Estimations of long-distance effects on $K^0 - \bar{K}^0$ mixing by chiral Lagrangian

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ABSTRACT

Long-distance contributions to $K_L - K_S$ mass difference are estimated with the chiral Lagrngian method proposed by Bardeen, Buras and Gérard. In this method the effects of strong interaction are divided into high energy part (above hadronic scale) and low energy part (below hadronic scale). The former is taken into account by using renormalization group eqations in quark picture, while the latter by chiral Lagrangian in meson picture.

Here estimations are made for single particle and two particles intermediate states which consist of the $U(3)$ pseudo scalar nonet. The sum of these contributions are estimated to be $8 \sim 36\%$ of the experimental value depending on the cutoff scale and s quark mass.

1. Introduction

K meson system is a very nice tool for the test of a model and the search of new physics. The studies of $K^0 - \bar{K}^0$ mixing, CP violation, rare decays and so on give us valuable informations on particle physics. However, we do not yet have a firm way of estimating strong interaction effects, which makes our analyses more or less uncertain. Here we evaluate long-distance effects on $K_L - K_S$ mass differnce by using the chiral Lagrangian method proposed by Bardeen, Buras and Gérard [1] as an approach to the estimation of strong interaction effects.

Following Wolfenstein [2] we express the long-distance effects in $K_L - K_S$ mass difference ΔM_K with two parameters, B and D.

$$\Delta M_K = B\Delta M_{box}^{VSA} + D\Delta M_K, \tag{1}$$

where ΔM_{box}^{VSA} is obtained from usual box diagram calculation with vacuum saturation approximation for the evaluation of the hadronic matrix element. [3] The parameter B shows the deviation from vacuum saturation approximation and D expresses the contriburion from hadron intermediate states such as π, η and $\pi\pi$. Bardeen, Buras and Gérad obtained $B = 0.66 \pm 0.10$ in thier approach. [4] We estimate the value of D following their approach with some modifications (inclusion of η' and so on) in this work.

2. Chiral Lagrangian

Our basic Lagrangian which express meson-meson strong interaction are given as [1, 5]

$$\begin{aligned}\mathcal{L}_0 =& \frac{f^2}{4}\left[\mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger) + r\,\mathrm{tr}(mU + U^\dagger m) - \frac{r}{\chi^2}\mathrm{tr}(m\partial^2 U + \partial^2 U^\dagger m)\right] \\ &+ (\text{mass terms for } \eta \text{ and } \eta'),\end{aligned} \tag{2}$$

where f, r and χ are constants, $m = diag(m_u, m_d, m_s)$ and

$$U = \exp[i\sqrt{2}\phi/f], \tag{3}$$

$$\phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^8}{\sqrt{6}} + \frac{\eta^0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^8}{\sqrt{6}} + \frac{\eta^0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta^8}{\sqrt{6}} + \frac{\eta^0}{\sqrt{3}} \end{bmatrix}. \tag{4}$$

This Lagrangian is different from that of Ref.1 in that $SU(3)$ singlet field η^0 are incorporated. The fourth term is necessary to obtain realistic masses and mixing of η and η'. We assume that it comes purely from QCD instanton effect [6] and gives no contribution to the weak currents which will be discussed later.

3. Incorporation of weak interaction

To incorporate weak interaction we first construct low energy effective Hamiltonian which are expressed as a sum of products of weak currents written in quarks. Then we translate these curents into meson picture.

The low energy $\Delta S = 1$ effective Hamiltonian are given in the large N (number of color) approximation as [1]

$$\mathcal{H}_{eff} = -(G_F/\sqrt{2}) \sum_{i=1,2,6} R_i Q_i + (\text{h.c.}), \tag{5}$$

where

$$\begin{aligned} &Q_1 = \bar{s}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)u, \qquad Q_2 = \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d, \\ &Q_6 = -8 \sum_{q=u,d,s} \bar{s}_L q_R \bar{q}_R d_L, \end{aligned} \tag{6}$$

and $R_i = z_i + \{s_2^2 + (s_2 s_3 c_2/c_1 c_3)e^{-i\delta}\}y_i$. The coefficients z_i and y_i are calculated from renormalization group equations. The effect of strong interaction from high energy ($\sim M_W$) to hadronic scale is contained in these coefficients.

