NEW PHYSICS AT TEV SCALE II

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Abstract

By following the Part I, where many important predictions mostly in composite models which have already been checked or will be tested in the future experiments at TeV energy scale are summarized, additional predictions are presented on the weak mixing angle (θ_w), the quark mixing matrix elements (V_{ub}, V_{cb}, V_{td}, etc.) and the Higgs scalar and top quark masses, mostly based on the principle of "triplicity" of hadrons, quarks and subquarks asserting that a certain physical quantity such as the weak current is taken equally well as either one of a composite operator of hadrons, that of quarks and that of subquarks.

1. Introduction

The history of modern particle physics may be best summarized as atomic physics in teens and twenties, nuclear physics in thirties and forties, hadron physics in fifties and sixties, and quark physics in seventies and eighties. What is the next? It seems most probably and naturally "subquark physics" in nineties and the first decade of the twenty-first century. If this is the case, we should fully prepare for the new era of composite models of quarks and leptons[1)] which will presumably come soon.

Last year at the Second Meeting on Physics at TeV Energy Scale, I summarized many important predictions mostly in composite models which have already been checked or will be tested in the future experiments at TeV energy scale.[2)] At the present Meeting, I will present additional predictions, mostly based on the principle of "triplicity" of hadrons, quarks and subquarks, which asserts that a certain physical quantity such as the weak current is taken equally well as either one of a composite operator of hadrons, that of quarks and that of subquarks.

The weak charged current, J_μ, provides one of the most instructive examples for physical quantities to which triplicity of hadrons, quarks and subquarks can be applied. It can be written in terms of hadrons (baryons and mesons) and leptons as the sum of over ten thousand terms,

$$J_\mu \cong \bar{\nu}_e\gamma_\mu(1-\gamma_5)e + \bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu + \bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\tau + \frac{G^\beta}{G^\mu}\bar{p}\gamma_\mu\left(1 - \frac{g_A^\beta}{g_V^\beta}\gamma_5\right)n + \frac{G^\Lambda}{G^\mu}\bar{p}\gamma_\mu\left(1 - \frac{g_A^\Lambda}{g_V^\Lambda}\gamma_5\right)\Lambda + \cdots \qquad (1)$$

where G^β/G^μ, g_A^β/g_V^β, $\cdots$ are over ten thousand parameters. It can also be written in terms of quarks and leptons as the sum of at least twelve terms,

$$J_\mu \cong \bar{\nu}_e\gamma_\mu(1-\gamma_5)e + \bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu + \bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\tau + V_{ud}\bar{u}_i\gamma_\mu(1-\gamma_5)d_i + V_{us}\bar{u}_i\gamma_\mu(1-\gamma_5)s_i + \cdots \quad (i=1,2,3) \qquad (2)$$

where V_{ud}, $\cdots$ are at least nine parameters called quark mixing matrix elements. Furthermore, it is now wellknown that in composite models of quarks and leptons it can be most simply written in terms of an iso-doublet of spinor subquarks with the charge of $\pm 1/2$, w_1 and w_2 (called wakems)[3] as a single term without any free parameters,[4]

$$J_\mu = \bar{w}_1\gamma_\mu(1-\gamma_5)w_2 \ . \qquad (3)$$

Some consequences of this triplicity for the weak charged current will be presented and discussed in detail later in Section 3.

2. Weak Mixing Angle

In the unified subquark model of quarks and leptons,[3),5)] not only quarks and leptons but also gauge bosons such as the weak bosons ($W^{\pm}$ and Z), the photon (A) and the gluons (G^a, a=1-8) can be taken as composite states of subquarks,

$$W_\mu^+ = \overline{w}_{2L}\gamma_\mu w_{1L}, \quad W_\mu^- = \overline{w}_{1L}\gamma_\mu w_{2L}, \tag{4}$$

$$A_\mu = \frac{\sqrt{3}}{2}\left(\frac{1}{2}\overline{w}_1\gamma_\mu w_1 - \frac{1}{2}\overline{w}_2\gamma_\mu w_2 - \frac{1}{2}iC_0^\dagger\overleftrightarrow{\partial}_0 C_0 + \frac{1}{6}iC_i^\dagger\overleftrightarrow{\partial}_\mu C_i \tag{5}$$

$$(= \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu),$$

$$Z_\mu = \frac{\sqrt{5}}{2}\left(\frac{1}{2}\overline{w}_{1L}\gamma_\mu w_{1L} - \frac{1}{2}\overline{w}_{2L}\gamma_\mu w_{2L}\right)$$

