Superstring Phenomenology

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Abstract

Realistic models from the heterotic superstring theory are reviewed. The emphasis is put on the connection between the structure of compactified manifolds and the four-dimensional effective theory. It is pointed out that the four-generation model associated with the maximally symmetric Calabi-Yau compactification is phenomenologically viable.

1. Calabi-Yau manifolds and effective theories

It is expected that the superstring theory provides a unified framework not only for all known fundamental interactions including gravity but also for matter and space-time.[1)2)] In the heterotic superstring theory in which "elementary particles" are not point-particles but tiny closed strings with their sizes of $O(M_{pl}^{-1})$, all of string states are represented as direct products of the left-moving 26-dimensional bosonic string and the right-moving 10-dimensional superstring. We are led to the $E_8 \times E_8'$ ten-dimensional superstring theory in the case of flat setting for ten-dimensional space-time. It is of great importance to explore the characteristic structure of the low-energy effective theory which follows from the superstring theory and to make a step towards the confrontation of superstring theory with experimental physics. We have already known some important properties of the standard model which are required for the low-energy effective theory as a "realistic" model. At present it is far from clear that one can find a "realistic" superstring model fulfillig all the requirements. The superstring theory and low-energy effective theory are closely connected with each other through the compactification. The topological and geometrical properties of the compactified manifold have an essential influence upon the low-energy effective theory.

In order that we get N=1 space-time supersymmetry in the four-dimensional effective theory, it is required that the ten-dimensional space-time is decomposed as $M_4 \times K$ where M_4 is four-dimensional Minkowsky space and that the six-dimensional compactified manifold K is a Ricci-flat ($SU(3)$ holonomy) Kähler manifold (Calabi-Yau manifold,[3] orbifold[4]). Furthermore, to obtain an anomaly-free theory, we introduce $SU(3)$ background gauge configuration in the first E_8 group and identify it with $SU(3)$ spin connection of K (the standard embedding). The second E_8' group is considered to belong to the hidden sector. The available gauge group G in this theory is E_6 for a simply connected Calabi-Yau manifold $K = K_0$ but becomes a subgroup of E_6 for a multiply connected manifold $K = K_0/G_d$ with a discrete symmetry G_d. If K is multiply connected, non-trivial Wilson loops $U(\gamma)$ on K cause further breaking of E_6, where γ is a non-contractible path on K.[5)6)] The non-trivial $U(\gamma)$ composes of the discrete symmetry $\overline{G_d}$, which is a homomorphic embedding of G_d into E_6. Then the available gauge group G is determined as $G = \{g \mid U \in \overline{G}_d, [g, U] = 0\}$. The $U(\gamma)$ means an Aharonov-Bohm phase in the non-abelian version. Only in the case of non-trivial $U(\gamma)$ realistic gauge hierarchies are possibly realized and we have rank$G$=5 or 6.[7)]

The four-dimensional massless fields are given by coefficient functions in six-dimensional harmonic form expansion of the ten-dimensional fields.[8)] For the sake of simplicity, if we take a simply connected Calabi-Yau manifold, the ten-dimensional super Yang-Mills fields (**248** in E_8) are decomposed in terms of $E_6 \times SU(3)$ as

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}; A_\mu) \oplus (\mathbf{27}, \mathbf{3}; \Phi) \oplus (\mathbf{27^*}, \mathbf{3^*}; \bar{\Phi}) \oplus (\mathbf{1}, \mathbf{8}; E). \tag{1}$$

$(\mathbf{78}, \mathbf{1}; A_\mu)$ represents the four-dimensional super Yang-Mills fields. $(\mathbf{27}, \mathbf{3}; \Phi)$, $(\mathbf{27^*}, \mathbf{3^*}; \bar{\Phi})$ and $(\mathbf{1}, \mathbf{8}; E)$ stand for chiral superfields whose numbers are h^{21}, h^{11}

and $\dim H^1(\mathrm{End}T_K)$, respectively. Here h^{pq}'s are topological invariants of K called the Hodge numbers, whereas $\dim H^1(\mathrm{End}T_K)$ depends on the geometry of K. The **27** chiral superfield Φ contains **16**, **10** and **1** representations in $SO(10)$ and each of them comprises the following fields.

