PROTON DECAY THEORY

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1. Coupling Constants Update

The standard $SU(3)\times SU(2)\times U(1)$ model of strong and electroweak interactions contains 18 independent couplings and masses. Grand Unified Theories (GUTS) correlate the three gauge couplings $g_3$, $g_2$, and $g_1$ by embedding the standard model in a compact simple group such as $SU(5)$, $SO(10)$, $E_6$ etc. Indeed, the high degree of symmetry naturally renders the bare couplings equal, explains charge-color quantization and promotes $\sin^2\theta_W$ from an infinite counterterm parameter to a rational number (generally 3/8). Unfortunately, GUTS have so far provided little new insight regarding the 15 mass and quark mixing parameters. Therefore, although GUTS represent a significant theoretical advancement, they cannot be the final word.

Before surveying proton decay predictions of GUTS, I will update the gauge coupling values. Those quantities are extremely important because they provide much of the basis for our belief in GUTS and a severe constraint on model building. In the case of the QCD coupling, the situation has not changed significantly during the last few years. Upsilon decays and high energy jet data are consistent with

$$A_{\text{MS}}^{(1)} \simeq 150^{+150}_{-75} \text{MeV}$$

(1.1)

Assuming $m_t \simeq 40 \text{GeV}$ and using $m_W = 80.8 \text{GeV}$ (UA1 and UA2 average), that range leads to

$$\alpha_3 (m_W) = 0.107^{+0.013}_{-0.008}$$

(1.2)

The analogous electroweak parameters, also defined by $\overline{\text{MS}}$ (modified minimal subtraction) have the short-distance values

$$\alpha^{-1} (m_W) = 127.76 \pm 0.30$$

(1.3)

$$\sin^2 \theta_W (m_W) = 0.228 \pm 0.004$$

(1.4)

It should be noted that the world average for the weak mixing angle in Eq. (1.4) has increased from last year's $0.219 \pm 0.006$ value primarily because of more precise deep-inelastic $\mu\nu$ scattering data and refinements in the $W^\pm$ and $Z$ mass determinations. That higher value has very important implications for GUTS, as we shall see.

Employing the relationships

$$\alpha_1 (m_W) = 5 \alpha (m_W) / 3 \cos^2 \theta_W (m_W)$$

(1.5a)

$$\alpha_2 (m_W) = \alpha (m_W) / \sin^2 \theta_W (m_W)$$

(1.5b)

leads to the gauge coupling values

$$\alpha_1 (m_W) = 0.0169 \pm 0.0001$$

(1.6a)

$$\alpha_2 (m_W) = 0.0343 \pm 0.0006$$

(1.6b)

Last year, the central value of $\alpha_2 (m_W)$ was 0.036. Assuming that there are no other new thresholds between the standard model's mass scale of $m_W$ and the grand unification scale of $m_X$, one can evolve the gauge couplings to higher energies using

$$\mu \frac{d}{\partial \mu} \alpha_i (\mu) = b_i \alpha_i^2 + \ldots, i = 1, 2, 3$$

(1.7a)

and the values of $\alpha_i (m_W)$ given above. If the three couplings meet at a single point, that would be clear evidence for grand unification. Last year when $\alpha_2 (m_W) \simeq 0.036$, they tended to meet near $\mu \sim 2 \times 10^{14} \text{GeV}$. That meeting was taken as strong confirmation of GUTS and perhaps an indication of no new physics thresholds at low or intermediate mass scales. Using the new value for $\alpha_2 (m_W)$ in Eq. (1.6b), one finds that is no longer the case. The couplings $\alpha_1 (\mu)$ and $\alpha_3 (\mu)$ continue to meet near $1.5 \times 10^{14} \text{GeV}$; however, $\alpha_2 (\mu)$ now crosses $\alpha_1 (\mu)$ at $1.5 \times 10^{13} \text{GeV}$ and meets $\alpha_3 (\mu)$ at $1.0 \times 10^{16} \text{GeV}$. Is grand unification ruled out? No, this development merely implies that new physics thresholds between $m_W$ and $m_X$ must change the evolution of the couplings such that they meet at a single value. In my opinion, the near equality of the couplings at high energies that we find using Eq. (1.7) should still be taken as a strong indication of grand unification. At issue is: What new physics rectifies the evolution and at what energy will it be manifested?

