## $\overline{\mathrm{p}}$ CAPTURE IN NEUTRAL BEAMS

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#### Abstract

The formation of bound exotic atomic systems in a $\bar{p}$-ion trap by Auger capture with neutral injected beams is discussed. From $\overline{\mathrm{p}}$ capture with $\mathrm{H}^{\mathbf{0}}$ for the $\overline{\mathrm{p} p} \rightarrow \pi^{+}+\pi^{-}$or $\mathrm{K}^{+}+\mathrm{K}^{-}$channel, the angular distribution between the preceding $L$ x-ray and the $\pi$ or $K$ would test whether annihilation to $\pi^{+}+\pi^{-}$ and to $K^{+}+K^{-}$are dynamically similar. From $\bar{p}$ capture with positronium, antihydrogen formation is greatly enhanced, our estimated cross section being $10^{5}$ larger than the cross section for capture from an equal-energy positron beam. This would lead to high antihydrogen-production rates.


## Introduction

For the past generation, research with 'low-energy' pions and muons has been extremely fruitful for physics, particularly for those fields outside the realm of particle physics. The antiproton will prove to be an even more remarkable tool for such areas because, unlike the $\pi$ and $\mu$ probes, the $\bar{p}$ annihilates to provide a unique signal for both atomic- and particle-physics
studies. Furthermore, as antimatter, it will offer possibilities in the future for the experimental completion of symmetry investigations with precise laser techniques.

In this paper, the discussion of low-energy $\bar{p}$ research will be restricted to just two lines of approach, both of which have as a premise that a low-energy $\vec{p}$ ion trap can be developed with high particle density $\left(\geqslant 10^{6} \overline{\mathrm{p}} / \mathrm{cc}\right.$ at $\left.E_{p}^{-\leqslant 1} \mathrm{keV}\right)$. By passing a neutral beam of particles such as $H^{0}, D^{0}$, etc., or positronium into the $\bar{p}$-ion trap (see Fig. 1), the consequent Auger capture, with its large (geometric) cross section, will yield a high production rate of atomic protonium, exotic deuterium, etc., or antihydrogen, respectively, in a simple table-top configuration.


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Fig. 1. Schematic arrangement for production of $\bar{p}$ atomic systems from $\bar{p}$ capture with neutral beams in a $\bar{p}$-ion trap. The $H^{0}, D^{0}$, etc. beams (solid line) would be collimated before entering the ion trap. This is not the case nor is it necessary for positronium (broken lines) which would be created in the ion trap wall at the site of a low energy positron moderator.

This paper will briefly cover some of the preconditions necessary to achieve such capture by concentrating on the atomic protonium (pp) and antihydrogen cases. As an example of $\bar{p} p$ research, angular-distribution measurements of the threshold annihilation $\overline{\mathrm{p} p \rightarrow \pi^{+}+\pi^{-}}$or $\overline{\mathrm{p} p} \rightarrow \mathrm{~K}^{+}+\mathrm{K}^{-}$will be discussed with the aim of investigating whether these decays are dynamically similar (as tests of quark models for the decays). The antihydrogen discussion will be restricted to an estimate of the positronium $-\bar{p}$ Auger-capture cross section and some preconditions for its utilization for antihydrogen production in a $\overline{\mathrm{p}}$-ion trap.

## General Considerations

## Sources

Hardware exists or is under development.
pp formation. Intense, highly cooled beams of $H^{0}$ (and $D^{0}$ ) from standard atomic-beam sources are available ${ }^{1}$; (at ETH, $I_{H} \cong 10^{17} / \mathrm{sec}$ at $E=0.02 \pm 0.01 \mathrm{eV}$. Of course the beams are also available polarized.)

Antihydrogen formation. A 0.5 Curie ${ }^{58} \mathrm{Co} \beta^{+}$source through a slow-positron moderator ${ }^{2}$ of single-crystal copper coated with a submonolayer of sulphur releases a slightly focussed beam of $2.5 \times 10^{6}$ slow positrons/sec at about 1 eV . In a typical case, positrons are emitted in three equal channels, slow-positron emission, free-positronium emission and positronium from surface-state trapping. The mean velocity of such slow positronium atoms is such that they can travel 2 cm (through the $\bar{p}$ trap) during one mean lifetime of the triplet state. The positron emitter ${ }^{64} \mathrm{Cu}$ is available with activities
$10^{3}$ higher than that of the ${ }^{58}$ Co source currently in use. Also higher fluxes of slow positrons can be made via cascade positrons produced at the beam dump of an electron accelerator or in pulsed form.

