Quark mass matrix and B_d^0-$\bar{B}_d^0$ Mixing

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Abstract

We propose a new quark mass matrix based on the BCS form . At the first stage, the desirable two mass gaps are derived and the generation mixing occurs between the second generation and the third one. At the second stage, the first generation quarks obtain masses and the CP violating phase appears. The interesting prediction is found in the V_{cb} element of the KM matrix, which is favorable to the heavy top quark. The B_d^0-$\bar{B}_d^0$ mixing is also discussed based on the new mass matrix.

I. Introduction

The generation problem of quarks and leptons is one of the most important subjects of particle physics. In particular, one has not yet understood the origin of the mass spectra of the quark-lepton system and the Kobayashi-Maskawa(KM) mixing angles[1]. In order to find out the clues to the fundamental theory beyond the standard model, it is important to investigate the quark-lepton mass matrices which give the observed KM matrix elements and the quark-lepton mass hierarchies. Two familiar mass matrices were proposed in past years: one is the Fritzch mass matrix[2] and other the Stech mass matrix[3], both having been investigated intensively[4,5].

These models have confronted with the recent data of the B_d^0-$\bar{B}_d^0$ mixing by ARGUS and CLEO[6]. Harari and Nir concluded[5] that the Stech model is inconsistent with the experimental constraints and the Fritzch model is consistent with them only if several experimental and theoretical quantities are taken at the limits of their allowed ranges. These rather unfavorable situation for these models follows from the fact that the top quark with the mass heavier than the previously expected one is called on by the observed B_d^0-$\bar{B}_d^0$ mixing[6] in the standard model.

Recently, a new approach for the quark mass matrix has been introduced by some authors[7,8,9,10]. The analogy to the BCS superconductivity and the nuclear pairing force[11] leads to a generation independent (so called democratic) quark mass matrix, which gives a large mass gap. Kaus and Meshkov[9] have obtained the quark mass matrix numerically based on the 'BCS quark mass matrix', and Koide[10] has proposed the mass matrix which is the BCS quark mass matrix with the diagonal correction terms. We believe that these approaches are important to take a step forward in understanding the origin of quark masses. However, their correction terms are not so transparent for us to understand these physical meanings. Instead of their mass matrices, we propose a new quark mass matrix with the simple structure based on the BCS form, and analyze it using the experimental quantity of the B_d^0-$\bar{B}_d^0$ mixing[6].

II. New quark mass matrix

If all six quark masses vanish, the Lagrangian of the standard model is invariant under the chiral symmetry $U(3)_L \times U(3)_R$, where $U(3)$ is the symmetry group connecting the three generations. This chiral symmetry is broken down to $U(2)_L \times U(2)_R$ by the mass matrix of the BCS form[7,8,9,10]

$$\frac{1}{3} M_0(q) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \qquad (1)$$

where q denotes the up-quark sector(u) or the down-quark sector(d). This matrix is diagonalized as diag[0, 0, $M_0(q)$] by using the orthogonal matrix U_0

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} . \qquad (2)$$

The up-quark mass matrix for quark state $u=(u_0, c_0, t_0)$ provides two massless quarks $u_1=(u_0-c_0)/\sqrt{2}$, $c_1=(u_0+c_0-2t_0)/\sqrt{6}$ and one heavy quark $t_1=(u_0+c_0+t_0)/\sqrt{3}$. For the down-quark sector, d_1, s_1 and b_1 are defined analogously. Thus, the BCS matrix form gives a large mass gap between $M_0(q)$ and zeros, which is favorable to the quark mass hierarchies. However, two mass gaps are demanded by the observed quark mass hierarchies. One of the simplest ways to get two mass gaps is to modify the BCS form in (1) as follows:

$$M(q) = \frac{1}{3} M_0(q) \begin{pmatrix} 1+\varepsilon_q & 1+\varepsilon_q & 1 \\ 1+\varepsilon_q & 1+\varepsilon_q & 1 \\ 1 & 1 & 1 \end{pmatrix} , \qquad (3)$$

where ε_q is a real correction factor and to be <<1. Mass matrix elements between the first two generations and the third generation are slightly different. Any exchanges of the column and row in (3) do not change following our results. In this stage the chiral symmetry $U(3)_L \times U(3)_R$ is violated and is reduced to the subsymmetry $U(1)_L \times U(1)_R$ acting on the (u_1, d_1) system. The mass matrix in (3) is no more diagonalized by $U_0 M(q) U_0^T$ as shown in the following:

$$U_0 \; M(q) \; U_0^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -B_q & \sqrt{2}\,B_q \\ 0 & \sqrt{2}\,B_q & A_q \end{pmatrix} , \qquad (4)$$

where $A_q = (1 + 4\varepsilon_q/9) \times M_0(q)$ and $B_q = -2\varepsilon_q/9 \times M_0(q)$. In order to diagonalize M(q), we introduce an orthogonal matrix U_{q2}, which is

$$U_{q2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{q2} & -\sin\theta_{q2} \\ 0 & \sin\theta_{q2} & \cos\theta_{q2} \end{pmatrix} . \qquad (5)$$