$$\begin{aligned} \mathbf{16} &: Q, \bar{u}, \bar{d}, l, \bar{e}, \bar{\nu}, \\ \mathbf{10} &: g, \bar{g}, h, h', \\ \mathbf{1} &: S, \end{aligned} \tag{2}$$

where Q and l are $SU(2)_L$-doublet quarks and leptons, respectively. g and $\bar{g}$ belong to **3** and **3*** in $SU(3)_c$ and to singlets in $SU(2)_L$. h, h' and S are Higgs-doublets and Higgs-singlet respectively. It should be noted that **27** (**27***) does not contain the Higgs fields in the adjoint representations of the gauge group. The ten-dimensional supergravity multiplet is also decomposed according as $M_4 \times K$. From this supermultiplet we derive three types of chiral superfields denoted by D, R and C.[9] The scalar component of D is related to a gauge coupling at the compactification scale M_c as $\langle \Re D \rangle = g^{-2}$. For R there are h^{11} fields called Kähler class moduli fields and $\langle \Re R_j \rangle$ $(j = 1, 2, \cdots, h^{11})$ give the size of K. And we have h^{21} C_j fields called complex structure moduli fields and their vev's $c_j = \langle C_j \rangle$ give complex structure moduli parameters of K. The imaginary parts of the scalar components of D and R_j behave as axion-like fields.[9] Thus we must consider the gauge hierarchy and also the low energy effective theory only within these ingredients which are determined by the topological structure of the compactified manifold K. This is in sharp contrast to the ordinary GUT models, in which the generation number and Higgs fields are introduced arbitrarily.

In general on the manifold K there can exist some discrete symmetries other than G_d. Such a symmetry H, which is also the symmetry of K_0, is given by[5]

$$H = \{h \mid \forall g \in G_d, \exists g' \in G_d \quad \text{s.t.} \quad hgh^{-1} = g'\}. \tag{3}$$

This discrete symmetry varies depending on the values of $c_j = \langle C_j \rangle$. The matter fields transform among themselves under the action of the discrete symmetry.

Therefore, non-vanishing couplings are restricted to invariant combinations of matter fields which respect the discrete symmetry. The discrete symmetry plays an important role in issues of low energy physics. In fact, the intermediate energy scale and Yukawa couplings are controlled by the discrete symmetry considerably.

2. General features of the effective theory

In order to get realistic gauge hierarchies, it is required that the gauge group G at the compactification scale M_c is smaller than E_6. Then the manifold K should be multiply connected. This is also in accord with a relatively small "low-energy" generation number. As mentioned above, we have h^{21} **27** chiral superfields Φ and h^{11} **27*** chiral superfields $\bar{\Phi}$ at the scale M_c. The discrete symmetry of K prevents Φ and $\bar{\Phi}$ to gain masses in pair. Thus there exist $(h^{21} + h^{11})$ families of chiral superfields at the scale M_c. When the discrete symmetry breaks down spontaneously at the intermediate energy scale M_I less than M_c, Φ and $\bar{\Phi}$ possibly get masses in pair at M_I or less than M_I. As a consequence, the "low-energy" generation number becomes $|h^{21} - h^{11}|$.

In the effective theory from the superstring theory there possibly exists the intermediate energy scale M_I which is determined by minimizing the scalar potential. The superpotential is expressed as

$$W = \lambda\Phi^3 + \bar{\lambda}\bar{\Phi} + \sum_{n=2}^{\infty} \lambda^{(n)} M_c^{3-2n} (\Phi\bar{\Phi})^n + \cdots \tag{4}$$

in the M_c^{-1} expansion. Here we write down explicitly the terms in which only Φ and $\bar{\Phi}$ participate. Coupling constants λ, $\bar{\lambda}$ and $\lambda^{(n)}$ are all dimensionless and of order one. The non-renormalizable terms in the superpotential are of great importance in determining the string vacuum. In particular, the minimum value of n for the non-vanishing $\lambda^{(n)}$'s denoted by p is directly related to the intermediate scale M_I. By minimizing the scalarpotential containing the soft supersymmetry breaking term, we obtain the scale M_I as

$$M_I \equiv \langle S \rangle = \langle \bar{S} \rangle \sim M_c \Big(\frac{M_s}{M_c}\Big)^{1/(2p-2)}, \qquad (5)$$

where we take $M_s \sim 10^3$ GeV so as to maintain the supersymmetry down to a TeV scale. The value of p is settled depending on the discrete symmetry of the manifold K.[10] In generic Calabi-Yau compactification, we have $p = 2$ and then $M_I \sim 10^{10}$ GeV. However, if the Calabi-Yau manifold has a certain high discrete symmetry, one can be led to $p = 3, 4, \cdots$. In the case $p = 3$, we obtain $M_I \sim 10^{14}$ GeV, and for $p = 4$ we have $M_I \sim 10^{15.5}$ GeV. As will be discussed later, the large M_I associated with $p \geq 4$ is preferable for the proton stability. Only the Calabi-Yau manifold with special discrete symmetries leads to $p \geq 4$.