## Why measure $\bar{p}$ 'capture-in-flight'?

(i) It has barely been measured in vacuum. Atomic capture and decay is gas-pressure dependent. It is difficult to design apparatus and detection equipment when even the lifetime of the $\overline{p p}$ is uncertain. One does not know the $n$ state of $\bar{p}$ capture; experimental values of the $\bar{p} p$ lifetime have ranged from $2-50 \mu \mathrm{sec}$.
(ii) The particle and $x$-ray ranges are large in vacuum. Clearly antihydrogen must be made and stored in vacuum.
(iii) The angular momentum of the annihilation partial waves are unambiguously tagged by gating with the previously emitted x-ray of the atomic state. For example, for $\bar{p} p \rightarrow \pi^{+}+\pi^{-}$or $\mathrm{K}^{+}+\mathrm{K}^{-}$, the $\mathrm{L} x$-rays tag the P -wave annihilation (which is 98\% abundant).
(iv) Threshold amplitudes are simpler and can be calibrated with fewer independent measurements. A later example should clarify this statement.
$\overline{\mathrm{p}}$-Ion Trap
With respect to $\overline{\mathrm{p}}$-ion trap characteristics, refer to Brown ${ }^{3}$ and Hynes ${ }^{4}$ (these proceedings) for Penning traps and Fischer ${ }^{5}$ and Dehmelt ${ }^{6}$ for r.f. traps. The $\bar{p}$-ion traps for the present utilization should have volumes $n 1$ cc, with $\overline{\mathrm{p}}$ densities in a range $10^{4}-10^{8} / \mathrm{cc}$, pressures $\leqslant 10^{-12}$ torr (residualgas pressure at $10^{-12}$ torr will result in a $\bar{p}$-annihilation background rate of about $10^{-4}$ per particle per second. See curves in Ref. 7). The positronium source and low-energy moderators for antiproton formation must
be placed at the wall of the trap. The electrodes should be meshed so that it is transparent to annihilation radiation and antihydrogen. The neutral beam-line sources must be placed far enough away so that there is no deterioration of the necessary vacuum conditions of the trap. (An atomic-beam hydrogen source has been successfully coupled to an r.f. trap.)

## Experimental Constraints, General

## Beam Handling and Counting Rates

The $\bar{p}$ capture on a neutral atomic system takes place when it is energetically favourable for the $\bar{p}$ to replace the electron which then drifts away with essentially zero velocity (see later calculation). At the slightly higher velocities (than those under consideration here) in a storage ring, this occurs roughly when the $\bar{p}$ and neutral beams are matched in velocity, and beam cooling becomes important. However, when the energy of the neutral beam is around the binding energy of hydrogen ( $\sim 10 \mathrm{eV}$ ), and the $\overline{\mathrm{p}}$ has an equivalent velocity (or below), then the rate of capture can be approximated by the simple well known formula,

$$
\begin{equation*}
R=N N_{-p}^{0}, \tag{1}
\end{equation*}
$$

where $N_{p}$ is the $\bar{p}$ density/cc in the trap, $N$ is the neutral current/ $\mathrm{cm}^{2} / \mathrm{sec}$ passing through the trap, and $\sigma$ is the Auger capture cross section for the process under consideration. A standard atomic beam source has a neutral hydrogen-emission rate of $\geqslant 10^{17} \mathrm{H}^{0} / \mathrm{cm}^{2} / \mathrm{sec}$ at E<0.02 eV . With estimates that $\mathrm{N}=10^{15} \mathrm{H}^{0} / \mathrm{cm}^{2} / \mathrm{sec}$ (at target), $\mathrm{N}_{\mathrm{p}}=10^{6} \overline{\mathrm{p}} / \mathrm{cm}^{3}$, and $\sigma=10^{-15} \mathrm{~cm}^{2}$ (Ref. 8), Eq. (1) yields the enormous $\overline{\mathrm{p}}$-production rate,
$\mathrm{R}=10^{6} / \mathrm{sec}$.
Since the trap can be refilled many times from a reservoir, the $\overline{\mathrm{p}}$ ion-trapdesign conditions ( $\mathrm{N}_{\mathrm{p}}=10^{6} \overline{\mathrm{p}} / \mathrm{cm}^{3}$ ) can be considerably lowered for continuous running.

