CP Violation Experiments using Hyperons at a \( p\bar{p} \) Machine*

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Abstract

I discuss ways to use counting type asymmetry measurements in \( p\bar{p} \rightarrow \Lambda\bar{\Lambda}, \Sigma\bar{\Sigma}, \Xi\bar{\Xi} \) as a way to uncover milliweak CP violation in the \( \Delta S = 1 \) interaction. Signals of order \( 10^{-4} \) are expected in many models, including the standard KM model. This is not far beyond the sensitivity of the present generation of experiments at LEAR, and serious study should be given to undertaking such experiments.

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I discuss ways to use counting type asymmetry measurements in \( p\bar{p} \rightarrow \Lambda\bar{\Lambda}, \Sigma\bar{\Sigma}, \Xi\bar{\Xi} \) as a way to uncover milliweak CP violation in the \( \Delta S = 1 \) interaction. Signals of order \( 10^{-4} \) are expected in many models, including the standard KM model. This is not far beyond the sensitivity of the present generation of experiments at LEAR, and serious study should be given to undertaking such experiments.

The origin of the observed violation of CP symmetry is still not known, but is one of the fundamental open questions in particle physics. There has been much discussion about valuable studies of CP violation in the kaon system using \( p\bar{p} \) machines. I would like to point out that new observations of CP violation in a different system, hyperons, seems tantalizingly within reach if enough statistics can be accumulated. The possibility of a major and fundamental breakthrough in this area is high enough to warrant serious study of these experiments.

Before describing the possible origin of the CP violation, let us have a first look at how it could be measured. Consider the reaction

\[
\begin{align*}
p(k) + \bar{p}(k) &\rightarrow \Lambda + \bar{\Lambda} \rightarrow p(p) + \pi^-(q) + \bar{p}(\bar{p}) + \pi^+(\bar{q}) \end{align*}
\]

Working in the center of mass, the triple product

\[
A = k \cdot (p \times q - \bar{p} \times \bar{q})
\]

is CP violating. This is just a correlation of the decay planes of the \( p\pi^- \) system \((\hat{n}_p = \hat{p} \times \hat{q})\) and \( \pi^+(\hat{n}_\pi) = \hat{p} \times \hat{q} \) with the beam direction. Triple vector products are well known from tests of \( T \) invariance. There they are most often not useful because, although naively \( T \) violating, they can be generated by final state interactions. The beauty of \( \bar{p} \) (or \( e^+e^- \)) machine is that, properly chosen,
these products can represent true CP violation. The initial state is related to itself under CP

\[ \text{PC}|p(k)\ p(-k)\rangle = Pe^{i\phi}|\bar{p}(k)\ \bar{p}(-k)\rangle \]

\[ = -e^{i\phi}|\bar{p}(-k)\ p(k)\rangle \]

\[ = e^{i\phi}|p(k)\ \bar{p}(-k)\rangle \]  

where the unessential phase \( \phi \) is related to arbitrariness in the definition of C and cancels when matrix elements are squared. The asymmetry in Eq. 2 is not the only CP odd observable, and others will be discussed later. The source of the CP violation would not be expected to be due to the strong interaction production of \( \Lambda\bar{\Lambda} \) but rather should be due to the weak decays of the \( \Lambda \) and \( \bar{\Lambda} \). Many models of CP violation have \( \Delta S=1 \) effects in hyperon decay. To see what these are and how they generate CP odd observables, we need to turn to an analysis of hyperon decay.

The matrix elements for \( B_i \rightarrow B_f \pi \) have the general form

\[ M = S + P \hat{\sigma} \cdot \hat{q} \]  

with \( S(P) \) being the S-wave (P-wave) parity violating (conserving) amplitude. From these we form the decay rate

\[ \Gamma \propto |S|^2 + |P|^2 \]  

and decay parameters

\[ \alpha = \frac{2 \text{Re}(S*P)}{|S|^2 + |P|^2} \]  

\[ \beta = \frac{2 \text{Im}(S*P)}{|S|^2 + |P|^2} \]  

which govern the final state properties

\[ W(\theta) = \frac{1}{4\pi} \left[ 1 + \alpha S_i \cdot \hat{P}_f \right] \]

\[ \langle \hat{\sigma} \rangle_f = \frac{1}{1 + \alpha S_i \cdot \hat{P}_f} \left\{ (\alpha + S_i \cdot \hat{P}_f)\hat{P}_f + \beta S_i \times \hat{P}_f + \sqrt{1-\alpha^2-\beta^2} \left[ \hat{P}_f \times (S_i \times \hat{P}_f) \right] \right\} \]
CP violation always arises from an interference, either between $S$ and $P$ amplitude or between $I = 1/2$ and $3/2$ final states. There have been a variety of signals discussed in the literature\(^1,2\), of which the useful ones are the partial rate asymmetry

