \times E_8'$
  $\hookrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times [U(1)]^4$

  extra $U(1) \longrightarrow Z'$

Rank of the gauge group is not changed and many extra U(1)'s survive.

Are large rank gauge groups a characteristic feature of chiral string models in 4-dimensions?

The purpose of this talk is

<u>to give a powerful method to construct chiral string models in 4-d with lower rank gauge groups.</u>

<u>Z orbifold</u>

6-dimensional torus $\quad T^6 = R^6/\Gamma_6$

Γ_6: lattice defining the torus

Z_3 invariance (point group)

$\theta: Z^\alpha \rightarrow \exp(2\pi i/3) Z^\alpha$

$\alpha = 1, 2, 3$ (R^6 in complex notation)

space group

$S: Z^\alpha \rightarrow (\theta Z)^\alpha + \zeta^\alpha$

$\zeta \in \Gamma_6$

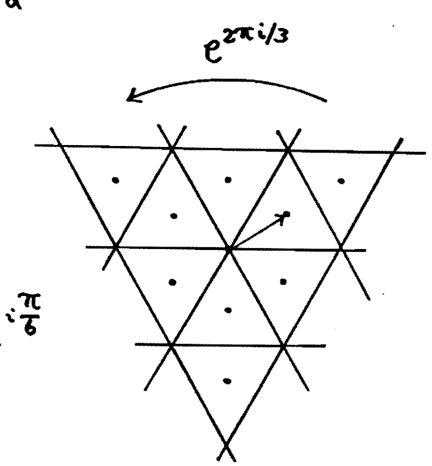
Z orbifold

$T^6/Z_3 \simeq R^6/S$

fixed points

$(0, 1, 2)\sqrt{\frac{1}{3}}\, e^{i\frac{\pi}{6}}$

mod 3



<u>Orbifold with commuting Wilson lines</u>

bosonic formulation

space group

$Z^\alpha \rightarrow (\theta Z)^\alpha + \zeta^\alpha \qquad \alpha = 1, 2, 3$

embedding in gauge degrees of freedom

$X^I \rightarrow (\Theta X)^I + V^I \qquad I = 1, \cdots, 16$

Θ: automorphism of group lattice

Weyl rotation

V^I: commuting Wilson lines (in Cartan subalgebra) corresponding to shifts ζ in Γ_6

<u>homomorphism</u>: $(\theta, \zeta) \rightarrow (\Theta, V)$

(rotation)×(shift) noncommutative

$\Rightarrow$ reduction of the rank of subgroup

It is rather difficult to construct realistic models on purely stringy basis.

$\Rightarrow$ Field theory approximation

- flat direction in the potential
- anomalous U(1)

§2 NONCOMMUTING WILSON LINES

fermionic formulation

☆ unified treatment of both Abelian and non-Abelian embedding
☆ systematic model building
☆ no field theoty approximation

orbifold with Wilson lines

(R^6 with background gauge field)/S

A x~~~~x B

fixed point

A = B up to shift in lattice

$$\psi(B) = \underbrace{e^{i\int_A^B A_\mu dx^\mu}}_{\text{Wilson line}} \psi(A)$$

fixed point ⇔ shift in lattice ⇔ Wilson line

fixed point ⇔ conjugacy class of S
⇔ twisted Hilbert space

○ Wilson lines corresponding to different fixed points are noncommuting.

This noncommutability may reduce the rank of the gauge group.

Fixed point on the Z orbifold is denoted by (p, q, r); p, q, r = 0, ±1 mod 3 specify location of the fixed point on the three complex z^α planes. $\alpha = 1, 2, 3$

$2\pi k/3$ rotation ⇔ shift $pe^1_k + qe^2_k + re^3_k$
$k = 1, 2$ for fixed point $f = (p, q, r)$

Wilson line corresponding to this shift

$$\Theta_{f,k} = \exp[2\pi i(pa_k + qb_k + rc_k)]$$

$$2\pi a_k = A_\mu \cdot (e^1{}_k)^\mu \quad \text{, etc.}$$

Internal degrees of freedom of the heterotic string is described by

$$\psi^i, \tilde{\psi}^i \quad (i = 1, \cdots, 16)$$

which transform as vectors of SO(16)×SO(16).