## Center-of-Mass Motion and Lifetime

The geometry of the trap and detection equipment for measurement is determined by the velocities and the lifetimes of the systems to be produced, as well as the velocities and lifetimes of the beams necessary for production.
$\mathrm{p} \overline{\mathrm{p}}$ and $\overline{\mathrm{p}}$. The lifetime of $\overline{\mathrm{p}}$ in the trap is determined by collisions with the residual hydrogen molecules in the trap and is given in Ref. 7 as a function of pressure. Actually, there are fewer collisions with the residual gas at the hotter $\bar{p}$ temperatures. The heating from an r.f. trap can be used advantageously for greater-density $\overline{\mathrm{p}}$ storage; capture conditions are even met at room temperatures. The neutral hydrogen beam is stable and is not a problem.

With respect to the created $\bar{p} p$ system, its lifetime depends critically upon the capture process. Present experimental values of lifetimes range from 2-50 $\mu \mathrm{sec}$ but have not as yet been published. The lifetime $\tau_{n}$ of the $\overline{\mathrm{p} p}$ system can be estimated from the width $\Gamma_{n}$ of the dipole transition. For large $n(n \gg 1)$,

$$
\begin{equation*}
\Gamma_{n} \approx \frac{2}{3} \alpha^{5} \mu c^{2} \frac{1}{n^{5}} \tag{2}
\end{equation*}
$$

thus,

$$
\begin{equation*}
\tau_{n}=\frac{3}{2} \frac{n^{5}}{\alpha^{5} \mu c^{2}} \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
\tau_{\text {total }}=\tau_{N}=\sum_{n=N_{\text {final }}}^{N} \tau_{n} \tag{4}
\end{equation*}
$$

where $\mu$ is the reduced mass of protonium, $\alpha=e^{2} / H c$ is the fine-structure constant, $c$ is the velocity of light, $n$ is the orbit of $\bar{p}$ at capture, and ( $N_{\text {final }}{ }^{-1}$ ) is the orbit from which annihilation takes place. The lifetime ${ }^{\boldsymbol{T}} \mathrm{N}$ of the $\bar{p} p$ system is plotted in Fig. 2 for a cascade down the yrast line, i.e., for circular-orbit transitions $n, l=n-1 \rightarrow(n-1),(l=n-2)$, where in this case (for annihilation from the atomic $P$ level).

$$
\begin{equation*}
{ }^{\top} N=\sum_{n=3}^{N}{ }^{\top}{ }_{n} \tag{5}
\end{equation*}
$$

As can be seen in Fig. 2, ${ }^{\tau} N$ is critically dependent upon the capture processes (into which n-level capture takes place). An experimental study of ${ }^{\top} N_{N}$ as well as o as a function of center-of-mass velocity will not only
elucidate the $\overline{\mathrm{p}}$ p system but will lead to practical information regarding experimental design.


Fig. 2. The $\overline{p p}$ total lifetime $\tau_{N}$ to the atomic $P$ level and partial lifetime $\tau_{n}$ for a cascade down the yrast line as a function of the $n$ level into which $\bar{p}$ capture takes place. This is an estimation from the dipole approximation (see Eqs. (2)-(5) in text).

Antihydrogen. Unlike pp, antihydrogen as such is stable. However, its detection to $a$ great extent depends upon the ratio of antihydrogen to $\bar{p} p$ formation, the latter from $\bar{p}$ capture with residual gas which would also give annihilation background-noise signals. Thus both the formation and detection of antihydrogen depend critically upon the $\bar{p}$-capture cross section with low-
energy positronium (later it will be quantitatively shown that such positronium capture dominates over the more usually considered positron capture).