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

and the decay asymmetries

$$A = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}$$  
$$B = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} \quad \text{or} \quad B' = \frac{\beta + \bar{\beta}}{\alpha - \bar{\alpha}}$$

Let us see how these arise. Each amplitude can be characterized by a magnitude, a CP violating phase due to the weak interaction $\phi_i$, and a phase which arises from strong final state interactions $\delta_i$. For example in $\Lambda$ decay we have

$$S = S_1 e^{i(\delta_1^S + \phi_1^S)} + S_3 e^{i(\delta_3^S + \phi_3^S)}$$
$$P = P_1 e^{i(\delta_1^P + \phi_1^P)} + P_3 e^{i(\delta_3^P + \phi_3^P)}$$

with the subscript indicating isospin. Because of the presence of the final state interactions, the CP violating phases cannot be isolated in $\Lambda$ decay alone. However the phases enter with different signs in $\bar{\Lambda}$ decay

$$\bar{S} = -S_1 e^{i(\delta_1^S - \phi_1^S)} - S_3 e^{i(\delta_3^S - \phi_3^S)}$$
$$\bar{P} = P_1 e^{i(\delta_1^P - \phi_1^P)} + P_3 e^{i(\delta_3^P - \phi_3^P)}$$

This will let us isolate the CP violating phases by comparing particle and antiparticle decays.

In these hyperon decays there are two small numbers\(^3\) which will produce a hierarchy of signal strengths. The $\Delta I = 1/2$ rule tells us that $\Delta I = 3/2$ amplitudes are typically twenty times smaller than those with $\Delta I = 1/2$. The other small number is $\sin \delta_1$ which is typically of order $1/10$. To leading order in these quan-
tities, we have

$$\Delta = \frac{\alpha_0}{\alpha_0} = -2 \frac{S_{3}^s}{S_{1}^s} \sin(\delta_3^s - \delta_1^s) \sin \phi_3^s - \phi_1^s) + S \leftrightarrow P$$

$$\Lambda = \frac{a + a}{a - a} = \sin(\phi_1^s - \phi_1^p) \sin(\delta_1^p - \delta_1^s)$$

$$B' = \frac{\beta + \bar{\beta}}{\alpha - \bar{\alpha}} = \sin(\phi_1^s - \phi_1^p)$$

$$B = \frac{B'}{\tan(\delta_1^p - \delta_1^s)}$$

$$A = \frac{A_3}{A_1} \approx \frac{1}{10} \chi$$

Here it is clear that in any model, $B'$ is the optimal signal as it is directly proportional to the CP odd strength. The others are suppressed by uninteresting small numbers. If we denote the strength of $\Delta S=1$ CP violation by $\chi$, we find

$$B' \approx \chi$$

$$A \approx \chi \times \sin(\delta_1^p - \delta_1^s) \approx \frac{1}{10} \chi$$

$$\Delta \approx \chi \times \sin(\delta_3^s - \delta_1^s) \times \frac{A_3}{A_1} \approx \frac{1}{100} \chi$$

The ease of measuring these quantities is unfortunately in the reverse order compared to the signal strength, but in cases where statistics is the limiting factor, it is most likely useful to attempt to measure the strongest signal.

If one wants to be general, one can ask how large $\chi$ could be. It is possible to have $\Delta S=1$ CP violation without generating the parameter $\epsilon'$. This occurs if the $\Delta S=1$ CP odd interaction is parity conserving or if it has the same isospin structure in $K \rightarrow \pi \pi$ as does the CP even interaction (as occurs in the isoscope conjugate left right model). The bound on $\chi$ then comes from $\epsilon$. The $\Delta S=1$ Hamiltonian can contribute to $\epsilon$ in the mass matrix through dispersive contributions, which for a parity conserving theory would primarily be $K^0 \rightarrow \pi, \eta, \eta' \rightarrow \bar{K}^0$, with one of the interactions being the usual CP conserving interaction. For SU(3) octet Hamiltonians the $\pi^0$ and $\eta$ intermediate states cancel in the SU(3) limit and the $\eta'$ becomes the dominant dispersive contribution. We would then have

$$\sqrt{2} \epsilon \Delta m = 2A_{K\eta'}^{(+)} \frac{1}{m_K^2 - m_{\eta'}^2} A_{K\eta'}^{(-)}$$
with \( A^{(+)}(\Lambda m, \Lambda m') \) being the CP conserving (nonviolating) matrix elements for \( K \rightarrow \eta' \).