Let us translate the operators Q_1, Q_2 and Q_6 into meson picuture. Suppose that $U_L(3) \times U_R(3)$ flavor symmetry is gauged, and introduce an external scalar field κ which transforms in the same way as U. The Lagrangian for u, d and s quarks are given as follws.

$$\mathcal{L} = i\bar{q}D_\mu\gamma^\mu q - \bar{q}_L\kappa q_R - \bar{q}_R\kappa^\dagger q_L, \tag{7}$$

where D_μ is the covariant derivative. We have mass terms of quarks in the limit $\kappa \to m$. The Lagrangian of U, Eq.(2), is also gauged and the mass m is replaced by κ. Then comparing the couplings of $U_L(3) \times U_R(3)$ gauge fields and κ to U

with those to quarks, we obtain

$$\bar{q}_L^a \gamma_\mu q_L^b = \frac{if^2}{4}[2\partial_\mu U U^\dagger + \frac{r}{\chi^2}(\partial_\mu U m - m\partial_\mu U^\dagger)]_{ba}, \tag{8}$$

$$\bar{q}_L^a q_R^b = -\frac{f^2}{4} r [U^\dagger - \frac{1}{\chi^2}\partial^2 U^\dagger]_{ba}. \tag{9}$$

By using these expressions we can obtain the $\Delta S = 1$ effective Hamiltonian written in mesons.

4.Estimations of long-distance contributions

With the $\Delta S = 1$ effective Hamiltonian obtained in the last section we can read off the vertices; $K^0 \to \pi^0,\ \eta,\ \eta',\ \pi\pi,\ \pi\eta$, and so on. Then we calculate Feynman diagrams in Figs 1~3. The following points should be noted.

1. We think that strong interaction corrections are already taken into account in the Lagrangian, Eq.(2). Only the corrections from weak interaction are considerd below. (Corrections from meson-meson strong interaction vertices are partly given in Ref.1, which we do not adopt here.)
2. Loop calculations have divergences, so we need regularization. Naive cutoff method is used in Ref.1, but it spoils Lorentz invariance as is well known. Here we adopt Pauli-Villars reguralization scheme: Propagator is modeified as follows,

$$\frac{1}{p^2 - m^2} \to \frac{1}{p^2 - m^2} - \frac{1}{p^2 - \Lambda^2}. \tag{10}$$

3. Derivative couplings give $O(\Lambda^4)$ terms. Chiral symmetry forbids quartic divergences. These are cancelled by the extra term which appear in the proper treatment of the quantization of the Lagrangian with drivative couplings. [7]

In the case of following Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a D^{ab}(\varphi)\partial^\mu \varphi_b - V(\varphi), \tag{11}$$

effective Lagrangian is given as

$$\mathcal{L}_{eff} = \mathcal{L} - \frac{i}{2}\delta^4(0)\ln(\det D). \tag{12}$$

4. The constants f and χ are determined from physical pion and kaon decay constants with the following relations;

$$F_\pi = f(1+\frac{m_\pi^2}{\chi^2}), \qquad F_K = f(1+\frac{m_K^2}{\chi^2}). \tag{13}$$

We get $f = 91$ MeV and $\chi = 890$ MeV. The constant r is related to meson and quark masses in the following way;

$$rm_s = 2m_K^2 - m_\pi^2, \tag{14}$$

where $SU(2)$ limit $m_u = m_d$ is taken. For $m_s = 150 \sim 250$ MeV we get $r = 2 \sim 3$ GeV.

The results of calculations are so lengthy and complicated that we do not give them here but numerical results of contributions to D parameter are shown in Table 1. (Details of the calculations will be given elsewhere. [8])

5. discussions

The results in Table 1 show us that the long-distance contributions give 8~36 % of the experimental value of $K_L - K_S$ mass difference for realistic masses of η and η'. Dependence on the parameter r is large, while the cutoff Λ dependence is not so significant for $\Lambda = 0.8 \sim 1$ GeV. The effect of Q_6 operator is very important. We find that $\pi\pi$ intermediate state gives major contribution.

Before closing this paper we give some problems to be discussed in future. We did not estimate 3π state contributions since it requires complicated 2 loop calculations. It may give not a little contribution as 3π state is the main decay mode of K_L. We have to study reguralization dependence of our calculations. We should invent a nice scheme where cutoff dependence is just cancelled by the scale dependence of the Wilson coefficients in the $\Delta S = 1$ effective Hamiltonian.