2. Minimal SU(5) Predictions

The original SU(5) Georgi-Glashow model provided much of the motivation for ongoing proton decay experiments as well as a theoretical framework for estimating expected rates and branching ratios. In the so-called minimal model, one assumes the existence of a great desert between $m_W$ and $m_X$, the unification mass scale. That simplistic assumption had an appealing consequence, it led to rather definite testable predictions. (The predictions hold in any GUT with a great desert.) Indeed, using $\alpha^{-1} (m_R) \simeq 127.76 \pm 0.30$ and $A_{\text{MS}}^{(0)} = 150^{+150}_{-75} \text{MeV}$, one predicts

$$m_X = (2.0^{+2.1}_{-1.6}) \times 10^{14} \text{GeV}$$

(2.1)

$$\sin^2 \theta_W = 0.214 \pm 0.004$$

(2.2)
Unfortunately, both of these predictions are now ruled out by experiment. As we shall see, proton decay bounds require \( m_X \gtrsim 7 \times 10^{14} \text{GeV} \), while the \( \sin^2 \theta_W (m_W) \) prediction conflicts with the world average in Eq. (1.4). The latter disagreement is, of course, just another quantitative way of describing the apparent lack of unification of gauge coupling when current \( \alpha_t (m_W) \) values are employed. These failures of the minimal SU(5) model do not rule out SU(5) as a viable grand unification group. They do indicate that new physics appendages in the form of additional scalars or fermions (perhaps supersymmetry) must be introduced to render \( m_X \geq 10^{15} \text{GeV} \) and increase the prediction for \( \sin^2 \theta_W (m_W) \). Another possibility is that a bigger GUT such as SO(10) or \( E_6 \) with intermediate stages of symmetry breaking must be employed. I subsequently describe how low energy supersymmetry may do the trick for SU(5) or any other GUT.

3. Gauge Boson Mediated Proton Decay

The SU(5) model\(^1\) contains a color triplet, \( SU(2)_L \) isodoublet of gauge bosons \( \left( X^{\pm \frac{3}{2}}, Y^{\pm \frac{1}{2}} \right) \) with \( m_X \approx m_Y \) which mediate proton decay. In higher rank groups SO(10), \( E_6 \) etc., a second color triplet, isodoublet \( \left( X'^{\pm \frac{3}{2}}, Y'^{\pm \frac{1}{2}} \right) \) with \( m_X' \approx m_Y' \) can also mediate such decays. (These are the only gauge bosons that mediate proton decay at the tree level.)

Virtual exchange of \( X, Y, X' \) and \( Y' \) bosons between quarks and leptons gives rise to the following to \( B \) and \( L \) violating (dimension 6) four Fermi Hamiltonian (B-L is conserved)\(^6\)

\[
H = \frac{g^2}{2m^2} \left( \frac{m_X^2 + m_Y^2}{m_X'^2} \right) A_{ijkl} \left[ \bar{\psi}_{kL} \gamma^\mu d_{iL} + \bar{\psi}_{kL} \gamma^\mu u_{iL} \right] + h.c.
\]

\[+\text{heavier generations}\]  

(3.1)

where \( g \) (of \( m_X \)) is the value of the gauge coupling at mass scale \( m_X \), \( A \approx 3 \) is an enhancement factor\(^6\) due to virtual gluon, \( W^\pm, \gamma \) and \( Z \) radiative corrections and \( r_e \approx 2m^2_{Z}/(m^2_X + m^2_{X'}) \). (In the SU(5) model one can set \( m_{X'} \to \infty \), \( r_e \approx 2 \)). Heavier generation interactions are obtained (for example) by the substitutions

\[
\bar{\psi}_{kL} \gamma^\mu d_{iL} \rightarrow \bar{\psi}_{kL} \gamma^\mu d_{iL} - \bar{\psi}_{kL} \gamma^\mu \tau^a s_{iL} + \bar{\psi}_{kL} \gamma^\mu u_{iL} \]

\[
r_{ei}\bar{\psi}_{kL} \gamma^\mu d_{iL} \rightarrow r_{ei}\bar{\psi}_{kL} \gamma^\mu d_{iL} - r_{ei}\bar{\psi}_{kL} \gamma^\mu \tau^a s_{iL} + r_{ei}\bar{\psi}_{kL} \gamma^\mu u_{iL} \]

