Color-Sextet Condensation*

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Abstract

The condensation of the color-sextet quarks are considered. We present the dynamical electroweak symmetry breaking model with high-color effect and four-fermion interactions where the color-sextet quark condensate $\overline{Q}Q$ acts as the Higgs field. The masses of sextet quarks are predicted to be 210∼280 GeV for the lighter sextet quark of the iso-spin doublet U and D, and 280∼330 GeV for the heavier partner. In the simplest model, the sextet quarks belong to $\underline{6}^*$ of $SU(3)_C$, and their charges are $+2/3$ for U, $-1/3$ for D.

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I. Motivation and Basic Assumptions

As is well known, the standard model (SM) of strong, weak and electromagnetic interactions is successful with $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries. However, there are some unsatisfactory features in this theory.

One of these features concerns with the representation of $SU(3)_C$. The strong sector of SM is described by Quantum Chromodynamics (QCD) with an SU(3) gauge group. Quarks belong to $\underline{\mathbf{3}}$ the fundamental representation of SU(3), and gluons to $\underline{\mathbf{8}}$ the adjoint representation. Mathematically there are also an infinite number of higher representations in SU(3): $\underline{\mathbf{6}}$, $\underline{\mathbf{10}}$, $\underline{\mathbf{15}}$, $\underline{\mathbf{15}'}$,$\cdots$. It has not been known whether the particles belonging to such representations exist. If these particles are not in nature, the reason which forbids such particles should exist. Marciano[1] suggested that the high-color effect of quark–anti-quark binding potential may increase in the energy scale of QCD confinement to a few hundred GeV or a few TeV and proposed the sextet quark condensation model.

Another unsatisfactory feature is the mass hierarchy problem. In SM, masses of all fermions and weak gauge bosons are originated from the Higgs field. Because the Higgs is a scalar field, the perturbative correction to masses is quadratically divergent. One can evade this problem by replacing the Higgs field by some fermion-field condensate in the dynamical electroweak symmetry breaking just as the technicolor model,[2] or Marciano's high-color quark model mentioned above. To produce the weak boson masses, the vacuum expectation value $\langle\overline{\Psi}\Psi\rangle$ of these fermion fields Ψ must be about 250 GeV. Then, in general, masses of fermions Ψ may lie in the range between a few hundred GeV and a few TeV.

Though once the technicolor-like models were nearly abandoned because of the well-known flavour changing neutral current problem, it has been shown that the models may be revived if the composite operator $\overline{\Psi}\Psi$ has a large anomalous dimension γ^* in these models.[3] In the technicolor model however, we were forced to introduce an extra fundamental fermion by hand. In the present paper we would like to circumvent this unsatisfactry situation by appearing to the color-sextet quarks.

On the other hand, being stimulated by recent experimental consequences that the top-quark mass may be very heavy as the W boson mass or more, several authors pointed out that the top quark may condense to be a substitute for the Higgs particle.[4],[5] Miransky, Tanabashi and Yamawaki[4],[6] observed that the composite operator $\bar{t}t$ acquires a large γ^* by using the Schwinger–Dyson equation in ladder approximation. This implies that the four-fermion operators such as $\bar{t}t\bar{t}t$, $\bar{t}t\bar{q}q$ and $\bar{t}t\bar{\ell}\ell$ become relevant and have to be included in the original Lagrangian, where q means u, d, s, c and b quark and ℓ the lepton. Their model is attractive because of the large γ^* and the economy: less particles and less free parameters than Higgs' scenario.

We pursue an alternative possibility that the color-sextet quark condensate to trigger the electroweak symmetry breaking. We start with the following assumptions to examine this possibility:

- The color-sextet quarks Q belonging to $\underline{\mathbf{6}}$ (or $\underline{\mathbf{6}}^*$) in $\mathrm{SU}(3)_\mathrm{C}$ exist.
- The bound states $\overline{\mathrm{Q}}\mathrm{Q}$ condensate.

We choose Q as an weak iso-doublet (U, D) for the second assumption. Q should be heavy as the weak boson masses to induce them. This may explain why the sextet quarks have not been found by experiments yet.