As mentioned before, the gauge group G at the scale M_c is determined via the Wilson-loop mechanism. If there exists the intermediate scale M_I, the non-vanishing vev $\langle S \rangle = \langle \bar{S} \rangle$ causes further breaking of G into G' at M_I. Since the fields S and $\bar{S}$ should not belong to the adjoint representation of G, the path of the symmetry breaking $G \to G'$ is severely limited. This is one of the important phenomenological implications from the superstring theory.[7]

The perturbative unification of gauge coupling constants are possible only for the cases $N_f = 3$ or 4 under specific gauge hierarchies and also under small numbers of the mater contents. This is due to the fact that we have many extra-matterfields other than quarks and leptons. In the framework of the perturbative unification non-abelian gauge couplings coincide with each other at the scale M_c . For abelian gauge couplings, however, the situation is quitely different. Since there potentially exists the kinetic-term mixing among two or more than two $U(1)$ gauge fields, these $U(1)$ couplings are not always unified even at the scale M_c.[11][12]

Now we discuss the problem of the proton stability. Baryon number non-conserving processes take place through gauge interactions (leptoquark gauge boson X-exchange) and/or Yukawa interactions (g, $\bar{g}$-exchange). The probability for these processes directly depends on masses of the leptoquark particles $(X, g, \bar{g})$. Experimentally, the most stringent lower bound for the proton lifetime is[13]

$$\tau(p \to \bar{\nu} K^+) \geq 7 \times 10^{31} \mathrm{yr}. \qquad (6)$$

The present limit places the conditions on masses[14)]

$$M_X, M_{g,\bar{g}} \geq 10^{16}\text{GeV} \tag{7}$$

Since leptoquark gauge bosons X can not get masses at the scale M_I via the Higgs mechanism, X should be massive at the scale M_c. On the other hand, g and $\bar{g}$ in **27** chiral superfields gain masses $M_{g,\bar{g}} \sim \lambda M_I$, where λ stands for the Yukawa coupling and $\lambda = 0(1)$. Therefore, the large intermediate scale scenario ($M_I \geq 10^{15.5}$ GeV with $p \geq 4$) is in line with the above phenomenological bound. Conversely, it is required that a realistic Calabi-Yau manifold should possess a special discrete symmetry which implies $p \geq 4$.

3. The three-generation model

The three-generation Calabi-Yau manifold is constructed using the Tian-Yau manifold which is an algebraic hypersurface defined as the zero locus of three polynomials in $CP^3 \times CP^3$.[15)] In terms of the homogeneous complex coordinates x_i and y_i ($i = 0, 1, 2, 3$) for each CP^3, we introduce the defining polynomials P_1, P_2 and P_3 with (3,0), (0,3) and (1,1) bi-degree, respectively. Three defining polynomials should satisfy the transversality condition

$$dP_1 \wedge dP_2 \wedge dP_3 \neq 0. \tag{8}$$

The Tian-Yau manifold (K_0) is a simply connected manifold with $h^{21} = 23$ and $h^{11} = 14$. If the defining polynomials are of the form

$$
\begin{aligned}
P_1 &= \sum_{i=0}^{3} x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3, \\
P_2 &= \sum_{i=0}^{3} y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3, \\
P_3 &= \sum_{i=0}^{3} c_i x_i y_i + c_4 x_2 y_3 + c_5 x_3 y_2,
\end{aligned} \tag{9}
$$

we have a freely acting Z_3 symmetry

$$
\begin{aligned}
(x_0, x_1, x_2, x_3) &\rightarrow (x_0, \alpha^2 x_1, \alpha x_2, \alpha x_3), \\
(y_0, y_1, y_2, y_3) &\rightarrow (y_0, \alpha y_1, \alpha^2 y_2, \alpha^2 y_3),
\end{aligned} \tag{10}
$$

where $\alpha^3 = 1$ and a_i, b_i and $c_i (c_0 = 1)$ are complex parameters. In this case a multiply connected Calabi-Yau manifold K can be constructed as $K = K_0/Z_3$ and we obtain $h^{12}(K) = 9$ and $h^{11}(K) = 6$.[16]