(3.2)

We shall assume that generation mixing is small.

The predicted proton decay rates that follow from Eq. (3.1) depend on \( g \) (of \( m_X \)), \( A \), \( r_e \) and most sensitively on \( m_X \) and \( m_{X'} \). In the minimal SU(5) model described in section 2, \( g^2 (m_X)/4\pi \approx 0.024 \), \( A \approx 2.7 \), \( r_e \approx 2 \) and \( m_X \approx 2 \times 10^{14} \text{GeV} \) for \( A \gapproxeq 150 \text{MeV} \); so, \( H \) is fully determined and proton decay rates can be estimated. Unfortunately, the calculation of hadronic matrix elements of \( H \) which interpolate from an initial nucleon to final state decay products is dependent on the model of hadronic structure employed. For example, in the case of \( p \rightarrow e^+ \pi^0 \), a survey\(^7\) of the various calculations suggests about a factor of 5 (perhaps as much as 10) uncertainty in the rate due to matrix element uncertainties. It is, of course, important to further refine such calculations by whatever means possible. Keeping \( m_X \) arbitrary but using the minimal SU(5) values for \( g (m_X) \), \( A \) and \( r_e \) leads to

\[
1/\Gamma \left( p \rightarrow e^+ \pi^0 \right) \approx 4 \times 10^{39} \text{yr} \]  

(3.3)

which is to be compared with the IMB experimental bound\(^8\)

\[
1/\Gamma \left( p \rightarrow e^+ \pi^0 \right) \lesssim 2.7 \times 10^{32} \text{yr}. \quad \text{(Exp.)} 
\]

(3.4)

From such a comparison one concludes

\[
m_X \gtrsim 7 \times 10^{14} \text{GeV} 
\]

(3.5)

which rules out the minimal SU(5) model. (Cf. Eq. (2.1)).

Even though the value of \( m_X \) becomes unspecified when we abandon minimal SU(5), the form of \( H \) in Eq. (3.1) still provides useful information and guidance with regard to branching ratios. Using isospin symmetry and several model dependent estimates, one finds the following (approximate) two body proton decay branching ratios (for \( m_{X'} \geq m_X \))

\[
\begin{array}{c|c|c|c|c}
\text{process} & \text{branching ratio} & \text{branching ratio} & \text{branching ratio} & \text{branching ratio} \\
 & \text{for } \nu_e \pi^0 & \text{for } \nu_e \pi^+ & \text{for } \nu_e \mu^+ K^0 & \text{for } \nu_e K^0 \\
\hline 
p \rightarrow e^+ \pi^0 & 0.40 & 0.30 & 0.01 & 0.02 \\
p \rightarrow \nu_e \pi^+ & 0.16 & 0.04 & 0.03 & 0.02 \\
\hline
\end{array}
\]

(3.6)

Note that \( p \rightarrow e^+ \pi^0 \) is expected to be the dominant gauge boson mediated two body decay mode. In the case of the neutron one obtains

\[
\begin{array}{c|c|c|c|c}
\text{process} & \text{branching ratio} & \text{branching ratio} & \text{branching ratio} & \text{branching ratio} \\
 & \text{for } e^- \pi^+ & \text{for } e^- \pi^0 & \text{for } e^- \mu^+ K^0 & \text{for } e^- K^0 \\
\hline 
n \rightarrow e^- \pi^- & 0.80 & 0.05 & 0.08 & 0.02 \\
n \rightarrow e^- \pi^0 & 0.80 & 0.05 & 0.08 & 0.02 \\
\hline
\end{array}
\]

(3.7)

The baryon number violating total decay rates of the proton and neutron are predicted to be about equal.