II. Dynamical Electroweak Symmetry Breaking by Sextet Quark Condensation

We start with the Lagrangian including a Nambu–Jona-Lasinio[7] type four-fermion interaction term to and study the mechanism of the dynamical electroweak symmetry breaking by the sextet quarks,

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\text{Electro-Weak}}(\text{without Higgs terms}) + \mathcal{L}_{4\mathrm{f}} \ ,$$

$$\mathcal{L}_{4\mathrm{f}} = \frac{1}{2} G^{(0)} \left[(\overline{Q}Q)^2 - (\overline{Q}\gamma_5 Q)^2 \right] \ . \tag{1}$$

One gets the self energy or the dynamical mass $\Sigma(p)$ of Q by solving the Schwinger–Dyson equation in the quenched planner approximation,

$$\begin{aligned}\Sigma(p) \ \simeq \ & m^{(0)} + \frac{g_{4\mathrm{f}}}{\Lambda^2} \int_{m_{\mathrm{Q}}^2}^{\Lambda^2} dq^2 \frac{\Sigma(q)}{q^2 + \Sigma(q)^2} \\ & + \int_{m_{\mathrm{Q}}^2}^{\Lambda^2} dq^2 \frac{q^2 \Sigma(q)}{q^2 + \Sigma(q)^2} \left[\frac{\lambda(q)}{q^2} \, \theta(q^2 - p^2) + \frac{\lambda(p)}{p^2} \, \theta(p^2 - q^2) \right] \ ,\end{aligned} \tag{2}$$

where

$$g_{4\mathrm{f}} = \frac{G^{(0)} N_c N_f \Lambda^2}{4\pi^2} \quad ,$$

$$\lambda(q) = \frac{3}{4\pi}\, \alpha_3^{(0)}(q) \quad .$$

Here N_c and N_f are the numbers of colors and flavours, respectively, Λ the ultraviolet cut-off, α_3 the QCD running coupling, and by the superscription (0) we mean the bare quantity. As shown in ref. [4], a non-trivial solution for $\Sigma(p)$ is obtained in the symmetry-broken phase and the anomalous dimension of the composite operator $\overline{Q}Q$ can be 2, if $g_{4\mathrm{f}}$ and $\lambda(q)$ approach along a certain path to the critical point $g_{4f} = 1$ and $\lambda(q) = 0$ in the continum limit $q \rightarrow \infty$. Therefore the four-fermion terms like $(\overline{\psi}\psi)(\overline{Q}Q)$ where ψ is any fermion field become relevant and it is justified that we have introduced the four-fermion terms into the original Lagrangian. According to the condensation of $\overline{\mathrm{Q}}\mathrm{Q}$, the vacuum expectation value $\langle \overline{Q}Q \rangle$ is non-vanishing, and the four-fermion terms play the role of the mass terms as

$$g_{4\mathrm{f}(\psi)}\, \langle \overline{Q}Q \rangle\, \overline{\psi}\psi$$

where $g_{4\mathrm{f}(\psi)}$ is a coupling corresponding to the four-fermion term $\overline{Q}Q\overline{\psi}\psi$. Note that $\overline{Q}Q$ behaves just like the Higgs field at low energy owing to the decrease of its scale dimension by γ^*.

III. Masses of Sextet Quarks

We will now evaluate the masses of Q. As in the Technicolor-like models[2], the W-boson mass m_{W} is induced by the condensation of the mass-generating fermions,

$$m_{\mathrm{W}}{}^2 \;=\; \frac{1}{4}\, g_2(m_{\mathrm{W}})^2\, F_{\pi^\pm}{}^2 \,, \tag{3}$$

where $g_2(m_{\mathrm{W}})$ is the weak coupling constant and $F_{\pi^\pm}$ the 'pion' decay constant of $\overline{\mathrm{Q}}\mathrm{Q}$. To estimate $F_{\pi^\pm}$, we use the following formula derived in ref. [4].

$$F_{\pi^\pm}{}^2 = \frac{N_c}{8\pi^2}\int_{m_{Q_1}^2}^{\Lambda^2} \frac{x\ dx}{(x+\Sigma_1^2)(x+\Sigma_2^2)}\left[(\Sigma_1^2+\Sigma_2^2) - \frac{1}{4}(\Sigma_1^2+\Sigma_2^2)' + \frac{x}{2}(\Sigma_1^2-\Sigma_2^2)\left\{\frac{1+(\Sigma_1^2)'}{x+\Sigma_1^2} - \frac{1+(\Sigma_2^2)'}{x+\Sigma_2^2}\right\}\right] \quad , \tag{4}$$

where $x \equiv p^2$, $' \equiv d/dx$, $m_{Q_1} = \Sigma_1(m_{Q_1})$, and by the subscript 1 and 2 we mean the heavier and lighter color-sextet quark in an iso-doublet, respectively.

At the energy scale $p \geq m_{Q_1}$, the behavior of Σ_i of with $i = 1, 2$ is computed by solving eq. (2).[6]

$$\Sigma_i(p) \simeq m_{Q_i}\left[\frac{\alpha_3(p)}{\alpha_3(m_{Q_i})}\right]^{A/2} . \tag{5}$$

The quantity A is 60 for present sextet quark model, while 8/7 for the top condensation model.