Embedding of Z_3 into $E_8 \times E_8$ is given by rotation matrix Ω for ψ

$$\Omega = \exp[2\pi i \zeta^\ell H_\ell], \quad 3\zeta^\ell = 0 \bmod 1$$

H_ℓ : Cartan subalgebra of $E_8 \times E_8$ $\quad \Omega^3 = 1$
$\ell = 1, \cdots, 16$

Boundary condition on orbifold

for gauge fermions

$$\psi(\sigma_1+\pi, \sigma_2) = (-1)^n \Omega^k \Theta_{f,k} \psi(\sigma_1, \sigma_2)$$

$$\psi(\sigma_1, \sigma_2+\pi) = (-1)^m \Omega^h \Theta_{f,h} \psi(\sigma_1, \sigma_2)$$

nondiagonal in matrix notation

n, m : spin structure

Automorphism of order 3

$$\theta_k: \exp[2\pi ik/3] \Leftrightarrow \omega_{f,k} = \Omega^k \Theta_{f,k}$$

# of automorphisms ω_f = # of fixed points f = # of twisted Hilbert spaces

Noncommuting ω_f and ω_g for different fixed points f, g

$[\omega_f, \omega_g] \neq 0 \Rightarrow$ reduction of the rank of gauge group

Quantum numbers invariant under ω_f determine the gauge symmetry of the subgroup.

§ 3 GAUGE SYMMETRY BREAKING

Commutability of $\omega_{f,k} = \Omega \Theta_{f,k}$ depends on the choice of background gauge fields A_μ

(i) Abelian embedding

A_μ in the Cartan subalgebra

all $\omega_{f,k}$ commutable
$\Rightarrow$ diagonalized simultaneously

This case is equivalent to the embedding of the space group by shifts in the $E_8 \times E_8$ lattice.

$$\Omega^k \Theta_{f,k} = \exp[2\pi ikv_f^\ell H_\ell] , \quad k=0, \pm 1$$

$$v_f^\ell = \zeta^\ell + (pa_1+qb_1+rc_1)^\ell \qquad \ell=1,\cdots,16$$

v_f corresponds to a shift in the $E_8 \times E_8$ lattice in the bosonic formulation.

Modular invariance in the presence of background gauge fields
$\Leftrightarrow$ the level matching condition.

condition of modular invariance

$$\begin{cases} N \sum_a \xi^a = 0 \mod 2 \\ N \sum_\ell v_f^\ell = 0 \mod 2 \qquad (N=3) \\ N\{\sum_\ell (v_f^\ell)^2 - \sum_a (\xi^a)^2\} = 0 \mod 2 \end{cases}$$

Here ξ^a detemines the boundary condition for the right-moving NSR fermions:

$$\lambda^a(\sigma_1+\pi, \sigma_2) = (-1)^n \exp(2\pi i k \xi^a)\lambda^a(\sigma_1, \sigma_2)$$

$$\lambda^a(\sigma_1, \sigma_2+\pi) = (-1)^m \exp(2\pi i h \xi^a)\lambda^a(\sigma_1, \sigma_2)$$

$$\xi^a = (1/3, 1/3, -2/3, 0) \qquad a=1,\cdots,4$$

condition of group invariance

string states on orbifold must be invariant under Z_3

$$(V^\ell + kv_f^\ell/2)v_f^\ell + (K^a - k\xi^a/2)\xi^a + m_k = 0 \mod 1$$

$V^\ell \in E_8 \times E_8$ lattice
$K^a \in SO(8)$ vector or spinor lattice
m_k : eigenvalue of the twist operator

$$\hat{g} = e^{2\pi i \hat{m}_k}, \quad \hat{g}\, z^\alpha \hat{g}^{-1} = e^{2\pi i/3} z^\alpha$$

Massless spectra

(1) untwisted sector

★ gauge boson
helicity $\pm 1 \in 8_v$ of SO(8)

$$K^a \xi^a = 0 \mod 1$$

group invariant condition

$$\Rightarrow \underline{V^\ell v_f^\ell = 0 \mod 1}$$

★ massless fermions
helicity $1/2 \in 8_s$ of SO(8)

$$K^a \xi^a = 2/3 \mod 1$$

$$\Rightarrow V^\ell v_f^\ell = 1/3 \mod 1$$

(2) twisted sector

★ massless fermion

$$\frac{1}{2}(V^\ell + v_f^\ell)^2 + N_L - \frac{2}{3} = 0$$

N_L: number of z_L oscillation

Mass spectrum is the same as the one obtained by embedding shifts in the $E_8 \times E_8$ lattice.