Some of the above relative rates are firm predictions because they depend only on isospin and the value of \( r_e \). For example\(^5\)

\[
\Gamma \left( p \rightarrow e^+ \pi^0 \right) \simeq \frac{1}{2} \Gamma \left( n \rightarrow e^+ \pi^- \right) \]

(3.8)

Measurements of ratios such as \( \Gamma \left( p \rightarrow e^+ \pi^0 \right)/\Gamma \left( p \rightarrow \nu_e \pi^+ \right) \) could therefore, in principle, be used to determine \( r_e \) and thus used to indicate the presence or absence of \( m_{X'} \). Effects. Another way of determining \( r_e \) would be to measure the \( e^+ \) polarisation which is predicted in \( \Delta A = 0 \) proton decays to be \( (1 - r_e^2)/(1 + r_e^2) \). It is clear that if proton decay is every observed, further measurements of branching ratios will provide a wealth of information.
4. Proton Decay via Higgs Scalars

Proton decay can also be mediated by Higgs scalars. At tree level, only the \( SU(3)_c \times SU(2)_L \times U(1) \) scalar multiplets \((3,1,-2/3), (3,3,-2/3)\) and \((3,1,8/3)\) induce proton decay. The first of these multiplets occurs in the \( SU(5) \) 5-plet while all three are present in the \( 45 \). If we allow arbitrary Higgs-fermion couplings, then there is no prediction for the decay rates. However, if we assume that those couplings are proportional to the fermion masses, then proton decays into kaons and muons should be favored. For example, Golowich found that only the Higgs 5-plet were used to provide fermion masses, the amplitudes for \((3,1,-2/3)\) scalar induced proton decay would have the following ratios

<table>
<thead>
<tr>
<th>( \nu_K^+ )</th>
<th>( \mu^+K^0 )</th>
<th>( \mu^+\pi^0 )</th>
<th>( e^+K^0 )</th>
<th>( e^+\pi^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{s,m_d} )</td>
<td>( m_{s,m_d} \sin^2 \theta_c )</td>
<td>( m_{s,m_d} \sin \theta_c )</td>
<td>( m_{s,m_d} \sin \theta_c )</td>
<td>( m_{s,m_d}^2 )</td>
</tr>
</tbody>
</table>

where \( \theta_c \) is the Cabibbo angle \( \sin^2 \theta_c \approx 0.05 \). Phase space suppresses the \( K \) modes somewhat while 3 quark fusion enhances the \( \pi^0 \) and \( K^\tau \) modes but not the \( K^0 \). Incorporating those effects with Golowich’s results, I expect the following two body branching ratios

\[
\begin{array}{c|ccccc}
\hline
& \nu_K^+ & \mu^+K^0 & \mu^+\pi^0 & e^+K^0 & e^+\pi^0 \\
\hline
\rho & 0.75 & 0.18 & 0.07 & 0.007 & 0.004 \\
\hline
\end{array}
\]

The \( \nu_K^+ \) and \( \mu^+K^0 \) decays dominate; however, the \( \mu^+\pi^0 \) (which is more easily observable in water detectors) is not insignificant. For the neutron, \( n \rightarrow K^0 l \mu \) dominates and \( \Gamma(n \rightarrow \pi^- l \mu^+) \approx \Gamma(p \rightarrow K^0 l \mu^+) \). In these scenarios, we expect the proton lifetime to be approximately

\[
\tau_{\text{Higgs}} \approx 10^{30} \left( \frac{m_H}{10^{11} \text{GeV}} \right)^4 \text{ yr}. \quad (4.3)
\]

where \( m_H \) is the mass of the scalar that mediates the decay. Some of the more basic bounds that are most relevant for these decays are \( \kappa \) (at 90% C.L.)

