Recent Topics on Technicolor Theory

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Abstract

After a short historical review of technicolor theory and the so called FCNC problem, recent two models, asymptotically free (AF) and asymptotically nonfree (ANF)gauge theories are introduced to solve the FCNC problem. The relation is discussed between the two models by using the solutions of improved planar Schwinger-Dyson equation. We observe two types of the ultraviolet fixed points, the one corresponds to the so-called trivial ultraviolet fixed point usually observed in AF gauge theories, and the other to the nontrivial ultraviolet fixed point. In the last part of this paper I would like to make a comment on the essence of FCNC problem comes from the momentum dependence of the dynamical generated fermion mass function.

Standard Weinbertg Salam theory is very successive in describing low energy electroweak interactions. We, particle physicists are still not yet content with it, however. because it does not explain the dynamical origin of the spontaneous breakdown of electroweak gauge symmetry. There elementary Higgs bosons are introduced phenomenologically which play dual roles to give the masses of W and Z on the one hand, and the masses of ordinary quarks on the other hand.

Techinicolor(TC) theory was proposed to address the above problem, in which TC gauge interaction, the grade-up version of QCD, induces the dynamical breaking of chiral symmetry, producing composite Higgs bosons as so called Nanbu-Goldstone bosons. This was a very attractive idea so far as the the gauge sector is concerned, giving the masses of W and Z bosons. However only the technifermions acquire the dynamical masses via TC interaction, leaving massless quarks. In order to get quark masses one must introduce the interacrtion (extended technicolor (ETC)) which connects the ordinary quark sector with the technifermions. Unfortunately ETC leads to a phenomenological disaster, the excessive flavor-changing neutral currents (FCNC problem). This had been the most serious difficulty of the TC theory and people regarded it not workable in spite of its beautifulness. The FCNC problem may be indeed not peculiar to the TC theory but common to any theory that has to do with the flavor problem or adress the dynamical origin of the breakdown of electroweak gauge symmetry. Anyway because of this the number of papers with keyward "TC" decreased quikly. The situation is visualized by Fig.1.

= Fig.1=

The proposals on how to solve the excessive FCNC probem in TC theories have inspired renewed interests in the idea of dynamical symmetry breakdown. One of the proporsals is a scale-invariant TC model in ANF theories[1-3] having nontrivial ultraviolet fixed point and the other is a walking TC model[4,5] in the framework of AF theories. A striking ferture of the former is that the large anomalous dimension automatically raised the dynamical fermion masses, preserving the enough suppression of FCNC. On the contrary the latter takes the view-point that we have not yet confirmed completely that such a nontrivial ultraviolet fixed point theory really exists. This is the reason why the latter model was seriously examined. Since then a special attention has been paid to the possibility of the existence in ANF theories of strong coupling phase which is separated from the usual perturbative weak coupling phase by a nontrivial ultraviolet fixed point. Theoretically it is suggested both from comprehensive studies of ladder Schwinger-Dyson (S-D) equation of QED[6-8] from recent lattice calculations of compact[9] and noncompact QED.[10] It seems quite natural to guess that such new phase, if it ever exists in QED anyhow, may be observed in a certain class of strongly coupled (provably ANF) gauge theories. On the other hand, QCD is an example which has a trivial fixed point, showing typical character of AF gauge theories.

For these several years there have been several critical arguments concerning the above two kinds of models.[2,5] Especially one may ask whether or not the former approach is simply the limiting case of the latter. In fact the momentum dependence of the running coupling constant in the AF gauge theories is

$$\alpha_{AF}(x) = \frac{g^2(x)}{4\pi} = \frac{\alpha_\mu}{1+\alpha_\mu b_0/\pi t}, \quad t = \ln(x/\mu^2); x = p^2, \tag{1}$$

where in the perturbative calculation $b_0 = 1/3(11C_2(G) - 4N_f T(F))$ for N_f-flavored fermions, $T(F)$ and $C_2(G)$ being Casimir invariants for the fermions of the representation F and gauge fields of the representation G. On the other hand in the scale invariant TC model the coupling constant approaches to the critical coupling $\alpha_c = \pi/3$ as

$$\alpha_{ANF}(x) = \alpha_c + (\frac{2\pi}{t})^2, \tag{2}$$

so, as far as we look at the behavior of running coupling constant, one may think that ANF gauge theory is the limit of AF gauge theory with $b_0 \to 0$, i.e., we arrive at the *standing* coupling constant from the *walking* one.