In this compactification the gauge group at the scale M_c becomes $G = SU(3)_c \times SU(3)_L \times SU(3)_R$. The chiral superfield Φ is decomposed as

$$
\mathbf{27} = (\mathbf{1}, \mathbf{3}, \mathbf{3}^*) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}^*, \mathbf{1}, \mathbf{3}^*). \tag{11}
$$

In this model there are $9 (= h^{21})$ $\Phi(\mathbf{1}, \mathbf{3}, \mathbf{3}^*)$'s and $6 (= h^{11})$ $\bar{\Phi}(\mathbf{1}, \mathbf{3}^*, \mathbf{3})$'s, whereas we have 7 colored Φ's and 4 colored $\bar{\Phi}$'s. Due to the discrete symmetry of K all these fields remain massless at the scale M_c. When the discrete symmetry breaks down spontaneously at the scale M_I, Φ and $\bar{\Phi}$ gain masses in pair and then the "low-energy" generation number becomes 3. This model implies $p = 2$ for the non-renormalizable terms in the superpotential. This means that the proton lifetime becomes too short. Furthermore, since we have too many matter fields above the scale $M_I \sim 10^{10}$ GeV, there are no solutions for the perturbative unification in renormalization group analysis. Therefore, the three-generation model is not realistic.

Although the Calabi-Yau compactification guarantees N=1 space-time supersymmetry at tree level, Ricci-flatness is not generally preserved for higher order corrections.[17)] To keep the manifold Ricci-flat at all order it is necessary to impose conformal invariance to the theory. Gepner has constructed such fully consistent theories algebraically by tensoring the N=2 minimal superconformal theories with the correct trace anomaly.[18)] Recently, it has been found the fundamental connection between the algebraic construction and the geometry by using the Landau-Ginzburg theory and the singularity theory.[19)] When we apply the connection to the three-generation case which is brought about algebraically by $1 \cdot 16^3$-model, the compactified manifold K is given by $K_0/Z_3 \times Z_3$, where K_0 is the Schimmrigk manifold, i.e.

$$P_1 = \sum_{i=0}^{3} z_i^3 = 0, \qquad P_2 = \sum_{i=1}^{3} z_i w_i^3 = 0, \tag{12}$$

in $CP^3(z_i) \times CP^2(w_j)$.[20)] Although this manifold K has the same Hodge numbers as the Calabi-Yau manifold mentioned above, this manifold possesses conical singularities and is different from the previous one. It is not clear whether or not the three-generation Calabi-Yau manifold mentioned above is conformally invariant.

4. The four-generation model

The four-generation Calabi-Yau manifold is constructed geometrically as $K = K_0/G_d$ with $K_0 = Y(4;5)$ and $G_d = Z_5 \times Z_5'$. $Y(4;5)$ is the manifold defined as the zero locus of the fifth-order polynomials in CP^4.[5)] The defining polynomial $P(z)$ is given by

$$P(z) = \frac{1}{5}\sum_{i=1}^{5} z_i^5 - \sum_{j=0}^{4} c_j P_j(z) = 0 \tag{13}$$

and

$$P_0(z) = z_1 z_2 z_3 z_4 z_5,$$
$$P_1(z) = \frac{1}{5}\sum_{i=1}^{5} z_i^3 z_{i+2} z_{i-2},$$
$$P_2(z) = \frac{1}{5}\sum_{i=1}^{5} z_i z_{i+2}^2 z_{i-2}^2,$$
$$P_3(z) = \frac{1}{5}\sum_{i=1}^{5} z_i z_{i+1}^2 z_{i-1}^2,$$
$$P_4(z) = \frac{1}{5}\sum_{i=1}^{5} z_i^3 z_{i+1} z_{i-1},$$

where $z_i (i = 1 \sim 5)$ are the complex coordinates of CP^4. The c_j's are complex parameters, which are vacuum expectation values of the complex structure moduli fields C_j.[9] For this defining polynomial the simply connected Calabi-Yau manifold K_0 has freely acting discrete symmetries

$$\begin{aligned} &S; \quad z_i \longrightarrow z_{i+1} \\ &T; \quad z_i \longrightarrow \alpha^i z_i \qquad (\alpha^5 = 1) \end{aligned} \tag{14}$$

for $\sum c_j \neq 1$. Then, by taking as $G_d = Z_5(S) \times Z_5'(T)$, we can construct the multiply connected Calabi-Yau manifold $K = K_0/G_d$ with $h^{21} = 5$ and $h^{11} = 1$.