$$- \frac{3\sqrt{5}}{10}\left(\frac{1}{2}\overline{w}_{1R}\gamma_\mu w_{1R} - \frac{1}{2}\overline{w}_{2R}\gamma_\mu w_{2R} - \frac{1}{2}iC_0^\dagger\overleftrightarrow{\partial}_\mu C_0 + \frac{1}{6}iC_i^\dagger\overleftrightarrow{\partial}_\mu C_i\right) \tag{6}$$

$$(= \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu),$$

$$G_\mu^a = i\sqrt{2}C_i^\dagger\overleftrightarrow{\partial}_\mu\left(\frac{\lambda^a}{2}\right)_{ij}C_j \tag{7}$$

where C_0 and C_i (i=1,2,3) are the Pati-Salam color-quartet of scalar subquarks[6)] with the charge of -1/2 and +1/6, respectively, A_μ^3 and B_μ are the third component of iso-triplet gauge bosons and the iso-scalar gauge boson in the Glashow-Salam-Weinberg theory of electroweak interactions,[7)] θ_W is the weak mixing angle and λ^a's are the Gell-Mann's matrices of color SU(3). These relations can be taken either as those derived from the unified subquark model[3)] of the Nambu-Jona-Lasinio type[8)] or as field-current identities[9)] for the gauge fields and subquark currents. In either way, it is now an elementary exercise to derive the Georgi-Glashow relations[10)] of

$$\sin^2\theta_W = \Sigma(I_3)^2/\Sigma Q^2 = \frac{3}{8} \tag{8}$$

and

$$f^2/g^2 = \Sigma(I_3)^2/\Sigma(\lambda^a/2)^2 = 1 \tag{9}$$

for the gluon and weak boson coupling constants (f and g) and the third component of isospin (I_3), the charge (Q) and the color-spin ($\lambda^a/2$) of subquarks from the relations (5)-(7) without depending on the assumption of grand unification of strong and electroweak interactions.

Similarly, in the unified quark-lepton model of the Nambu-Jona-Lasinio type[3)] or in field-current identification for the gauge fields and quark-lepton currents, the gauge boson fields can be taken at least approximately as composite operators made of quarks and leptons,

$$W_\mu^+ \cong \frac{1}{\sqrt{4N_g}}(\bar{e}_L\gamma_\mu\nu_{eL}+\bar{d}_{iL}\gamma_\mu u_{iL}+\cdots),\ W_\mu^- \cong \frac{1}{\sqrt{4N_g}}(\bar{\nu}_{eL}\gamma_\mu e_L+\bar{u}_{iL}\gamma_\mu d_{iL}+\cdots) \tag{10}$$

$$A_\mu \cong \frac{\sqrt{3}}{4\sqrt{N_g}}(-\bar{e}\gamma_\mu e + \tfrac{2}{3}\bar{u}_i\gamma_\mu u_i - \tfrac{1}{3}\bar{d}_i\gamma_\mu d_i + \cdots) \tag{11}$$

$$\begin{aligned} Z_\mu \cong\ & \frac{\sqrt{5}}{4\sqrt{N_g}}(\tfrac{1}{2}\bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L + \tfrac{1}{2}\bar{u}_{iL}\gamma_\mu u_{iL} - \tfrac{1}{2}\bar{d}_{iL}\gamma_\mu d_{iL} + \cdots) \\ & - \frac{3\sqrt{5}}{20\sqrt{N_g}}(- \tfrac{1}{2}\bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R \\ & \qquad +\tfrac{1}{6}\bar{u}_{iL}\gamma_\mu u_{iL}+\tfrac{1}{6}\bar{d}_{iL}\gamma_\mu d_{iL}+\tfrac{2}{3}\bar{u}_{iR}\gamma_\mu u_{iR}-\tfrac{1}{3}\bar{d}_{iR}\gamma_\mu d_{iR}+\cdots)\ , \end{aligned} \tag{12}$$

$$G_\mu^a \cong \frac{1}{2\sqrt{N_g}}(\bar{u}_i\gamma_\mu(\frac{\lambda^a}{2})_{ij}u_j + \cdots \tag{13}$$

where N_g is the number of generations (≥3). It is also almost trivial to derive the Georgi-Glashow relations (8) and (9) from these approximate identities.