The relationship between the above two models, ANF and AF gauge theories can be most easily demonstrated from the viewpoint of the modern renormalization theory.[11-14] The basic viewpoint is that *the parameters of the original lagrangian should be arranged so that the physical quantities, $\{M_j\}$ evaluated from the original lagrangian are kept invariant and finite when we let the cut-off Λ go to infinity.* It must stressed that there correspond a set of parametes of the lagrangian space to a set of low-energy physical quantities as functions of the cut off:

$$\begin{aligned} (\lambda_1, \lambda_2, \cdots, \lambda_n) &\longrightarrow (M_1, M_2, \cdots, M_m) \\ M_j &= \Lambda^{d_j} F_j(\{\alpha_i\}), \quad i = 1 \sim n, \ j = 1 \sim m. \end{aligned} \tag{3}$$

where d_j is the dimension of the physical quantity M_j. This clearly implies that in the cases of $n \geq m$ the requirements of the finiteness of the measurable quantities, $(M_1, M_2, \cdots, M_m)$ in the limit $\Lambda \to \infty$ are almost trivially satisfied so far as the set of coupled eqs., $F_j(\{\alpha_i\}) = 0$ (for $d_j > 0$) in (3)have solutions. Thus in order to show the possibility of renormalizability of the theory, *it is necessary (but not sufficient, of course) to show that the Λ-dependence of the set of parameters $\{\alpha_i\}$ can be chosen so that the calculated physical quantities $\{M_i\}$ are all finite and independent of Λ and the number m should be at least more than the freedom of relevant parameters, n.* The renormalization flow is determined in such a way that the low-energy physical quantities M may be kept constant independently of the cutoff parameter Λ and is described by the contour line in the λ-plane and the phase structure and fixed points are determined as the criticality of this parameter plane.

Now the following improved planar S-D equation[15] for fermion selfenergy function is used as a tool to treat both AF and ANF gauge thories:

$$\Sigma(x) = m_q + \frac{\lambda(x)}{4x}\int_0^x \frac{y\Sigma(y)dy}{y+\Sigma^2(y)} + \int_x^\infty \frac{\lambda(y)\Sigma(y)dy}{4(y+\Sigma^2(y))}, \tag{4}$$

with the effective (running) gauge coupling function which is parametrized as

$$\lambda(x) = \frac{3C_2(F)}{\pi}\alpha(x) = \frac{\lambda_0}{1+At}\theta(t) + \lambda_0\theta(-t), \quad \lambda_0 \equiv \lambda(t=0), \tag{5}$$

where μ is the momentum scale where the spontaneous chiral symmetry breakdown occurs and is here set equal to $\Sigma(0)$ as a kind of self-consistency condition. The eq.(5) can be rewritten as

$$\lambda(t) = \frac{\tilde{\lambda}}{1+\tilde{A}(t+\ln\mu^2/\Lambda^2)}\theta(t) + \lambda_0\theta(-t), \tag{6}$$

with

$$\tilde{\lambda} = \frac{\lambda_0}{1+A\ln(\Lambda^2/\mu^2)}, \quad \tilde{A} = \frac{A}{1+A\ln(\Lambda^2/\mu^2)}. \tag{7}$$

We regard the above $\tilde{A}$ in eq.(4), as well as the strength $\tilde{\lambda}$ of the gauge interaction, is the parameter ($\{\lambda\}$ in eq.(3)) which specifies the effective interaction of the system at scale Λ. (In order to see the ultraviolet structure, we analyze in $(\tilde{\lambda}, \tilde{A})$ plane instead of (λ, A).) Note that it facilitates us to treat the AF and ANF gauge theories on the same footing. The phase diagram is determined as the criticality of this two-parameter plane which divides the region of having the nontrivial solution of eq.(4) from that of trivial solution. The renormalization flow structure is obtained, triggered by certain physical quantities. The ultraviolet fixed points are those to which the renormalization flow converges in the limit $\Lambda \to \infty$. I here show an typical graph of the result in Fig.2, in which f_π together with $\Sigma(0)$ are taken as physical quantities M.

= Fig.2 =

From this figure we see that *there exist two ultraviolet fixed points* $(\tilde{\lambda} = 1, \tilde{A} = 0)$ *and* $(\tilde{\lambda} = 0, \tilde{A} = 0)$*, which we shall call as "AF point" and "ANF point", respectively.* Each flow line corresponds to a physical theory with some renormalized value of f_π and $\Sigma(0)$. The direction of each flow from the ultraviolet fixed point determines the low energy physical value f_π. The ANF point $(1, 0)$ supports the range of $0.37\mu \leq f_\pi \leq \infty$, while the AF point $(0, 0)$ supports the region $0 \leq f_\pi < 0.37\mu$.

We could take other low-energy physical quantities. The results are found to be almost similar.[16] These two kinds of ultraviolet fixed points can be regarded as split halves of the point which supports all the low-energy physical systems. In this sense AF and ANF gauge theories share the physical systems each other and there seems to exist no drastic singular gap between them.