Positronium ( $\mathrm{e}^{+} \mathrm{e}^{-}$) itself exists in two states; fortunately $3 / 4$ of the slow $e^{+} e^{-}$emitted from moderators at $E \leqslant 1 \mathrm{eV}$ are in the triplet state, with a mean velocity such that the $\mathrm{e}^{+} \mathrm{e}^{-}$travels about 2 cm during its mean lifetime of $1.42 \times 10^{-7} \mathrm{sec}$. As stated previously, this is of sufficient length to penetrate $a \sim 1 \mathrm{~cm}^{3} \overline{\mathrm{p}}$ ion trap. Note again that in a typical case, there is about equal emission of slow $\mathrm{e}^{+} \mathrm{e}^{-}$and $\mathrm{e}^{+}$after the moderator. To velocitymatch a $1-\mathrm{eV} \mathrm{e}^{+} \mathrm{e}^{-}$beam, the trap should contain $4-\mathrm{keV} \overline{\mathrm{p}}$ 's with proportionally lower energy $\bar{p}^{\prime}$ s at lower $\mathrm{e}^{+} \mathrm{e}^{-}$energy, but velocity matching need not be perfect.

## Examples

## 

The analysis of the angular correlation of the following weak ( $\sim 0.2 \%$ ) annihilation branch will serve as an example of new measurements that can be performed with a $\bar{p}$ ion trap at threshold energies,

Specifically, one determines $W(\theta)$, the angular correlation between the outgoing $\pi^{ \pm}$or $K^{\ddagger}$ and the direction of the $L$ x-ray (emitted just before annihilation) (see Fig. 3 and Appendix A for details). Asterix experiments ${ }^{9}$ have determined that 98\% of the annihilation occurs from $P$ waves; the preceding $L$ x -ray average energy is 1.7 keV .



#### Abstract

Fig. 3. The $\bar{p} p \rightarrow \pi^{+}+\pi^{-}$decay in the center-of-mass coordinate system. The $0 Z$ axis represents the direction of the last emitted atomic photon (L x-ray) detected before annihilation. The allowed atomic E1 transitions $(\Delta m= \pm 1, \Delta m \neq 0)$ between the substates along the 02 quantum axis create an atomic p-state alignment which yields an $L$ x-rayangular correlation.


The role of the L x-ray is vital, and its detection performs three independent functions: (i) It helps determine the annihilation vertices. (ii) It tags the annihilation-orbital angular momentum ( $P$ wave). (iii) It aligns the P atomic state so that angular-correlation effects can be measured by inflight capture.

As calculated in Appendix $A$, the angular correlation between the $\mathrm{L} x$-ray and the $\pi$ meson has the form

$$
\begin{equation*}
W(\theta)=C_{0}\left[1+C_{1} \cos ^{2} \theta\right], \tag{7}
\end{equation*}
$$

where $\theta$ is the angle between the $L$ - ray and the $\pi$ (or $K$ ) direction (as shown in Fig. 3), and the C parameters are defined in Appendix A. See also Ref. 10.

The experiment determines the $C_{0}$ and $C_{1}$ parameters which are functions of the $J=0$ and $J=2$ partial-wave intensities for the process $\bar{N} N \rightarrow \pi^{+} \pi^{-}$or $K^{+} K^{-}$ extrapolated to threshold energies. Under the assumption that the atomic $D$ state $m$ sublevels are equally populated (i.e., there is no atomic D-state alignment), that the $P$ state is entirely populated via E1 decays and using extrapolated values of the $S$ and $D$ annihilation partial waves given in $A p-$ pendix $A$, our theoretical estimates yield $C_{1}=-0.046$. In a situation, where the proton is polarized, $C_{1}$ can vary from -0.046 when the proton spin is parallel to the $x$-ray direction, to +0.092 when the proton spin is perpendicular to the $x$-ray direction. However, the main point is that a determination of the $C$ coefficients yields the threshold partial-wave intensities. Such a determination for both branches ( $\pi^{ \pm}, \mathrm{K}^{ \pm}$) would reveal whether annihilation to $\pi^{+} \pi^{-}$and $\mathrm{K}^{+} \mathrm{K}^{-}$are dynamically similar. Thus the data would provide important tests for quark models of these annihilations.