By neglecting all fermion masses, the asymptotic form of $\alpha(p)$ at the energy scale above m_{Q_1} is

$$\alpha_3(p)^{-1} \cong \frac{1}{6\pi}\ln\frac{p}{M_Q} , \tag{6}$$

where M_Q is a scale parameter. Note that this expression for $\alpha_3(p)$ is applicable only for $p \geq m_{Q_1}$ and is different from the ordinary $\alpha_3(p)$ in low energy scale.

We would like to evaluate M_Q in terms of the low energy QCD parameter M_4 (for four light quarks). For this purpose, we take into account the quark mass effects so that we employ the beta function for massive quarks given by Georgi and Polizer,[8]

$$\beta_3(g_3, \tfrac{m}{p}) \cong -\frac{g_3{}^3}{16\pi^2}\left(11 - \frac{2}{3}\sum_{\text{triplets}}\frac{1}{1+\frac{5m^2}{p^2}} - \frac{10}{3}\sum_{\text{sextets}}\frac{1}{1+\frac{5m^2}{p^2}}\right) , \tag{7}$$

where $g_3{}^2 = 4\pi\ \alpha_3$. This equation leads another expression for $\alpha_3(p)$ applicable to the energy scale $p \geq m_c$,

$$\alpha_3(p)^{-1} \cong \frac{1}{6\pi}\left(25\ln\frac{p}{M_4} - \sum_{i=b,t}\ln\frac{p^2+5m_i{}^2}{M_4{}^2+5m_i{}^2} - 5\sum_{i=Q_1,Q_2}\ln\frac{p^2+5m_i{}^2}{M_4{}^2+5m_i{}^2}\right) . \tag{8}$$

For $p \geq m_{Q_1}$, eq. (8) should reproduce eq. (6) and we have the relation

$$\ln M_Q = 25\ln M_4 - 2\sum_{i=b,t}\ln m_i - 10\sum_{i=1,2}\ln m_{Q_i} - 12\ln 5 . \tag{9}$$

There have been reported many experimental estimations for M_4. Since our

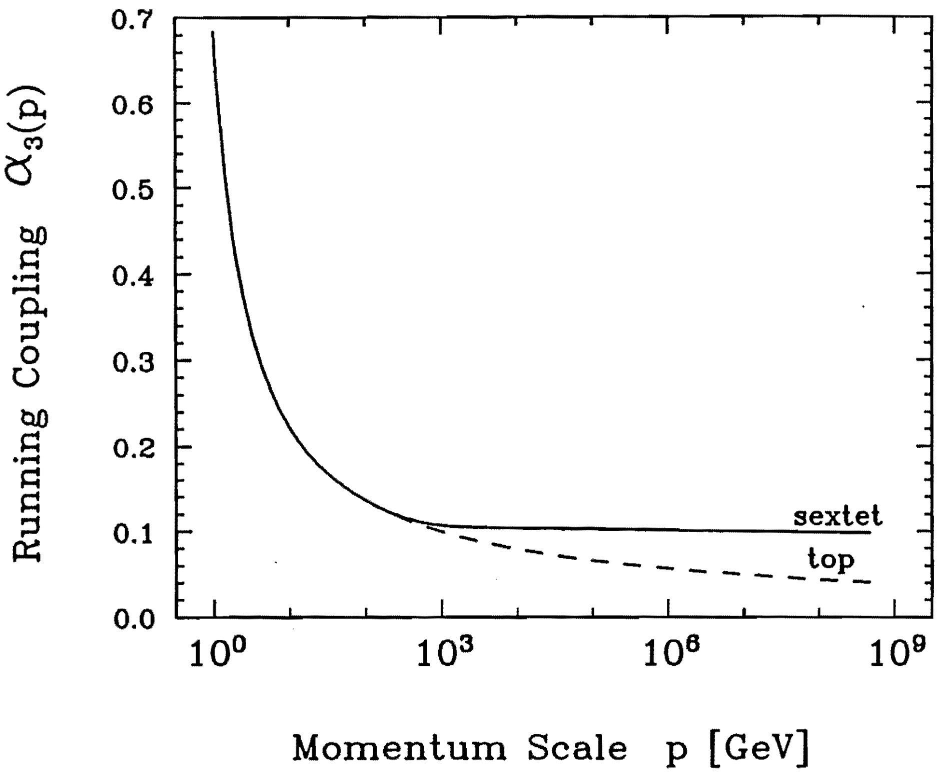

Fig. 1 The behavior of QCD running coupling $\alpha_{3(p)}$. The solid line corresponds to the present sextet model, while the dashed line represents the case without sextet quarks. Here m_{Q_1}, m_{Q_2} and m_t are taken to be 300 GeV, 250 GeV and 150 GeV, respectively.