Furthermore, all these gauge bosons except for the gluons can also be taken as composite operators made of hadrons (baryons and mesons). By ignoring not only quark mixing but also all hadrons other than the ground-state baryons of spin 1/2 and weak-isospin 1/2, they can be most roughly written as

$$W_\mu^+ \cong \frac{1}{\sqrt{2N_g}}(\bar{e}_L\gamma_\mu\nu_{eL}+\bar{n}_L\gamma_\mu p_L+\cdots),\quad W_\mu^- \cong \frac{1}{\sqrt{2N_g}}(\bar{\nu}_{eL}\gamma_\mu e+\bar{p}_L\gamma_\mu n_L+\cdots), \tag{14}$$

$$A_\mu \cong \frac{1}{2\sqrt{N_g}}(-\bar{e}\gamma_\mu e + \bar{p}\gamma_\mu p + \cdots)\ , \tag{15}$$

and $$Z_\mu \cong \frac{\sqrt{3}}{4\sqrt{N_g}}(\bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \bar{e}_L\gamma_\mu e_L + \cdots)$$

$$- \frac{\sqrt{3}}{12\sqrt{N_g}}(-\bar{\nu}_{eL}\gamma_\mu\nu_{eL}-\bar{e}_L\gamma_\mu e_L-2\bar{e}_R\gamma_\mu e_R+\bar{p}_L\gamma_\mu p_L+\bar{n}_L\gamma_\mu n_L+2\bar{p}_R\gamma_\mu p_R+\cdots). \tag{16}$$

It is again trivial to derive the following Georgi-Glashow relation from these very rough identities:

$$\sin^2\theta_W = \sum(I_3)^2/\sum Q^2 = \frac{1}{4}\ . \tag{17}$$

The numerical result for the weak mixing angle in the subquark picture remarkably coincides with that in the quark picture (as in (8)) but differs from that in the "hadron picture" of (17). This coincidence (or "duality"), seems more than a mere coincidence as it is caused by the same degrees of freedom due to the four wakems (w_{1L}, w_{2L}, w_{1R}, w_{2R}) and four chroms (C_0, C_1, C_2, C_3) forming "subquark-superquartet". The experimental value is $\sin^2\theta_W = 0.230\pm0.0048$ in world-average.[11] The disagreement between the value of 3/8 predicted either in the subquark model or in the quark model and the experimental value might be excused for by insisting that the predicted is viable as the running value renormalized a la Georgi, Quinn and Weinberg[12] at extremely high energies (as high as 10^{15}GeV), given the "desert hypothesis". On the other hand, it is more comfortable to find that the value of 1/4 obtained in the hadron picture remarkably well agrees with the experimental value. The agreement becomes even better and probably too good if it is compared to $\sin^2\theta_W(m_W) = 0.253\pm0.005$ which is the experimental value renormalized at the $W^\pm$ mass ($\cong$ 80GeV).

3. Quark Mixing Matrix

As the "hadron-quark duality relations" of (1) and (2) for the weak charged current mass-produce the approximate relations of

$$\frac{G^{\beta}}{G^{\mu}}\bar{p}\gamma_{\mu}(1-\frac{g_A^{\beta}}{g_V^{\beta}}\gamma_5)n \cong V_{ud}\langle p|\bar{u}\gamma_{\mu}(1-\gamma_5)d|n\rangle ,$$

$$\frac{G^{\Lambda}}{G^{\mu}}\bar{p}\gamma_{\mu}(1-\frac{g_A^{\Lambda}}{g_V^{\Lambda}})\Lambda \cong V_{us}\langle p|\bar{p}\gamma_{\mu}(1-\gamma_5)s|\Lambda\rangle , \cdots, \tag{18}$$

the "quark-subquark duality relations" of (2) and (3) do those of

$$V_{ud}\bar{u}\gamma_{\mu}(1-\gamma_5)d \cong \langle u|\bar{w}_1\gamma_{\mu}(1-\gamma_5)w_2|d\rangle ,$$

$$V_{us}\bar{u}\gamma_{\mu}(1-\gamma_5)s \cong \langle u|\bar{w}_1\gamma_{\mu}(1-\gamma_5)w_2|s\rangle , \cdots. \tag{19}$$

By using the algebra of subquark currents,[4),5)] the unitarity of quark mixing matrix, $VV^{+} = V^{+}V = 1$ has been demonstrated.