Diagonalizing M(q) by $U_{q2} U_0 M(q) (U_{q2} U_0)^T$, we obtain diag(0, z_{q2}, z_{q3}), where

$$z_{q2} + z_{q3} = A_q - B_q \quad , \quad z_{q2} z_{q3} = -A_q B_q - 2\,B_q^2 \quad ,$$

$$\tan 2\theta_{q2} = \frac{2\sqrt{2}\;B_q}{A_q + B_q} . \qquad (6)$$

Then, the parameters A_q and B_q are expressed in terms of the eigenvalues z_{q2} and z_{q3} as follows:

$$A_q = z_{q2} + z_{q3} + B_q \quad ,$$

$$B_q = -\frac{1}{6}(z_{q2} + z_{q3})\{ 1 - [1 - 12\, z_{q2} z_{q3} (z_{q2} + z_{q3})^{-2}]^{\frac{1}{2}} \} . \qquad (7)$$

The approximate values of z_{q2} and z_{q3} are given as

$$z_{q2} \approx -B_q \approx \frac{2}{9}\varepsilon_q M_0(q) \;, \qquad z_{q3} \approx A_q \approx (1 + \frac{4}{9}\varepsilon_q) M_0(q) \;, \qquad (8)$$

where we have chosen the solution of $z_{q2} \ll z_{q3}$ because of the quark mass hierarchies. Thus, we find that M(q) in (3) shares a nice feature to give the quark mass hierarchies in three generations. Moreover, we remark that the generation mixing between the second generation and the third one occurs at this stage due to U_{q2}.

At this stage, the first generation quarks are massless and do not mix with the other generation quarks. Now, we assume that the masses of the first generation quarks follow from the other origin, which breaks residual $U(1)_L \times U(1)_R$ chiral symmetry. We may consider the Yukawa coupling of the Higgs particle in the first generation sector. At the next stage, we give simple correction factors on the base of diag(0, z_{q2}, z_{q3}) as follows:

$$M^{phys}(q) = \begin{pmatrix} 0 & C_q \exp(i\alpha_q) & 0 \\ C_q \exp(i\beta_q) & z_{q2} & 0 \\ 0 & 0 & z_{q3} \end{pmatrix}, \qquad (9)$$

where C_q is a real number and α_q, β_q are phases. The correction is given at off diagonal elements in order to derive the Cabbibo angle in terms of the quark masses[2].

Since $M^{phys}(q)$ is a complex matrix, we get the CP violating phase in the KM matrix at this stage. For our convenience, we redefine the left-handed and the right-handed quark fields to eliminate the phase factor in (9). Then, the mass matrix in (9) is brought to the real 3×3 matrix as follows:

$$M_R^{phys}(q) = \begin{pmatrix} 0 & C_q & 0 \\ C_q & z_{q2} & 0 \\ 0 & 0 & z_{q3} \end{pmatrix}, \qquad (10)$$

but the phase matrix appears in deriving the KM matrix such as

$$P_0 = \begin{pmatrix} \exp(i\sigma) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (11)$$

with the definition $\sigma=\alpha_d-\alpha_u$.

The mass matrix $M_R^{phys}(q)$ in (10) is easily diagonalized by $U_{q1}M_R^{phys}(R)U_{q1}^T$, where

$$U_{q1} = \begin{pmatrix} \cos\theta_{q1} & -\sin\theta_{q1} & 0 \\ \sin\theta_{q1} & \cos\theta_{q1} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad (12)$$

In terms of the eigenvalues; diag$(-m_{q1}, m_{q2}, m_{q3})$, we get

$$\tan 2\theta_{q1} = \frac{2\,C_q}{z_{q2}} , \qquad C_q = \sqrt{m_{q1}m_{q2}} ,$$

$$z_{q2} = -m_{q1} + m_{q2} , \qquad z_{q3} = m_{q3} . \qquad (13)$$

From Eqs.(2),(5),(11) and (12), we obtain the KM matrix as follows:

$$V_{KM} = U_{u1}P_0\,U_{u2}U_0\,(U_{d1}U_{d2}U_0)^T = U_{u1}P_0\,U_{u2}U_{d2}^TU_{d1}^T . \qquad (14)$$

It is remarked that the CP violating phase follows from the phase matrix P_0.

Since σ is a free parameter, it should be determined so as to be consistent with the experimental value of $|V(1,2)|$. The simplest choice is to take $\exp(i\sigma)=i$ (or $-i$) as already discussed by other authors[4]. So we take $\sigma=90^\circ$ in the following numerical calculations.