(ii) <u>Non-Abelian embedding</u>

A_μ not restricted to Cartan subalgebra

➡ $\omega_{f,k}$ noncommutable

Quantization of string states needs to diagonalize the boundary conditions:

$$\Omega^k \Theta_{f,k} = U_f^{-1} \exp[2\pi i k v_f^\ell H_\ell]\, U_f$$

transformation matrix $U_f \in SO(16)\times SO(16)$

eigenvalue v_f: $3v_f^\ell = 0$ mod 1 $\quad Z_3$ invariance

String states associated with each fixed point are expressed in the different basis.

String Hilbert space is invariant under

$$\omega_{f,k} = \Omega^k \Theta_{f,k}, \text{ automorphism of } Z_3$$

★ Gauge symmetry is determined by invariance under $\omega_{f,k}$ <u>for all f</u>

$$\Rightarrow \begin{cases} V^\ell v_f^\ell = 0 \mod 1 \\ U_f E_V U_f^{-1} = E_V \to \text{always satisfied} \\ \underline{U_f H_\ell U_f^{-1} = H_\ell} \to \text{reduce the rank of subgroup} \end{cases}$$

<u>Chiral fermions</u>

(1) untwisted sector

$$V^\ell v_f^\ell = 1/3 \mod 1$$

$$U_f H_\ell U_f^{-1} = H_\ell$$

Singlets with the noninvariant U(1) charge are projected out.

(2) twisted sector

massless condition

$$\frac{1}{2}(V^\ell + v_f^\ell)^2 + N_L - \frac{2}{3} = 0 \qquad \text{unchanged}$$

N_L: number of z_L oscillation

<u>Modular invariance of the truncated theory</u>

partition function for the left-moving gauge fermions for the boundary condition $(k,h)_f$

$$Z_{(k,h)_f} = \mathrm{Tr}[\Omega^h \Theta_{f,h} \exp(2\pi i \tau H_{f,k})]$$
$$= \mathrm{Tr}[\exp(2\pi i h v_f^\ell H_\ell)\exp(2\pi i \tau H_{f,k})]$$

$H_{f,k}$: Hamiltonian in the $(k,h)_f$ sector invariant under $SO(16)\times SO(16)$

Partition function does not depend on U_f.

(Inner)automorphism corresponding to U_f is commutative with modular transformation.

⇒ the level matching condition for eigen values v_f is sufficient for modular invariance

The discarded zero modes are singlets of the unbroken non-Abelian group.

⇒ Anomaly cancellation with respect to unbroken subgroup is not changed.

Remark

◎ Gauge symmetry and mass spectra are determined by the eigen value v_f of the boundary conditions for gauge fermions.

⇒ Different Wilson lines with the same v_f give the same symmetry and mass spectra.

⇒ degenerate orbifold

◎ When the subgroup contains U(1), some of them might be anomalous.

§4 MODEL BUILDING

(1) Z_3 embedding

$$\Omega = \exp[2\pi i \zeta^{\ell} H_{\ell}]$$

$$\zeta = \tfrac{1}{3}(2,1,1,0^5;\ 0^8)$$ standard embedding

(2) Possible Wilson lines

$$\Theta_{f,k} = \exp[2\pi i (pa_k + qb_k + rc_k)^{IJ} T_{IJ}]$$

T_{IJ}: SO(16)×SO(16) generators

Electroweak symmetry should not be violated.

SO(16) adjoint $\underline{120} \supset$ SO(10)×SU(4)

$\underline{120}$ = (45, 1) + (10, 6) + (1, $\underline{15}$)

↳ $SU(2)_R$ breaking ↳ SU(4) or SU(3) breaking

★ electroweak symmetric Wilson lines

(3, 1) + (1, 15) of $SU(2)_R \times SU(4)$

Wilson lines		reduction of rank
$SU(2)_R$, SU(2)	···	1
$SU(2)_R \times SU(2)$, SU(3)	···	2
$SU(2)_R \times SU(3)$, SU(4)	···	3
$SU(2)_R \times SU(4)$	···	4

★ <u>Simple model</u>

$$E_8\times E_8' \to SU(3)_c\times SU(3)_L\times SU(3)_R$$
$$\times SO(14)'\times U(1)'$$

(1) Z_3 embedding

$$\zeta = \frac{1}{3}(e_1+e_2+e_5-2e_6+e_7+2e_1')$$

e_i: orthogonal basis

ζ is invariant under $SU(9)\times SO(14)\times U(1)$.

(2) one Wilson line from SU(3)

$$a^{IJ}T_{IJ} \to \frac{1}{3}(2H_1+H_2+H_3)$$

diagonalized by $SO(16)\cap SU(9)$ rotation which leaves ζ invariant.