argument is based on the leading order approximation in α_3, we adopt as M_4 the average value of the scale parameter determined by using the leading order QCD predictions.[9] Our value taken here is

$$M_4 \;=\; (\; 330 \;\pm\; {}^{210}_{130}\;)\;\; \mathrm{MeV}\;.^{1)} \tag{10}$$

The behavior of $\alpha_3(p)$ is shown in fig. 1. Note that α_3 walks very slowly in the energy region above m_{Q_1}.

Now let us go back to eq. (3),

$$m_W{}^2 \;=\; \frac{1}{4}\, g_2(m_W)^2\; F_{\pi^\pm}(m_{Q_1}, m_{Q_2}, m_t; M_4, \Lambda)^2\;. \tag{11}$$

[1)] Its five-light-quark version is $M_5 = (\; 240 \;\pm\; {}^{170}_{100}\;)$ MeV with $m_b = 4.9$ GeV.

The recent experimental values for the input parameters are

$$m_W = (\ 80.0\ \pm\ 0.6\ )\ \text{GeV}\ ,^{[10]}$$

$$g_2(m_W)\ :\ \alpha_2(m_W) = \frac{g_2(m_W)^2}{4\pi} \quad = \ 0.0344 \pm 0.0007\ ,^{[11]} \tag{12}$$

and M_4 given in eq. (10). We take Λ to be 10^{15} GeV for calculation, but it makes no significant change, even if it is 10^{12} GeV or 10^{19} GeV. Remaining unknown masses are m_{Q_1}, m_{Q_2} and m_t to which eq. (11) imposes a constraint.

There is another experimental restriction coming from the ρ parameter,

$$\rho \ \equiv\ \frac{{F_{\pi^\pm}}^2}{{F_{\pi^0}}^2}\ , \tag{13}$$

with

$${F_{\pi^0}}^2 = \frac{N_c}{8\pi^2}\int_{m_{Q_1}^2}^{\Lambda^2} dx\ x \left[\frac{\Sigma_1^2 + \frac{x}{4}(\Sigma_1^2)'}{(x+\Sigma_1^2)^2} + \frac{\Sigma_2^2 + \frac{x}{4}(\Sigma_2^2)'}{(x+\Sigma_2^2)^2}\right] \ . \tag{14}$$