\[
\begin{align*}
1/\Gamma(p \rightarrow \nu_K^+ K^+) &> 5 \times 10^{31} \text{ yr} \\
& \quad \text{(Kamiokande)} \quad (4.4a) \\
1/\Gamma(p \rightarrow \mu^+K^0) &> 4 \times 10^{31} \text{ yr} \\
& \quad \text{(Kamiokande)} \quad (4.4b) \\
1/\Gamma(p \rightarrow \mu^+\pi^0) &> 1.8 \times 10^{32} \text{ yr} \\
& \quad \text{(IMB)} \quad (4.4c) \\
1/\Gamma(n \rightarrow K^0 l \mu) &> 4.3 \times 10^{31} \text{ yr} \\
& \quad \text{(Kamiokande)} \quad (4.4d) \\
1/\Gamma(n \rightarrow \mu^+\pi^-) &> 6 \times 10^{31} \text{ yr} \\
& \quad \text{(Frejus)} \quad (4.4e)
\end{align*}
\]

Those bounds illustrate quite nicely how the various big detectors complement each other. Comparing Eq. (4.3) and (4.4) leads to

\[
m_H \gtrsim 2 \times 10^{11} \text{ GeV} \quad (4.5)
\]

(Given the theoretical uncertainties, that bound is still very flexible.) Of course, the mass of the Higgs scalar is arbitrary. We usually assume for exotic scalars \( m_H \approx m_X \) in which case the Higgs mediated decay rates should be insignificant. However, in some locally supersymmetric theories an intermediate mass scale of order \( 10^{10} \sim 10^{11} \text{ GeV} \) is quite natural.\(^{10}\) So, there is good reason to push the bounds in Eq. (4.4) as far as possible.

5. Supersymmetry

The basic idea of supersymmetry is that each known boson (fermion) has a fermion (boson) partner. In those scenarios, the \( \alpha(M_W) \) evolution equations Eq. (1.7) change when we pass the supersymmetry thresholds. In leading order, one finds for three generations of fermions and \( N_H \) light Higgs doublets \(^{11}\)

\[
\begin{pmatrix}
\delta_1 \\
\delta_2
\end{pmatrix} = - \frac{1}{2\pi} \begin{pmatrix}
-6 & \frac{3}{4} N_H \\
-\frac{1}{2} N_H & 3
\end{pmatrix}
\]

Taking \( N_H = 2 \) (the minimal value) and using the \( \alpha(M_W) \) values in section 1 as input, we can solve for \( m_{\text{SUSY}} \) and \( m_X \). One finds in leading order

\[
\ln \left( \frac{m_X}{m_{\text{SUSY}}} \right) \approx \frac{\pi}{2} \left( \frac{1}{\alpha_3(M_W)} - \frac{1}{\alpha_3(M_W)} \right)
\]

independent of \( m_{\text{SUSY}} \). Using the values of \( \alpha_3(M_W) \) in Eq. (1.6a) and \( \alpha_3(M_W) \) in Eq. (1.1) then gives the range of predictions

\[
\frac{m_X}{m_{\text{SUSY}}} \approx 2 \times 10^{14} \sim 2 \times 10^{16} \text{ GeV}
\]

The lower mass range corresponds to very large \( m_{\text{SUSY}} \) while the higher values require \( m_{\text{SUSY}} \) to be nearer \( m_W \). In SUSY GUTS, one expects the gauge coupling \( g(m_X) \) to be somewhat larger, so the gauge boson mediated decay rate in Eq. (3.3) becomes

\[
\frac{1}{\Gamma(p \rightarrow e^+\pi^0)} \approx 1.3 \times 10^{30} \pm 0.7
\]

\[
\times \left( \frac{m_X}{2 \times 10^{14} \text{ GeV}} \right)^4 \text{ yr} \quad \text{(SUSY)}
\]