Now that we have seen the relation between the AF and ANF gauge theories from the viewpoint of ultraviolet fixed points structure; they can be elegantly described by the interaction parameters, in the $(\tilde{\lambda}, \tilde{A})$ plane. Next I would like to explain the essence which was proposed by the so-called FCNC problem. In any dynamical model to adress the electromagnetic symmetry breaking, the behavior of the dynamical-generated fermion mass function plays an essential role. Usually we call "hard mass" in the case where fermions have bare masses and "soft mass" if it is dynamically generated: their behaviors are up to $\ln(x)$ factor,

$$
\begin{aligned}
&\Sigma(x) = m, && \textit{soft mass}, \\
&\Sigma(x) = \mu\sqrt{x}^{-1}\Sigma(\mu), && \textit{ANF hard mass } (\gamma^* = 1), \\
&\Sigma(x) = \mu^2 x^{-1}\Sigma(\mu), && \textit{AF hard mass } (\gamma^* = 0),
\end{aligned}
\tag{8}
$$

where γ^* is the anomalous dimension. However it must be noted that we get the same hard mass function even when it is dynamically generated, for example in the Nanbu-Jona-Lassinio model or usual Higgs model and in this case γ^*=2. Also Marciano obtained the same behavior from the coupling reduction approach.[17,18] It is known that the gauge boson masses are determined by the scale of so-called f_π, in which the low energy part of $\Sigma(x)$ dominates in the integral. On the other hand, since the ordinary quarks acquire their masses from the above behaved dynamical mass via ETC and the main contribution comes from the integrand in the region around the ETC scale (about 1000μ). Thus the more slowly $\Sigma(x)$ damps, the larger quark mass becomes. This is the reason why the elementary Higgs or equivallently the NJL type model are free from the FCNC problem. We shall see how it becomes a key point to keep the above fact clearly in mind when we want to construct any realistic dynamical model for Higgs sector, especially when we pay an attention to the custodial isospin violation in composite Higgs models.

I would like to thank K.I.Aoki, T.kugo, K.Hasebe and H.Nakatani for kind cooperations.

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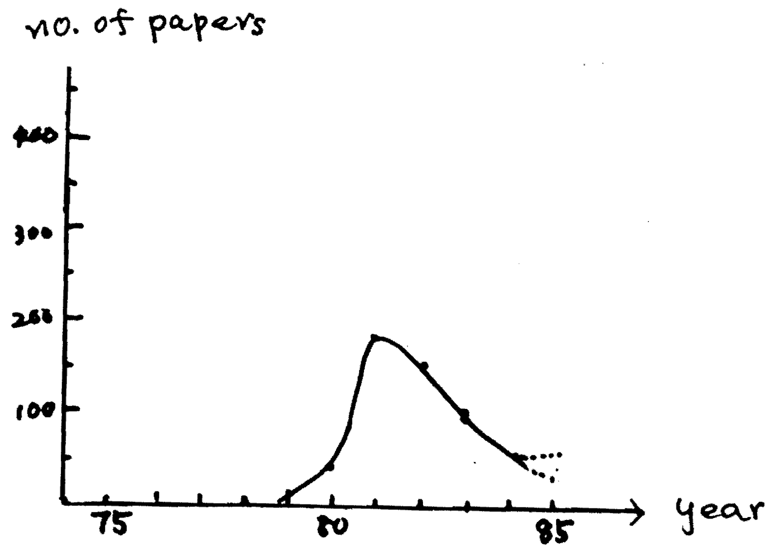

Fig.1
The number of the produced papers per year which attach the keyward "TC"

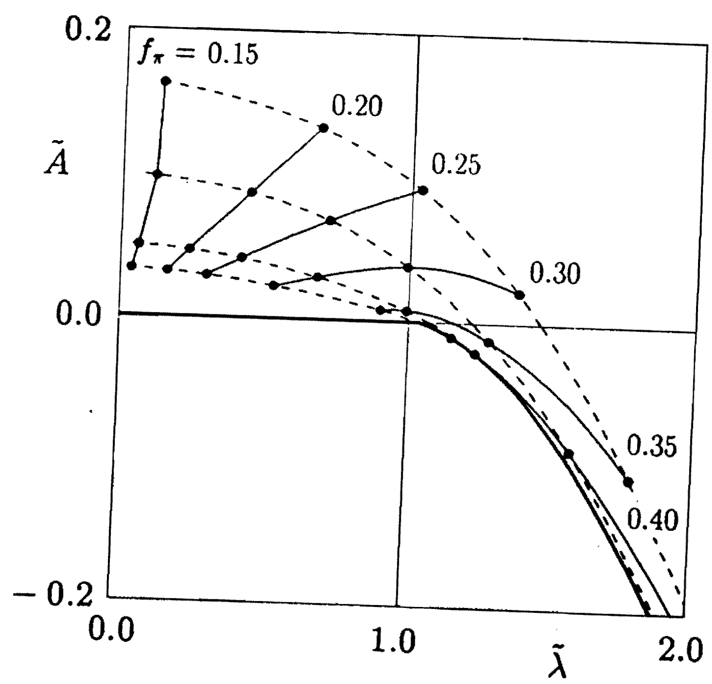

Fig.2
Ultraviolet fixed point structure in $(\tilde{\lambda}, \tilde{A})$ plane