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		(a)	(b)	(c)	(d)
$Fig.1$	π^0	0.21	0.13	0.12	0.07
	η	−0.24	−0.16	−0.19	−0.15
	η'	−0.006	−0.001	−0.0007	−0.012
	sum	−0.04	−0.03	−0.07	−0.09
$Fig.2$		−0.16	−0.10	−0.16	−0.16
$Fig.3$	$\pi^+\pi^-$	0.25	0.26	0.16	0.11
	K^+K^-	−0.02	−0.04	−0.01	−0.011
	$\pi^0\pi^0$	0.26	0.23	0.14	0.07
	$\eta\eta$	−0.01	−0.02	−0.008	−0.005
	$\eta'\eta'$	0.01	0.0006	0.01	0.006
	$\pi^0\eta$	0.05	0.06	0.02	0.01
	$\pi^0\eta'$	−0.01	0.002	−0.007	−0.008
	$\eta\eta'$	−0.02	0.004	−0.004	−0.004
	sum	0.51	0.49	0.30	0.17
	total	0.31	0.36	0.08	−0.09

Table 1

Contributions to D parameter from each processes. The parameters are taken as follows in each cases:

$$m_\pi = 137,\ m_K = 496,\ m_\eta = 549,\ m_{\eta'} = 958\ (\mathrm{MeV}),\ \sin\theta_{\eta-\eta'} = 0.182.$$

(a) $r = 3$ GeV, $\Lambda = 0.8$ GeV.

(b) $r = 3$ GeV, $\Lambda = 1$ GeV.

(c) $r = 2$ GeV, $\Lambda = 0.8$ GeV.

(d) No Q_6 contribution. $\Lambda = 0.8$ GeV.

FIGURE CAPTIONS

1. **Single particle intermediate states contributions.**
2. **Corrections to $\Delta S = 2$ effective Hamiltonian.**
3. **Two particles intermediate states contributions.**

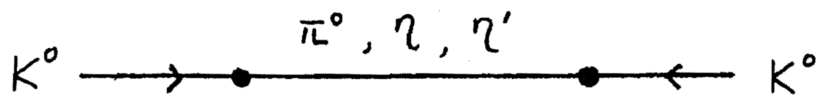

Fig. 1

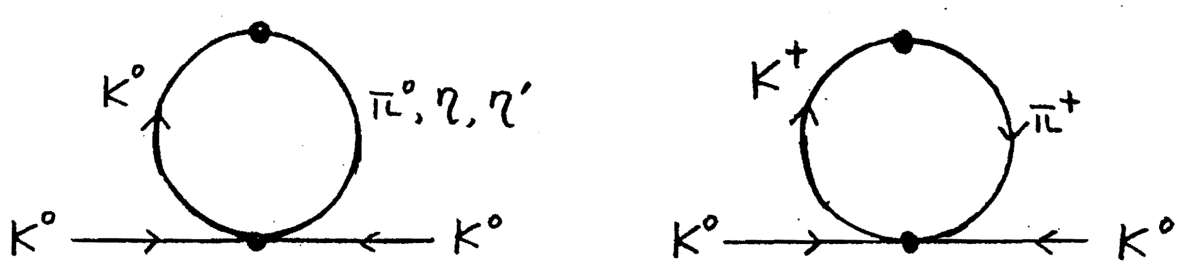

Fig. 2

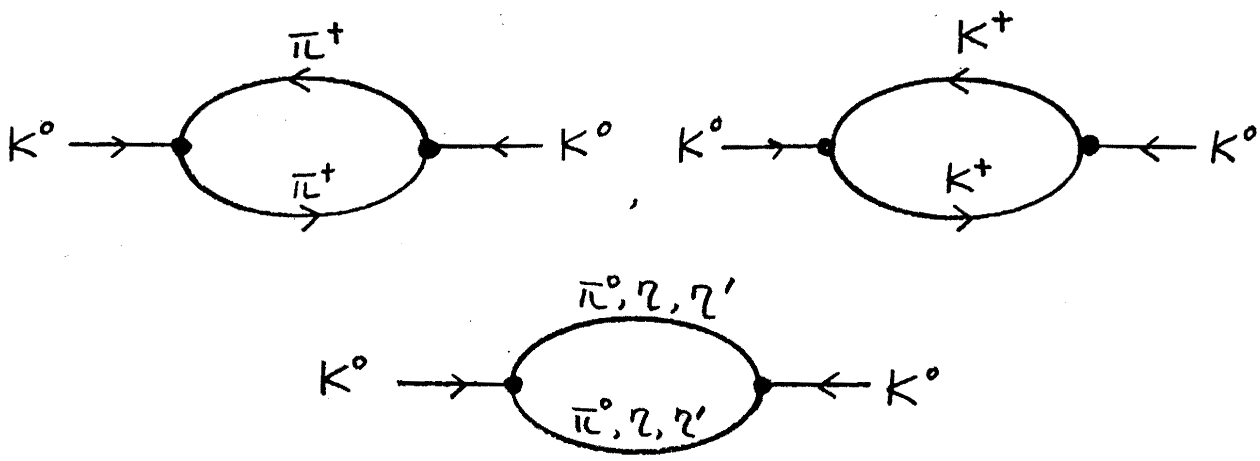

Fig. 3