Using \( A^{(+)}(\Lambda m, \Lambda m') \sim A^{(+)}(\Lambda n, \Lambda n') \) and taking the latter from data via the soft pion theorem, we find \( A^{(-)}_{\Lambda n} \sim 2 \times 10^{-10} \text{ GeV}^2 \). To translate this into an estimate of \( \chi \), I divide it by \( 2m_K \) to account for the different normalizations of baryon and meson matrix elements, and compare it with the parity conserving \( A_{\Lambda \Lambda} \) matrix element (again using PCAC)

\[
\chi \sim \frac{(A_{\Lambda n, \Lambda n'}/2m_K)}{A_{\Lambda \Lambda}} \sim 3 \times 10^{-3}
\]

Given the rough nature of this estimate, one could perhaps allow a value for \( \chi \) up to \( 10^{-2} \) but much larger would be in conflict with the constraint imposed by the magnitude of \( \epsilon \). This means that measurements become significant contributions to the field for \( \beta + \bar{\beta} \sim 10^{-2} \), \( \alpha + \bar{\alpha} \sim 10^{-3} \) and \( \Delta \sim 10^{-4} \).

The analysis above has been model independent. The various models of CP violation will produce differing signal strengths. The KM model generates \( \Delta S=2 \) CP violation primarily through the box diagram, but \( \Delta S=1 \) effects enter due to the penguin interaction, with strength roughly \( \chi/\epsilon \sim 20 \epsilon'/\epsilon \sim 1/10 \). In contrast, the Weinberg Higgs model has primarily \( \Delta S=1 \) effects even in the kaon system. Here the strength is nominally \( \chi/\epsilon \sim 1 \) but comes out somewhat less in practice. In a version of the left right model one also has \( \chi/\epsilon \sim 1/10 \). Of course, in superweak models \( \chi = 0 \). Detailed calculations supporting these estimates have been carried out using bag model matrix elements. These are described in detail elsewhere, but I quote the results for \( \Lambda \) and \( \Xi \) decay in the Table below. One should note that bag models generally provide smaller nonleptonic matrix elements than either other quark models or experiment (by a factor of two or more). Because of this, these estimates could well be too low. In addition any particular number is sensitive to the D/F ratio in both the S waves and P waves, which is not well understood. Thus these numbers should be taken as rough guides only.
\[
\begin{array}{ccc}
\Lambda \rightarrow p\pi^- \\
\beta + \bar{\beta} & \alpha + \bar{\alpha} & \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \\
\hline
\text{KM} & 5 \times 10^{-4} & -0.8 \times 10^{-4} & -5.4 \times 10^{-7} \\
\text{Higgs} & 3 \times 10^{-4} & -4 \times 10^{-5} & -7.8 \times 10^{-6} \\
\text{Left-Right} & 1 \times 10^{-4} & -1.4 \times 10^{-5} & 0 \\
\end{array}
\]

\[
\Xi^- \rightarrow \Lambda^0\pi^- \\
\beta + \bar{\beta} & \alpha + \bar{\alpha} & \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \\
\hline
\text{KM} & 2 \times 10^{-4} & -0.7 \times 10^{-4} & 0 \\
\text{Higgs} & 1 \times 10^{-3} & -3 \times 10^{-4} & 0 \\
\text{Left-Right} & -0.6 \times 10^{-4} & 2 \times 10^{-5} & 0 \\
\end{array}
\]

Table I. Estimates of CP violating quantities in hyperon decay

If we now return to pp, we can see how these decay parameters translate into CP odd asymmetries. By rewriting the correlation in Eq. 1, we see

\[
A_\Lambda = \vec{k} \cdot (\vec{p} \times (\vec{p} + \vec{q}) - \vec{p} \times (\vec{p} + \vec{q}))
\]

\[
= -\vec{k} \times (\vec{p} + \vec{q}) \cdot [\vec{p} + \vec{p}]
\]

since \(\vec{p} + \vec{q} = -\vec{p} + \vec{q}\). The first cross product defines the production plane in the center of mass. The asymmetry just counts the numbers above and below the production plane. In general the \(\Lambda\) and \(\bar{\Lambda}\) will be produced with equal polarization normal to the production plane. Given this polarization (in magnitude of order 50% for much of the range of momentum transfer\(^5\)) the \(\alpha\) parameter will generate a nonzero value for \(A_\Lambda\). The angular distribution is

\[
W(\theta, \bar{\theta}) = \frac{1}{16\pi^2} \left[ 1 + P(\alpha \cos \theta_y + \bar{\alpha} \cos \bar{\theta}_y) \right]
\]

\[
+ \sum_{i,j=x,y,z} C_{ij} \cos \theta_i \cos \bar{\theta}_j
\]
Integrating the asymmetry we find

\[ \Lambda_{\Lambda} \equiv \frac{N_{\text{up}} - N_{\text{down}} + N_{\text{up}} - N_{\text{down}}}{N_{p} + N_{\bar{p}}} \]

\[ = \frac{1}{2} P \cdot (\alpha + \bar{\alpha}) \]