## Antihydrogen. Comparison of Positronium ( $\left.e^{+} e^{-}\right)$to Positron ( $e^{+}$) Capture Cross Sections

As a first step for antihydrogen research, a practical means of antihydrogen production must be attained. If the $e^{+}$penetrates the $\bar{p}$-ion trap as low-energy $e^{+} e^{-}\left(E_{n} \underline{1} \mathrm{eV}\right.$, see earlier sections), a $10^{5}$ enhancement in the antihydrogen formation can be attained via Auger capture, as compared to the
radiative capture from the unbound $e^{+}$. The cross sections can be estimated as follows:

The cross section for the process $\bar{p}+e^{+} e^{-}+\bar{p}^{+}+e^{-}$at very low energy ( $E \varepsilon_{2} \frac{1}{2}_{\infty}=6.8 \mathrm{eV}$ ) can be crudely estimated ${ }^{8.11}$ in a simple adiabatic model. For distances $r>R_{0}=0.64 a_{0}\left(a_{0}=\mu^{2} / e^{2} m_{e}\right)$ between the positron and the antiproton, the energy eigenvalue $\varepsilon(r)$ of the electron is negative, corresponding to a bound state. For a (static) value of $r$ less than $R_{0}$, the electron is no longer bound, i.e., it can move to large distances from the $\bar{p} e^{+}$system without additional supply of energy.

The adiabatic potential between $\bar{p}$ and $e^{+}$is then given by $V(r)=\varepsilon(r)-$ $e^{2} / r+\frac{1}{2} R_{\infty}$, where the last term is the binding energy of the $e^{+} e^{-}$system. For a given initial energy $E$ and impact parameter $B$, the classical distance $R$ of closest approach is given by $E=V(R)+E B^{2} / R^{2}$. The requirement that $R \leqslant R_{0}$ results in an upper limit for the impact parameter $B<B_{0}$ where

$$
\begin{equation*}
o_{A}^{(1)} \equiv \pi B_{0}^{2}=\pi R_{0}^{2}\left(1+\left(\frac{e^{2}}{R_{0}}-\frac{1}{2} R_{\infty}\right) / E\right) \tag{8}
\end{equation*}
$$

All $B \& B_{0}$ lead to conditions where the electron is unbound and under these adiabatic assumptions therefore will drift away with essentially zero velocity. For $\mathrm{E} \leqslant \frac{1}{2} \mathrm{R}_{\infty}$, the $\overline{\mathrm{p}}{ }^{+}$system must then be left in a bound state of negative energy. This Auger-capture cross section is then given by $o_{A}^{(1)}$ which, after insertion of $R_{0}$, becomes

$$
\begin{equation*}
\sigma_{A}^{(1)} \simeq 0.32 \times 10^{-16} \mathrm{~cm}^{2}(1+36 \mathrm{eV} / E) \tag{9}
\end{equation*}
$$

At low energies, the antiproton may be reflected from the barrier in the radial potential. For large distances, $V(r)$ can be calculated in pertur-
bation theory as ${ }^{8} \quad V(r) \underline{q} / 4 \quad e^{2}\left(2 a_{0}\right)^{3} / r^{4}$. The barrier height is $V_{b}=E^{2} B^{4} /\left(72 e^{2} a_{0}^{3}\right)$ at the distance $R_{b}^{2}=36 e^{2} a_{0}^{3} /\left(E B^{2}\right)$. The condition $E \geqslant V_{b}$ then leads to

$$
\begin{equation*}
\sigma_{A}^{(2)}=12 \pi a_{0}^{2} \sqrt{R_{\infty} / E}=35 \times 10^{-16} \mathrm{~cm}^{2} \sqrt{1 \mathrm{eV} / \mathrm{E}}, \tag{10}
\end{equation*}
$$

which equals $\sigma_{A}^{(1)}$ for $E \approx 0.1 \mathrm{eV}$. Thus for energies below 0.1 eV , we use $\sigma_{A}^{(2)}$, and for energies above 0.1 eV , we use $\sigma_{\mathrm{A}}^{(1)}$, i.e.,

$$
\begin{equation*}
\sigma_{A} \equiv \operatorname{Min}\left\{\sigma_{A}^{(1)}, \sigma_{A}^{(2)}\right\} . \tag{11}
\end{equation*}
$$