Now, we give the approximate KM matrix elements as follows:

$$V_{ud} \approx 1 , \qquad V_{us} \approx \left(\frac{m_d}{m_s} + \frac{m_u}{m_c} \right)^{\frac{1}{2}} ,$$

$$V_{ub} \approx \left(2\frac{m_u}{m_c} \right)^{\frac{1}{2}} \left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \exp[-i(\sigma+\delta-180^\circ)] ,$$

$$V_{cd} \approx -\left(\frac{m_d}{m_s} + \frac{m_u}{m_c} \right)^{\frac{1}{2}} , \qquad V_{cs} \approx 1 , \qquad V_{cb} \approx \sqrt{2}\left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right) ,$$

$$V_{td} \approx \left(2\frac{m_d}{m_s} \right)^{\frac{1}{2}} \left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \exp(i\delta) \ ,$$

$$V_{ts} \approx -\sqrt{2} \left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \ , \quad V_{tb} \approx 1 \ , \tag{15}$$

where δ is defined to make $V(1,2)$ real. One of remarkable feature in our KM matrix is found in V_{cb}, which depends on the difference between the m_s/m_b ratio and the m_c/m_t one. Remind that $V_{cb} \approx \left(\frac{m_s}{m_b} \right)^{\frac{1}{2}} - \left(\frac{m_c}{m_t} \right)^{\frac{1}{2}}$ and $\left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right)^{\frac{1}{2}}$ in the Fritzsch model[2] and Stech model[3], respectively. Our V_{cb} is favorable to the heavy top quark. Since the ratio m_s/m_b is roughly 1/30, the ratio m_c/m_t is expected to be much smaller than 1/30 by the experimental value $|V_{cb}|=0.045\pm0.08$[12].

Another remarkable feature is found in the $|V_{ub}|/|V_{cb}|$ ratio. This ratio is equal to $\sqrt{m_u/m_c}$, which is independent of the top quark mass.

Let us show the numerical results. We calculate the V_{KM} without any approximations, using the central values of the following quark masses at the energy scale of 1 GeV as[13]

$$m_u=5.1\pm1.5 \text{ MeV} , \quad m_d=8.9\pm2.6 \text{ MeV} , \quad m_c=1.35\pm0.05 \text{ GeV},$$
$$m_s=175\pm55 \text{ MeV}, \quad m_b=5.3\pm0.1 \text{ GeV} . \tag{16}$$

The top-quark mass at the 1 GeV scale is a free parameter. The physical top quark mass is obtained by the QCD evolution formula, where we take the QCD parameter Λ as $\Lambda=0.1$ GeV.

We show in Fig.1 the predicted numerical value of $|V_{cb}|$ versus the physical top-quark mass m_t^{phys}. The present experimental limit $0.037<|V_{cb}|<0.053$[12] is also shown in Fig.1, from which the top-quark mass is suggested to be larger than 90 GeV. Since the predicted value of $|V_{cb}|$ depends on the s-quark mass considerably, we show the allowed region in the m_s - m_t^{phys} plane for the experimental region of $|V_{cb}|$ in Fig.2.

The $|V_{ub}|/|V_{cb}|$ ratio is predicted to be $\sqrt{m_u/m_c}$ = 0.0615, which is independent of m_s and m_t^{phys}, and so this prediction is crucial for our mass matrix. The CP violating phase depends on the value of m_s. In the parametrization $V_{ub}=|V_{ub}|\exp(i\Phi)$, we predict

Φ=77.28°, 74.76°, 72.68° for m_s=120 MeV, 175 MeV, 230 MeV, respectively.

III. B_d^0-$\bar{B}_d^0$ mixing

Another physical quantity crucially depending on the top quark mass is the B_d^0-$\bar{B}_d^0$ mixing. In the standard model, the mixing parameter $x_{Bd}=\Delta m_{Bd}/\Gamma_{Bd}$ is given as[14]

$$x_{Bd}=1.49\times10^5\,\mathrm{GeV}^{-2}\times B_{Bd}f_{Bd}^2\,\eta_{tt}E(y_t)\,|V_{td}^*V_{tb}|^2\ . \qquad (17)$$

The QCD correction factor η_{tt} is taken to be 0.85 and $E(y_t)$ is the loop-integral function given by Inami and Lim[15]. We use the typical value $B_{Bd}f_{Bd}^2=(0.16\ \mathrm{GeV})^2$[14].