In the first order perturbation of isospin breaking (the Hamiltonian H_I), the relations of

$$V_{us} = \frac{\langle u|H_I|c\rangle}{m_u-m_c} + \frac{\langle d|H_I|s\rangle}{m_s-m_d} , \quad V_{cd} = \frac{\langle c|H_I|u\rangle}{m_c-m_u} + \frac{\langle s|H_I|d\rangle}{m_d-m_s} ,$$

$$V_{cb} = \frac{\langle c|H_I|t\rangle}{m_c-m_t} + \frac{\langle s|H_I|b\rangle}{m_b-m_s} , \quad V_{ts} = \frac{\langle t|H_I|c\rangle}{m_t-m_c} + \frac{\langle b|H_I|s\rangle}{m_s-m_b} ,\cdots \tag{20}$$

have been obtained. From these follow immediately the antisymmetry relations of

$$V_{us} = -V_{cd}^{*} , \quad V_{cb} = -V_{ts}^{*}, \cdots, \tag{21}$$

which agree well with the experimental values of $V_{us} = 0.217 \sim 0.223$ and $V_{cd} = -(0.217 \sim 0.223)$.[11] They also produce some other relations such as

$$|V_{cb}| \cong (m_s/m_b)|V_{us}| \cong 0.02 , \tag{22}$$

which roughly agrees with the latest experimental value of $|V_{cb}| = 0.046^{+0.008}_{-0.010}$ (Argus).[11]

In the second order perturbation, the relations of

$$|V_{ub}/V_{cb}| \cong (m_s/m_c)|V_{us}| \cong 0.08 \tag{23}$$

and $$|V_{td}| \cong |V_{us}V_{cb}| \cong 0.01 \tag{24}$$

have been predicted. The relation (23) agrees remarkably well with the latest experimental observation of $|V_{ub}/V_{cb}| = 0.09 \pm 0.02$ (Argus) and 0.10 ± 0.03 (CLEO).[11] It is highly desirable to test the relation (24) when the top quark is found.

Before closing this Section, I wish to remind you of the simple picture of quark mixing in four-fold way.[13] Suppose there are 4×4 wakems (w_{aA}) and 4×4 chroms ($C_\alpha{}^A$) for 4 isospin-handedness (a = 1L, 2L, 1R, 2R), 4 colors ($\alpha = 0,1,2,3$) and 4 subcolors ($A = 1,2,3,4$). For simplicity, let w_{aA} and $C_\alpha{}^A$ be bosonic commuting operators, satisfying

$$w_{aA}\bar{w}^{bB} = \delta_a{}^b \delta_A{}^B \quad \text{and} \quad C_\alpha{}^A C^{\dagger\beta}{}_B = \delta_\alpha{}^\beta \delta^A{}_B . \tag{25}$$

Then, an infinite number of candidates for quark-lepton states can be written with the normalization of $\bar{f}f = 1$ as

$$f_{a\alpha}^{(1)} = \frac{1}{\sqrt{4}} w_{aA} C_\alpha{}^A ,$$

$$f_{a\alpha}^{(2)} = \frac{1}{\sqrt{4}(3!)^{3/2}} \varepsilon_{abcd}\varepsilon_{ABCD}\bar{w}^{bB}\bar{w}^{cC}\bar{w}^{dD}C_\alpha{}^A ,$$

$$f_{a\alpha}^{(3)} = \frac{1}{\sqrt{4}(3!)^{3/2}} w_{aA}\varepsilon_{\alpha\beta\gamma\delta}\varepsilon^{ABCD}C^{\dagger\beta}_B C^{\dagger\gamma}_C C^{\dagger\delta}_D ,$$

$$f^{(4)}_{a\alpha} = \frac{1}{\sqrt{4}(3!)^3} \varepsilon_{abcd}\varepsilon_{ABCD}\bar{w}^{bB}\bar{w}^{cC}\bar{w}^{dD}\varepsilon_{\alpha\beta\gamma\delta}\varepsilon^{AEFG}C^{\dagger\beta}_{E}C^{\dagger\gamma}_{F}C^{\dagger\delta}_{G} , \ldots . \tag{26}$$

The quarks and leptons of the first, second and third generation, f_1, f_2 and f_3, can be identified with $f^{(1)}$, $f^{(2)}$ or $f^{(3)}$, and $f^{(4)}$, respectively, which is closely related to the line of Miyazawa's hypersymmetry.[14] This assignment is good as it provides the plausible reason why[15]

$$m_u < m_d, \quad m_c > m_s \quad \text{and} \quad m_t > m_b \quad \text{if} \quad m_{w_1} < m_{w_2} . \tag{27}$$