The geometric cross section of the $e^{+} e^{-}$system is $\sigma_{G}=\pi\left\langle r^{2}\right\rangle=2 \pi a_{0}^{2}=9.4 \times 10^{-16}$ $\mathrm{cm}^{2}$ which is the value of $\sigma_{A}$ at about 1 eV . Thus, at lower energies, $\sigma_{A}$ is significantly larger than $\sigma_{G}$. The above estimate uses the procedure developed for processes ${ }^{8,11}$, where a heavier (classical) particle is involved instead of the positron considered here. One should therefore be cautious with the above results, especially in the region E 20.1 eV , although comparable processes lead to cross sections of the same orders of magnitude ${ }^{8,11,12}$.

The cross section for the radiative capture process of $e^{+}$by $\bar{p}$ into levels of principal quantum number $n$ is given by ${ }^{13}$,

$$
\begin{equation*}
\sigma_{\gamma}(n)=1.96 \pi^{2} a_{o}^{2} \alpha^{3} R_{\infty}^{2} /\left(E n\left(R_{\infty}+E n^{2}\right)\right) \tag{12}
\end{equation*}
$$

The total cross section is then approximately
$\sigma_{\gamma}=\sum_{n=1}^{\infty} \sigma_{\gamma}(n) \approx \int_{1 / 2}^{\infty} \sigma_{\gamma}(x) d x$
$=0.98 \pi^{2} a_{0}^{2} \alpha^{3} R_{\infty} \frac{1}{E} \ln \left(1+4 R_{\infty} / E\right)$
$\approx 1.3 \mathrm{eV} \times 10^{-21} \mathrm{~cm}^{2} \frac{1}{\mathrm{E}} \ln (1+54 \mathrm{eV} / \mathrm{E})$.


Fig. 4. Estimated capture cross sections for antihydrogen production from (radiative) positron capture $\sigma_{\gamma}$ or (Auger) positronium capture $\sigma_{A}$ with $\bar{\rho}$ as a function of energy $E$ (see Eqs. (8)-(13) in text).

In Fig. 4, we show both $\sigma_{\gamma}$ and $\sigma_{A}$ as function of energy. The positronium Auger-capture cross section $\sigma_{A}$ exceeds the positron radiative capture cross section $\sigma_{\gamma}$ by about five orders of magnitude.

# By use of Eq. (1) with the following estimates: $\sigma_{A}=10^{-15} \mathrm{~cm}^{2}$ (for $E_{e^{+}} e^{-\approx 1} \mathrm{eV}$ ), $N_{p}^{-}=10^{6} / \mathrm{cc}$, and $N=10^{8} \mathrm{e}^{+} \mathrm{e}^{-} / \mathrm{cm}^{2} / \mathrm{sec}$ (from a $50-\mathrm{C}^{64} \mathrm{Cu}$ source with low-energy $\beta^{+}$moderator) an antihydrogen-formation rate, 

$$
R_{H}=0.1 / \mathrm{sec}
$$

can be achieved without enhancement or source development*. At lower $\mathrm{e}^{+} \mathrm{e}^{-}$ energies and by brute force, i.e., by $e^{+} e^{-}$source development, by $\bar{p}$ ion-trap technology, or by enhancement tricks, several orders of magnitude increases of antihydrogen-production rates above the previous modest estimate may be possible.

The $\bar{p}+e^{+} e^{-}$capture cross section in an ion trap can be measured by the equivalent $p+e^{+} e^{-}$rate, which should be the same. Note that for a $1-e V e^{+} e^{-}$, velocity matched to the proton, the $H^{0}$ produced would have $4-k e v$ energy, easjily measurable by a channeltron. The $\bar{p}$-ion trap for antihydrogen production and detection could be developed and tested with protons without the use of expensive $\bar{p}$ facilities. The apparatus can be simple and of table-top

* This formation rate was questioned at the workshop due to a misunderstanding about collimation of the positronium "beam". Positronium is created at the low energy positron moderator in the wall of the $\bar{p}$ ion trap and all positronium entering the trap through the forward $2 \pi$ solid angel are utilized for antihydrogen formation. The fact that there is no need for collimation was the reason for the misunderstanding and there was general agreement at discussions after the session that the formation rate given here is essentially correct.
dimensions. The production of antihydrogen opens up the possibility of experiments with antimatter, for example laser experiments which would test CPT invariance to a high degree of accuracy.