A few comments on this result are in order. Background processes cannot generate this asymmetry, so that experimenters do not need to make tight cuts to eliminate background. In this way the counting asymmetries are better than comparing separate measurements of \( \alpha \) and \( \bar{\alpha} \). In addition when averaging over various values of \( t \), the asymmetry can be weighted to take maximum use of the polarization. For example if one knows that the polarization reverses sign at some \( t_c \) one can use

\[ \Lambda'_{\Lambda} = \Lambda_{\Lambda} \theta(t - t_c) - \Lambda \theta(t_c - t) \]

in order to have the contribution add instead of cancelling. Finally one doesn't have to use only those events where both \( \Lambda \) and \( \bar{\Lambda} \) decay into charged particles. One can use the \( p, \bar{p} \) information in

\[ \Lambda \bar{\Lambda} + (p\pi^- - \bar{p}\pi^+) + (N\pi^0 - \bar{p}\pi^+ + p\pi^- - N\pi^0) \]

equally well.

The asymmetry above did not involve the optimal parameter \( \beta + \bar{\beta} \). It is probably impossible to observe this with enough sensitivity in \( \Lambda\bar{\Lambda} \) decay because it would involve measuring the final proton or neutron's spin. The best hope would appear to be \( pp \rightarrow E\Xi \) since the final \( \Lambda \) in \( E \rightarrow \Xi\pi \) easily analyzes its own spin. Here we would be after a term

\[ \vec{s}_{\Xi} \cdot \vec{s}_{\Lambda} \times \vec{p}_{\Lambda} \]

The spin of the \( \Xi \) will be in the direction \( \vec{k} \times \vec{p}_{\Xi} = \vec{\eta}, \) while the spin of the \( \Lambda \) will be manifest by the final proton direction \( \vec{q}_{p} \). This says that the asymmetry

\[ B_{\Xi} \equiv \eta \cdot (\vec{p} \times \vec{q} - \vec{q} \times \vec{\eta}) \]

\[ \propto p_{\Xi} \alpha_{\Lambda}(\beta + \bar{\beta})_{\Xi} \]
will be sensitive to \((\beta + \bar{\beta})\) and may represent the largest signal. We are presently classifying the various types of signals and attempting to give estimates of their strength.\(^6\)

It is always valuable for experimenters to look at CP odd signals at any sensitivity that they can muster. However it would be a real breakthrough to actually measure a nonzero value. How feasible is this? From my point of view (however remember that I am a theorist) it does not seem that it is the systematic effects which are the problem. My benchmark here is the field of parity violation in nuclear physics where experimenters have been especially ingenious in measuring small signals. For example the asymmetry in the scattering of polarized protons has been measured at the level of a few times \(10^{-7}\). Since the stakes are much higher in the case of CP violation, I would think that experimenters would be able to do very well here also. The limit will then be most likely statistical. In a preprint by K. Killian, he says that at LEAR they will be able to measure the partial rate asymmetry to a level of \(10^{-3}\) in 10 days with \(10^6\) antiproton/sec. If this accuracy can be transferred to other asymmetries this is close to being significant in some models. I do not know what options are available to increase the statistics in this reaction, but if one is designing a new machine, this subject is important enough that enhancing this sensitivity should be a design goal. From the discussion at this workshop it seems that this may in fact be possible.

The observation of CP violation in hyperons would be much more than just another number. Its very existence would imply that CP violation is a milliweak phenomena, not superweak. The rough magnitude provides a discrimination between theories. However the hyperon system is very rich and many effects can be studied. The decays of \(\Sigma^+, \Lambda, \Xi\) all provide different tests of the CP odd interaction. Each model has a different SU(3) structure. It is likely that if these systems were well studied, the true theory of CP violation could be
determined. I see no other measurements which could be as powerful or as promising as these. Combined with the studies of the kaon system, a good \( p\bar{p} \) machine could be the optimal "CP violation machine".

References