The weak current is given by the operator of

$$J^{\dagger} = w_{1LA}\bar{w}^{2LA} \tag{28}$$

and the quark mixing occurs due to the "condensation" of multi-subquark states which can be expressed in terms of the transition states which can be expressed in terms of the transition operator of

$$T = \frac{\varepsilon}{(4!)^2} \varepsilon^{abcd}\varepsilon^{ABCD}w_{aA}w_{bB}w_{cC}w_{dD}$$
$$+ \frac{\varepsilon}{4(4!)^2} \varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{ABCD}C_{\alpha}{}^{A}C_{\beta}{}^{B}C_{\gamma}{}^{C}C_{\delta}{}^{D} \tag{29}$$

where ε and η are constants. If this is the case, the quark mixing matrix given by

$$V_{mn} = \bar{f}_m J^{\dagger} f_n \tag{30}$$

can be calculated to be

$$V_{mm} = \begin{pmatrix} 1- & \frac{\varepsilon}{\sqrt{4!}} & \frac{\varepsilon\eta}{8(4!)} & \cdots \\ -\frac{\varepsilon}{\sqrt{4!}} & 1- & \frac{\eta}{4\sqrt{4!}} & \cdots \\ \frac{\varepsilon\eta}{8(4!)} & -\frac{\eta}{4\sqrt{4!}} & 1- & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{pmatrix} \cong \begin{pmatrix} 1- & 0.204 & 0.0052 & \cdots \\ -0.204 & 1- & 0.051 & \cdots \\ 0.0052 & -0.051 & 1- & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{pmatrix}$$

$$\text{for} \quad \varepsilon=\eta=1, \qquad (31)$$

which is to be compared with the experimental values of V_{us} = 0.220±0.003, V_{ub} = 0.0041±0.0016 and $V_{cb} = 0.046^{+0.008}_{-0.010}$. The remaining problem is to explain why the parameters ε and η are so close to unity.

4. Mass Sum Rules

The unified subquark model predicts[3),5)] the following two sum rules:

$$m_W = [3(m_{W_1}^2+m_{W_2}^2)/2]^{1/2} \qquad (32)$$

and $$m_H = 2[(m_{W_1}^4+m_{W_2}^4)/(m_{W_1}^2+m_{W_2}^2)]^{1/2} \qquad (33)$$

where m_H is the mass of the physical Higgs scalar in the Glashow-Salam-Weinberg theory. Also, the unified quark-lepton model of the Nambu-Jona-Lasinio type[5)] predicts the following two sum rules:

$$m_W = [3\langle m_{q,\ell}^2\rangle]^{1/2} \qquad (34)$$

and $$m_H = 2[\sum m_{q,\ell}^4/\sum m_{q,\ell}^2]^{1/2} \qquad (35)$$

where $m_{q,\ell}$'s are the quark and lepton masses and < > denotes the average value for all the quarks and leptons. Notice that the second sum rules (33) and (35) are essentially the same as the

Nambu relation[16] of $m_\xi : m_\psi : m_\eta = 0:1:2$ or $m_\xi^2 + m_\eta^2 = 4m_\psi^2$ where ξ, η and ψ are the Nambu-Goldstone boson, the physical scalar and the constituent fermion, respectively and that they are the consequences of the Nambu's supersymmetry and, therefore, less model-dependent.

By combining the sum rules (32) and (33), the following relation can be obtained for $m_{w_1} = m_{w_2} = m_w$:

$$m_w : m_W : m_H = 1 : \sqrt{3} : 2 \,. \tag{36}$$

From this relation, the wakem and Higgs scalar masses can be predicted as

$$m_w = (1/\sqrt{3})m_W = (46.8\pm0.8)\mathrm{GeV} \tag{37}$$

and $m_H = (2/\sqrt{3})m_W = (93.5\pm1.5)\mathrm{GeV}$ for $m_W = (81.0\pm1.3)\mathrm{GeV}$, (38)

which is subject to a future experimental test probably at LEPII. More precisely, from the two sum rules, the Higgs mass can be bounded as

$$(93.5\pm1.5)\mathrm{GeV} = (2/\sqrt{3})m_W \le m_H \le (2\sqrt{6}/3)m_W = (132.3\pm2.1)\mathrm{GeV}. \tag{39}$$