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## Appendix

The annihilation of an $\overline{p p}$ bound state to $\pi^{+} \pi^{-}$is closely related to the scattering process $p+\bar{p} \rightarrow \pi^{+}+\pi^{-}$. This scattering process is usually described in terms of the helicity amplitudes $f_{ \pm}^{J}$ of Frazer and Fulco ${ }^{14}$ and these same helicity amplitudes can also be used to calculate the bound state annihilation probabilities. While scattering measurements provide information on the total scattering amplitude, which is a sum over all angular momentum states, the bound state annihilation probabilities have the advantage of providing information on particular angular momentum states. For
example the annihilation probability from the ${ }^{3} S_{1}$ state can be related to the threshold values of $f_{ \pm}^{1}$, while the annihilation probabilities from the ${ }^{3} P_{0}$ and ${ }^{3} P_{2}$ states can be related to the threshold value of $f_{+}^{0}$ and $f_{ \pm}^{2}$ respectively. Here we concentrate on annihilation from the ${ }^{3} P_{0}$ and ${ }^{3} P_{2}$ states and use the threshold behaviours of $f_{+}^{0}$ and $f_{ \pm}^{2}$ which are of the form

$$
\begin{align*}
& f_{+}^{0}{\underset{p}{n} \rightarrow 0}^{C_{+} p^{2}}  \tag{14}\\
& f_{+}^{2}{\underset{p}{m} \rightarrow 0}^{\frac{2 M}{\sqrt{6}} C_{-} \quad f_{-}^{2} \quad p^{\sim} \rightarrow 0} C_{-} \tag{15}
\end{align*}
$$

where $p$ is the proton momentum in the $p \bar{p} C . M$. system.
There is a technical complication because we are dealing with a $\mathrm{p} \overline{\mathrm{p}}$ bound state. If the binding energy is $E_{B}$ then the total energy of the system is $2 M \mathrm{c}^{2}-E_{B}$ and the momentum of the free pjons is given by

$$
\begin{equation*}
2\left(q^{2}+m_{\pi}^{2}\right)^{1 / 2}=2 M c^{2}-E_{B} . \tag{16}
\end{equation*}
$$

The proton momentum cannot be calculated in this way since the py system is bound and in principle $p$ varies from 0 to $\infty$. However for the $1 P$ state $E_{B} \ll$ $\mathrm{Mc}^{2}$ and the r.m.s. value $\left\langle\mathrm{p}^{2}\right\rangle^{1 / 2}$ is $\sim 3 \mathrm{MeV} / \mathrm{c}$. Since this value is so small we assume that the same threshold behaviour holds in this off mass shell situation and we end up with an angular distribution of the $\pi^{+}$with respect to the preceeding $\mathrm{L} x$-ray direction of the form

$$
\begin{align*}
& W\left({ }^{3} P_{0} \rightarrow \pi^{+} \pi^{-}\right)=\frac{1}{8 \pi^{5}} q \frac{|k|^{2}}{M^{3}}\left|C_{+}\right|^{2}  \tag{17}\\
& W\left({ }^{3} P_{2} \rightarrow \pi^{+} \pi^{-}\right)=\frac{41}{192 \pi^{5}} q^{5} \frac{|k|^{2}}{M}\left|C_{-}\right|^{2}\left(1-\frac{3}{41} \cos ^{2} \theta\right) \tag{18}
\end{align*}
$$

where

$$
\begin{equation*}
k=\int_{0}^{\infty} \tilde{R}_{1 p}(p) p^{3} d p \tag{19}
\end{equation*}
$$