The IMB bound, Eq. (3.4) then rules out the \( m_X \lesssim 10^{16} \text{ GeV} \) region in Eq. (5.3) but leaves open the possibility of \( m_{\text{SUSY}} \lesssim 10^6 \text{ GeV} \) as the “new physics” we are looking for. This example illustrates how a new physics threshold (supersymmetry in this case) can bring GUTS into agreement with low energy phenomenology. It also demonstrates the complementarity between proton decay and high energy experiments. If \( m_{\text{SUSY}} \lesssim 10^{10} \text{ TeV} \), it is likely to be discovered at the SSC, and in this example, the proton decay rate is too slow to observe. On the other hand, if \( m_{\text{SUSY}} \) is beyond 10 TeV, the value of \( m_X \) is lower and the detection of proton decay is more likely. Of course, the use of a single supersymmetry mass scale is rather simplistic. Nevertheless, this example points out the importance of pushing the search for proton decay as far as possible.
6. Dimension 5 Operators and Proton Decay

The gauge boson and Higgs scalar mediated proton decays previously described result from tree diagrams which lead directly to dimension 6 baryon violating four-fermi operators. The resulting decay rates are proportional to \(1/m_X^6\) or \(1/m_H^6\) and hence naturally suppressed by the large values of \(m_X\) and \(m_H\). It was, however, pointed out by Weinberg and Sakai and Yanagida\(^{15}\) that \(B\) and \(L\) violating dimension 5 operators may be induced by Higgsino mixing (Higgsino is the spin - 1/2 fermion partner of the Higgs scalar) in some supersymmetric GUTS. The dimension 5 operators entail fermion-fermion scalar-scalar couplings. Through loop effects such operators can give rise to dimension 6 \(B\) and \(L\) violating four-fermi amplitudes suppressed only by \(1/m_X\) and mixing factors. The resulting proton lifetime is of \(O(10^{10^{3}}\text{yr.})\) with \(p \to \nu_e K^+\) and \(n \to \nu_e K^0\) being the dominant decay modes. (In superstring models other modes may dominate.) The expected rate appears to be in conflict with the bounds in Eq. (4.4) by more than an order of magnitude.\(^{14}\) However, there is considerable uncertainty in the theoretical prediction. Furthermore, the dimension 5 operators can be exorcised by symmetries.\(^{18}\) Nevertheless, this scenario supports the analysis in section 4 where it was shown that \(p \to \nu K^+\) and \(n \to \nu K^0\) appear to be the best modes for detecting scalar induced \(B\) and \(L\) violation.

7. Conclusion and Outlook

A few years ago, proton decay was considered a far-out speculation. Now because of GUTS, it has become a part of theoretical lore. Furthermore, the observed matter-antimatter asymmetry of the universe strongly suggests that baryon number violation must have been quite large in the very early universe. We now ask not whether the proton decays, but whether or not we will be able to experimentally observe its decay. Unfortunately, lifetimes beyond \(10^{34}\) yr. would be hidden by backgrounds.

The minimal SU(5) model now appears to be ruled out on two fronts. It predicts too small a value for \(\sin^2 \theta_W (m_W)\) and too short a proton lifetime. Both failures can, however, be easily rectified by the addition of new particles with masses between \(m_W\) and \(m_X\). Supersymmetry is a realistic example of how that can be done. As new physics is uncovered, we will be in a better position to predict \(\sin^2 \theta_W (m_W)\) and \(\tau_p\). In the meantime, precision determinations of low energy couplings should be an important priority. Searches for proton decay should also be pushed as far as possible. At present, both the gauge boson favored \(p \to e^+\pi^0\) and Higgs mediated \(p \to \nu K^+\) decay modes appear to be good bets. That suggests the need for very large water Cherenkov detectors as well as fine grained detectors with good tracking ability. Such detectors can also search for magnetic monopoles, \(n \to \pi\) oscillations and anomalous neutrino sources. They are multi-purpose facilities rather than single experiments.

GUTS have earned a permanent place in high energy physics due to their elegance. They have motivated a number of interesting experiments which have so far only produced stringent limits and a few candidate proton decays which are consistent with expected neutrino backgrounds. Uncovering proton decay will certainly be harder than some people anticipated a few years ago; but the prize is worth the effort.

Acknowledgments

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References

8. D.S. Ayers et al., in these proceedings.