Notice that the lower bound corresponds to the case of $m_{w_1} = m_{w_2}$ while the upper one to that of $m_{w_1}/m_{w_2} = 0$ or ∞. Therefore, it seems more likely that the physical Higgs scalar will be found close to the lower bound, i.e. $m_H \cong 94\mathrm{GeV}$. The reliability of this prediction may be enhanced by the following independent observation: Suppose that the subquark dynamics is described by "quantum subchromodynamics (QSCD)", the Yang-Mills gauge theory of subcolors which is an analogy to QCD. Then, the masses of $W^\pm$ and H are scaled by Λ_{sc}, the mass scale of QSCD, while the masses of the corresponding hadrons, $\rho^\pm$ and σ, are scaled by Λ_c, the mass scale of QCD. If this is the case, the Higgs scalar mass can be estimated as

$$m_H \cong \frac{m_\sigma}{m_\rho} m_W \cong \frac{\sim 900\text{MeV}}{770\text{MeV}}(81.0\pm1.3)\text{GeV} = (94.7\pm1.5)\text{GeV} \qquad (40)$$

which amazingly produces a similar prediction, $m_H \cong 95\text{GeV}$.

If there exist only three generations of quarks and leptons, the sum rules (34) and (35) completely determine the top quark and Higgs scalar masses as

$$m_t \cong (2\sqrt{6}/3)m_W = (132.3\pm2.1)\text{GeV} \qquad (41)$$

and $m_H \cong 2m_t \cong (4\sqrt{6}/3)m_W = (264.5\pm4.2)\text{GeV}$ for $m_W = (81.0\pm1.3)\text{GeV}$.

(42)

If indeed the top quark is this heavy, production of the topponium and top-antitop pairs is unfortunately beyond the reach of LEPII. If, instead, there are four generations, the sum rule (34) gives an estimate for the average mass of the fourth generation of quarks and leptons as

$$[\langle m_{q,\ell}^2 \rangle_{N_g=4}]^{1/2} \cong (2/\sqrt{3})m_W = (93.5\pm1.5)\text{GeV for } m_W=(81.0\pm1.3)\text{GeV}.$$

(43)

Triplity of hadrons, quarks and subquarks tells us that these sum rules can be further extended to the approximate sum rules of

$$m_W \cong [3\langle m_{B,\ell}^2 \rangle]^{1/2} \qquad (44)$$

and

$$m_H \cong 2[\sum m_{B,\ell}^4 / \sum_{B,\ell}^2]^{1/2} \qquad (45)$$

where $m_{B,\ell}$'s are the "canonical baryon" and lepton masses and < > denotes the average value for all the canonical baryons and leptons. The "canonical baryon" denotes either one of p, n and other ground-state baryons of spin 1/2 and weak-isospin 1/2 consisting of a quark heavier than u and d quarks and a scalar and isoscalar diquark made of u and d quarks. These sum rules can be derived, in the same way as those of (32)-(35), in the "unified hadron-lepton model" of the Nambu-Jona-Lasinio type which is written in terms of the canonical baryons and leptons as fundamental fermions.

If there exist only three generations of quarks and leptons, the sum rules (44) and (45) completely determine the masses of the canonical topped baryon, T, and Higgs scalar as

$$m_T \cong 2m_W = (162.0\pm2.6)\mathrm{GeV} \tag{46}$$

and $m_H \cong 2m_T \cong 4m_W = (324.0\pm5.2)\mathrm{GeV}$ for $m_W = (81.0\pm1.3)\mathrm{GeV}$. (47)

If, instead, there are four generations, the sum rule (44) gives an estimate for the average mass of the fourth generation of the canonical baryons and leptons as the same as in (43). Notice that the predicted value for the canonical topped baryon mass in (46) is by 22 percent larger than that for the top quark mass in (41) and that the predicted values for the Higgs scalar mass in (38), (42) and (47) are $1:2\sqrt{2}:2\sqrt{3}$. Especially the latter may indicates either that there exist at least four generations of quarks and leptons or that both the unified quark-lepton model of the Nambu-Jona-Lasinio type and the unified hadron-lepton one are in a very bad approximation for describing the Higgs scalar. An answer will be given by future high-energy experiments.

5. Conclusion

We have re-interpreted and discussed in detail the weak mixing angle, the quark mixing matrix and the mass sum rules in triplicity of hadrons, quarks and subquarks. We have presented many predictions for the weak mixing angle, the quark mixing matrix elements and the top quark and Higgs scalar masses. Some of which have already been checked experimentally and the others will be tested in the near future. We hope that the notion of triplicity will become more useful in particle physics after much more applications are found.

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