In the moduli parameter space with respect to c_j's, there is a special point on which the theory possesses a high discrete symmetry. In fact, for K_0 we have a maximal discrete symmetry $S_5 \times Z_5^5/Z_5$ at $c_j = 0$ for all j, in which case K_0 is just the manifold given by 3^5-model in the algebraic approach.[21)22)] Here we take up the maximally symmetric Calabi-Yau manifold. In the compactification, according to the charge conservations coming from the discrete symmetries, the structure of the superpotential is strongly constrainted.[23)]

In this model the gauge group G at the scale M_c becomes $SU(3)_c \times SU(2)_L \times SU(2)' \times U(1)^2$ or $SU(4) \times SU(2)_L \times U(1)^2$. Here we confine ourselves to the former case with $SU(2)' = SU(2)_R$ as the most interesting case. In this case we have 5 and 1 sets of h, h' and S in **27** and **27*** chiral superfields respectively. On the other hand, quarks, leptons, g and $\bar{g}$ appear in 4 sets for **27** but not for **27***. One of the most important phenomenological implications of this model is that the intermediate energy scale M_I becomes very large, i.e. $M_I \sim 10^{15.5}$GeV. This is due to the fact that the maximal discrete symmetry $S_5 \times Z_5^5/Z_5$ yields $p = 4$ for the non-renormalizable term in the superpotential.[10)]

The discrete symmetry constrains the dependence of the superpotential also on the complex structure moduli fields $C_j (j = 0 \sim 4)$. The Kähler class moduli field $R(h^{11} = 1)$ does not participate in the superpotential at the perturbation level, but appears as the suppression factor through the world-sheet instanton effect.[24)] For the unbroken supersymmetry the discrete symmetry implies $c_j = \langle C_j \rangle = 0$. When the supersymmetry breaks down via the soft supersymmetry breaking, the Calabi-Yau manifold is deformed spontaneously and we get $c_j \neq 0$. In fact, we have an interesting solution for the spontaneous deformation[25)]

$$c_1 = c_4 = O(1),$$

$$c_0, \quad c_2 = c_3, \quad \langle S \rangle = \langle \bar{S} \rangle \sim 10^{-2.5}. \tag{15}$$

As a result, all four sets of g and $\bar{g}$ chiral superfields gain masses in pair at the order of the scale M_I. This fact guarantees the stability of the proton. Furthermore, one light sector h and h' remains massless and the other sets get masses at M_I. This is in favor of the perterbative unification of gauge couplings for $SU(3)_c$ and $SU(2)_L$ at the scale M_c. Therefore, it appears that the four-generation model with the maximal discrete symmetry is phenomenologically viable.

5. Conclusion

The superstring theory implies that it is of great importance to clarify the fundamental structure of matter in close conjunction with the structure of space-time. Through the recent investigations we have the geometrical approach based on the topology and the geometry of the compactified manifold and also the algebraic approach based on N=2 superconformal field theory. By uniting both the approaches via the Landau-Ginzburg theory and the singurarity theory, we are in a position to do with many solutions for fully consistent superstring theories. Since we have no theoretical criterions to select among them, it is efficient for us to combine the theoretical requirements with phenomenological requirements. As a matter of fact, the realistic superstring theories must pass the some rigorous check points from phenomenological viewpoints. By studying the gauge symmetry and the matter contents in the effective theory, the possibilities of realistic models have been investigated. Furthermore, the problem of the proton stability requires that the manifold possesses a specific discrete symmetry which brings about the absence of $(\Phi\bar{\Phi})^2$- and $(\Phi\bar{\Phi})^3$-terms in the superpotential. The requirement of such a specific discrete symmetry confines the compactified manifolds to a considerably small extent. The interesting scenario for a realistic superstring model is the spontaneous deformation of the maximally symmetric Calabi-Yau manifold triggered by the soft supersymmetry breaking.

At present we need further studies of characteristic features of the effective theory from the superstring theory such as gauge couplings, Yukawa couplings, quark/lepton mass matrices, smallness of neutrino mass, weak CP violation and so on. For the future it is necessary to verify the superstring theory as a "grand unified theory" from the following two standpoints. The one is to reproduce the standard model as the low energy effective theory and to give a unified interpretation for the above issues. The other is to predict new phenomena beyond the standard model. In the present situation interesting experimental searches for the superstring theory are to look for a signature of extra electroweak gauge symmetries, superparticles, the proton decay,etc. Unfortunately, even if we get such an evidence, it does not always mean the direct experimental verification of the superstring theory because such phenomena are not peculiar to the superstring theory. However, it should be emphasized that the superstring theory is able to give the most unified interpretation and is theoretically the most persuasive.

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