The predicted value of x_{Bd} versus m_t^{phys} is shown in the case of m_s=175 MeV in Fig.1, where the experimental value x_{Bd}=0.70±0.13[6] is also shown. From Fig.1, the top-quark mass is suggested to be 150 GeV~190 GeV. This value is consistent with the allowed region of m_t^{phys} obtained from the analysis of V_{cb}. We also show the allowed region in the m_s - m_t^{phys} plane for the experimental value of x_{Bd} in Fig.2. Combining two regions for V_{cb} and x_{Bd} in the m_s - m_t^{phys} plane , we get the allowed region of m_t^{phys}=130 GeV~200 GeV and m_s=155 MeV~210 MeV.

IV.Summary

We have proposed a new quark mass matrix based on the BCS form. At the first stage, the desirable two mass gaps are derived and the generation mixing occurs between the second generation and the third one. At this stage, no CP violating phase appears. At the second stage, the first generation quarks obtain masses and the CP violating phase appears. The phase in the KM matrix follows from the difference between the two phases of the down quark mass matrix element and the up quark one associated with the first generation. This difference σ is expected to be ±90°. The most prominent prediction is found in the V_{cb} element, which is favorable to the heavy top quark. The top quark mass is predicted to be 130 GeV~200 GeV. It is important for our model to compare the predicted $|V_{ub}|/|V_{cb}|$ ratio with the experimental one. To derive our mass matrix on the basis of the some dynamical mechanism will be the interesting work .

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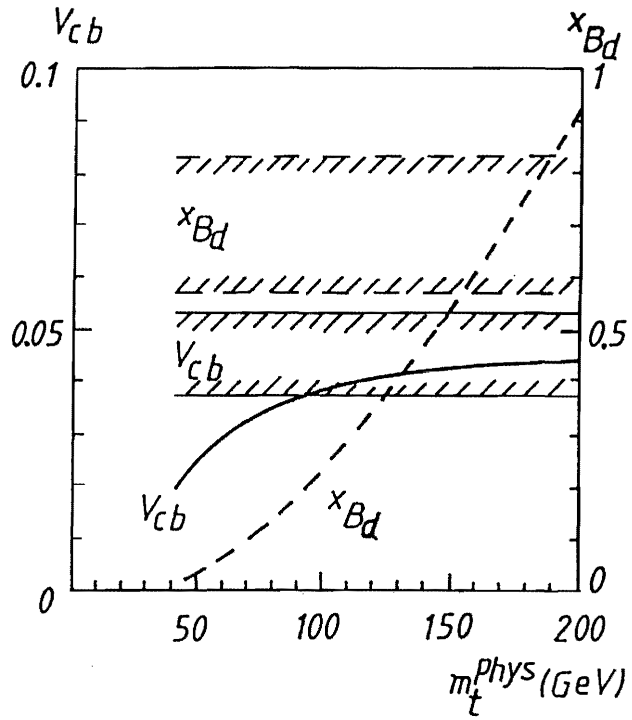

Fig.1

The predictions of $|V_{cb}|$ and x_{Bd} versus the physical top quark mass in the case of m_s=175 MeV. The solid line denotes $|V_{cb}|$ by the scale of the left side vertical axis and the dashed line denotes x_{Bd} by the scale of the right side vertical axis.

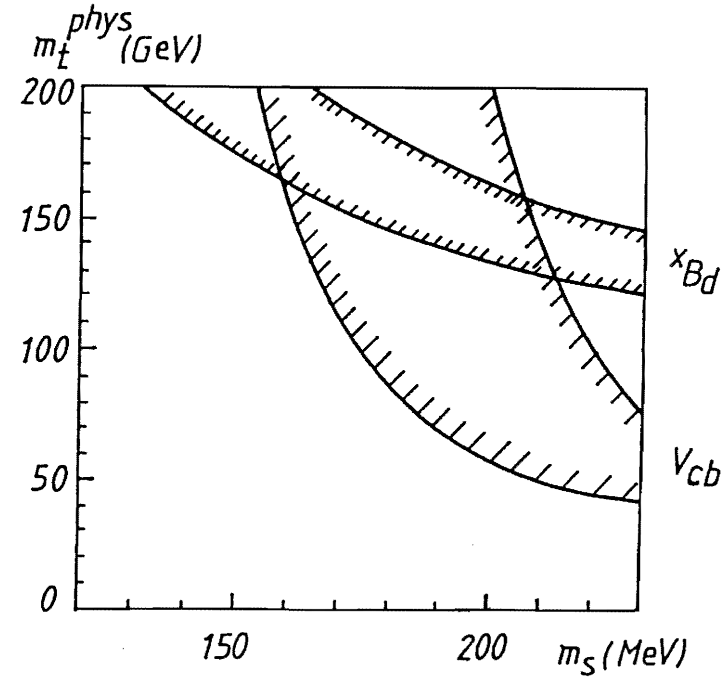

Fig.2

The allowed regions of m_s and m_t^{phys} for $|V_{cb}|$ and x_{Bd}.