$\tilde{R}_{1 p}(p)$ being the Fourier transform of the 1 P radial wave function. We have also assumed that the $m$ sublevels of the ${ }^{3} P_{2}$ state are populated by electric dipole transistions so that the relative populations are

$$
(m= \pm 2):(m= \pm 1):(m=0)=42: 39: 38
$$

If the energy resolution is good enough then it is possible to resolve the ${ }^{3} \mathrm{P}_{0}$ and ${ }^{3} \mathrm{P}_{2}$ levels and make separate measurements of their annihilation probabilities to $\pi^{+} \pi^{-}$, thus determining $\left|C_{+}\right|$and $\left|C_{-}\right|$. This is probably difficult in practice and so it may well be better to measure a total pstate annihilation probability to $\pi^{+} \pi^{-}$and use the $\cos ^{2} \theta$ term in the angular distribution of $\pi^{+}$w.r.t. the $L x$-ray to separate $\left|C_{+}\right|$and $\left|C_{-}\right|$.

To give a feeling for the sizes involved we have estimated $\left|C_{+}\right|$and $\left|C_{1}\right|$ by extrapolating down to threshold the $\pi \pi-N \bar{N}$ helicity amplitudes of Martin and Morgan ${ }^{15}$. If we define the total p-state annihilation distribution by

$$
\begin{align*}
W\left(P \rightarrow \pi^{+} \pi^{-}\right) & =\frac{5}{12} W\left({ }^{3} P_{2} \rightarrow \pi^{+} \pi^{-}\right)+\frac{1}{12} W\left({ }^{3} P_{0} \rightarrow \pi^{+} \pi^{-}\right)  \tag{20}\\
& =C_{0}\left(1+C_{1} \cos ^{2} \theta\right) \tag{21}
\end{align*}
$$

(the ' $P_{1}$ and ${ }^{3} P_{1}$, components of the $P$-state cannot annihilate to $\pi^{+} \pi^{-}$), then we obtain $C_{0}=0.023 \mathrm{meV} /$ steradian and $C_{1}=-0.046$.

Another approach is to try to construct a quark model description of the annihilation process. If we take the results of Kohno and Weise ${ }^{16}$ then $C_{+}$is about 3 times larger and $C_{-}$about half as large as the extrapolated Martin and Morgan values. This enhances the isotropic ${ }^{3} p_{0}$ part giving $C_{0}=$ $0.099 \mathrm{meV} /$ steradian and $C_{1}=-0.0045$. These authors also use their model to calculate the annihilation to $K^{+} K^{-}$, and in this case they find that the ${ }^{3} P_{2}$ part is totally dominant giving $C_{0}=0.00038 \mathrm{meV} / \mathrm{steradian}$ and $C_{1}=-0.0732$. Thus measurements of the values of $C_{1}$ in the $p-s t a t e$ annihilation to $\pi^{+} \pi^{-}$ and to $\mathrm{K}^{+} \mathrm{K}^{-}$can provide a valuable check on such a quark model calculation of these processes.

A similar analysis can be carried out in the case where the proton of the pp bound state is polarized. If $100 \%$ polarization survives until the annihilation then the angular distribution becomes modified to

$$
\begin{equation*}
W\left({ }^{3} P_{2} \rightarrow \pi^{+} \pi^{-}\right)=\text {const }\left(1-\frac{3}{41}\left(1-3 \sin ^{2} \alpha\right) \cos ^{2} 8\right) \tag{22}
\end{equation*}
$$

where $\alpha$ is the angle between the proton spin direction and the $L$ x-ray. (Earlier we considered the same process in the situation where there is no spin-orbit splitting of the $1 P$ level. In this, unfortunately unrealistic, situation more complicated interference effects occur ${ }^{17}$.) Thus the $\cos ^{2} \theta$ coefficient can vary from $-3 / 41$ to $+6 / 41$ depending on the value of $\alpha$. A study of the variation of the $C_{1}$ coefficient as $\alpha$ is varied can provide information on how much of the initial proton polarization survives during the cascade from the high $n$ level where the $p \bar{p}$ bound state is initially formed to the $n=2$ 1p level where the annihilation takes place. Such information would be of interest in atomic physics studies of this cascade process.

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