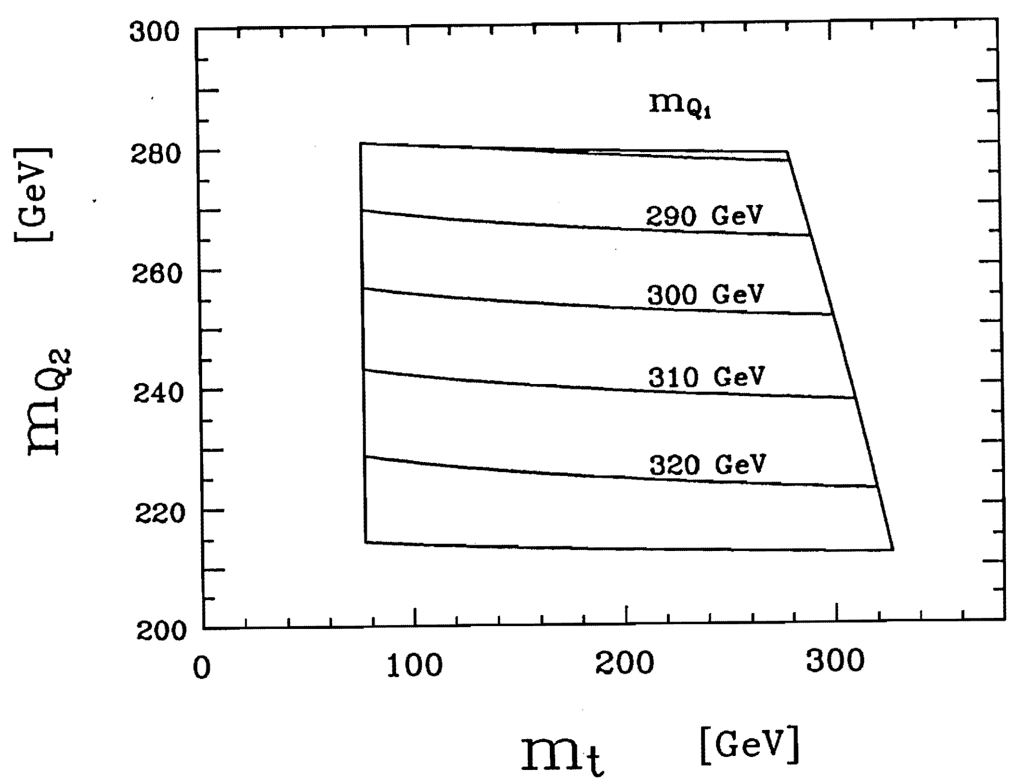

Fig. 2 The allowed region of three masses m_{Q_1}, m_{Q_2} and m_t. Here m_{Q_1} is represented by the iso-mass lines in the plane of m_{Q_2} and m_t.

Its experimental value is

$$\rho \; = \; 0.998 \; \pm \; 0.0086 \; . \quad (15)$$

[11]

Three quark masses must be chosen to reproduce this value.

The contour map of m_{Q_1} is shown in fig. 2. The weak dependence of m_{Q_1} on m_t comes through eq. (8). The upper bound of the graph corresponds to $m_{Q_1} \geq m_{Q_2}$, the right one to $m_{Q_1} \geq m_t$ (assumed), the lower one to the ρ parameter and the left boundary to the recent $p\bar{p}$ collider experiment at Fermi Lab.[12]

The allowed ranges of the masses are

$$\begin{aligned} m_{Q_1} &: \; 280 \sim 330 \text{ GeV} \,, \\ m_{Q_2} &: \; 210 \sim 280 \text{ GeV} \,, \\ m_t &: \; 77 \sim 330 \text{ GeV} \,. \end{aligned} \quad (16)$$

The uncertainties caused by the errors of input data in eqs. (10) and (12) are about 10 GeV for each of boundaries except for the lower limit of the top quark mass.

These predictions are consistent with the lower limit of the sextet quark mass given by the recent experiment: $m_Q \geq 84$ GeV (95% C.L.) for the long lifetime case.[13]

IV. Quantum Number Assignments for Sextet Quarks

Now we discuss quantum number assignments for Q briefly. For this purpose, we need a new assumption in addition to the ones given before:

- The sextet quarks can decay into the ordinary quarks and/or leptons via $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions.

This requirement may protect our scenario from the cosmological constraints. Although there are numerous sets of quantum numbers and decay modes of Q satisfying the present assumptions, we choose the simplest solution among many possibilities:

∘ The sextet quarks belong to $\underline{\mathbf{6}}^*$ and the anti-sextet-quarks belong to $\underline{\mathbf{6}}$.

- The sextet quarks are an $SU(2)_L$ doublet.
- The charges of the sextet quarks are $+2/3$ for U and $-1/3$ for D, respectively, just as the ordinary quarks.
- The decay modes of the sextet quarks are

$$Q \longrightarrow q + q + \overline{q} \qquad \text{or} \qquad Q \longrightarrow \overline{q} + \overline{q} + \overline{\ell}\,. \tag{17}$$

  The baryon number and the total lepton number of Q are $1/3$ and 0 for the former mode, respectively, $-2/3$ and -1 for the latter mode, respectively, if they are conserved. The heavier sextet quark Q_1 decays into the lighter one through the weak interaction, too. (If the mass splitting permits, real $W^{\pm}$ will be produced.)
- It is needed two additional iso-doublet leptons for the chiral anomaly cancellation. The additional leptons are sequential, i.e. the same quantum numbers as the ordinary leptons except for the lepton numbers associated with the generation. (These additional neutrinos must have masses heavier than 45 GeV to be consistent with the Z-decay experiments.[14])

Any additonal quarks will violate the QCD asymptotic freedom.

Possible bound states are $\overline{Q}Q$, $\overline{Q}qg$, etc. for bosons and QQQ, Qqq ,etc. for fermions where g represents gluon. Note that the charges of these bound states are integral.

V. Conclusion

The consequences of the dynamical electroweak symmetry breaking by the color-sextet-quark condensate with the four-fermion interactions is presented. Due to the large anomalous dimension of the composite operator, the four-fermion operators like $\overline{Q}Q\overline{\psi}\psi$ become relevant and the condensate $\overline{Q}Q$ play the role of the Higgs field. The masses of two color-sextet iso-doublet quarks are 280$\sim$330 Gev and 210$\sim$280 GeV. The simplest quantum number assignments is that the sextet quarks belong to $\underline{6}^*$ of $SU(3)_C$ and have charge $+2/3$ and $-1/3$ for the iso-spin up and down particle, respectively.

In this talk, we have given just a sketch of our work. For the details, please, see ref. [0].

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