<u>Symmetry breaking</u> :

$$V\cdot\zeta = 0 \Rightarrow E_8\times E_8' \to SU(9)\times SO(14)'\times U(1)'$$
$$V\cdot a = 0 \Rightarrow SU(9) \to SU(3)_c\times SU(3)_L\times SU(3)_R$$
$$\times[U(1)]^2$$

$\hookrightarrow$ killed by $SO(16)\cap SU(9)$ rotation

H_1+H_2

$2H_1+H_2+H_3$

<u>Chiral fermions</u>

◆ untwisted sector

$$3\,\{(\bar{3},\bar{3},\bar{3}) + 3(1,1,1)\}$$

$\hookrightarrow \begin{cases} -H_1' & \longrightarrow (1,1,1) \\ H_1+H_2 & \times \\ 2H_1+H_2+H_3 & \times \end{cases}$

two of them are projected out by $SO(16)\cap SU(9)$ rotation

◆ twisted sector

$$27\,\{(1,3,1) + (3,1,1) + (1,1,3)\}$$

More elaborate examples are constructed by choosing Z_3 embedding, ζ and Wilson lines.

(2.0) orbifold

rank $E_8\times E_8' \rightarrow$

rank	gauge groups	Wilson lines	massless fermions
7	$E_6\times U(1)\times SO(14)'\times U(1)'$	$SU(2)$	$12\ \underline{27}+81\ \underline{1}$
7	$SU(6)\times SU(3)\times SO(14)'\times U(1)'$	$SU(2)_R$	$3(15,3)+9(15,1)+36(\bar{6},1)+45(1,\bar{3})$
6	$E_6\times SO(14)'\times U(1)'$	$SU(3)$	$3\ \underline{27}+3\ \underline{\overline{27}}+54\ \underline{1}$
6	$SU(6)\times U(1)\times E_7'\times U(1)'$	$SU(2)_R\times SU(2)$	$9\ \underline{15}+36\ \underline{\bar{6}}+18\ \underline{6}+81\ \underline{1}$
5	$SO(10)\times SO(14)'\times U(1)'$	$SU(4)$	$3\ \underline{16}+6\ \underline{10}+3\ \underline{\overline{10}}+36\ \underline{1}$
5	$SU(6)\times E_7'\times U(1)'$ $\hookrightarrow SU(5)\times U(1)$	$SU(2)_R\times SU(3)$	$3\ \underline{15}+3\ \underline{\overline{15}}+24\ \underline{6}+24\ \underline{\bar{6}}+54\ \underline{1}$
4	$SU(4)_c\times SU(2)_L\times E_7'\times U(1)'$	$SU(2)_R\times SU(4)$	$3(4,2)+12(\bar{4},1)+6(4,1)+30(1,2)+6(6,1)+3(\bar{6},1)+36(1,1)$
4	$SU(3)_c\times SU(2)_L\times U(1)_Y\times E_7'\times U(1)'$	$SU(2)_R\times SU(4)\times U(1)_Y$	$6(3,2)+3(\bar{3},2)+33(1,2)+9(\bar{3},1)+3(3,1)+36(1,1)$

$(0,0,r)$: $3\{(3,2)+3(1,2)+3(\bar{3},1)+(3,1)+3(1,1)\}$ + 27 singlets

$$\left.\begin{array}{ll} r=0 & t,\ b,\ \tau,\ \cdots \\ r=1 & u,\ d,\ e,\ \cdots \\ r=-1 & c,\ s,\ \mu,\ \cdots \end{array}\right\}\ \text{3 generation}$$

$$\left.\begin{array}{ll} (0,1,r): & 3\{4(1,2)+(\bar{3},2)\} \\ (0,-1,r): & 3\{4(1,2)+(3,2)\} \end{array}\right\}\ \text{mirror conjugate}$$

§5 CONCLUSIONS

1. We have proposed a powerful method to construct orbifold models with lower-rank gauge groups.

   non-commuting Wilson lines

   $\Rightarrow$ automorphism of Z_3, ω_f for twisted Hilbert space $\mathbf{H}_f$

   $[\omega_f,\ \omega_g]\neq 0$ for $f\neq g$

2. Unified treatment for Abelian (no rank-reduction) and non-Abelian embedding

   more general than bosonic formulation

3. Purely stringy construction of models with lower-rank gauge group

   no field theory approximation

   flat direction of potential $\Rightarrow \langle\phi\rangle\neq 0$

4. Need more study to construct realistic models

   choice of Z_3 embedding and Wilson lines