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FORWARD ELASTIC SCATTERING AND TOTAL CROSS-SECTION AT VERY HIGH ENERGIES

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1. Introduction

The successful cooling technique [1] of antiproton beams at CERN has recently allowed the acceleration of proton and antiproton bunches simultaneously circulating in opposite directions in the SPS. Hadron-hadron collisions could so be produced at a centre-of-mass energy one order of magnitude higher than previously available, thus opening a new wide range of energies to experimentation. This technique also made it possible to replace one of the two proton beams in the ISR by a beam of antiprotons, allowing a direct precise comparison, by the same detectors, of $p p$ and $\bar{p} p$ processes at the same energies

The aim of this report is to summarize the recent results on the forward elastic scattering and total cross-section in this new energy domain. For a more complete discussion of this subject the interested reader is referred to a recent review [2] by G. Sanguinetti and myself.

## 2. Forward Elastic Scattering

At very high energy, hadron elastic scattering is believed to be well described by a single scalar, mainly imaginary, amplitude and one usually assumes that, as energy increases, spin tends to play a negligible role. The interaction becomes mostly absorptive, dominated by the many open inelastic channels. Elastic scattering is then essentially the shadow of the inelastic cross-section.

Therefore the elastic amplitude has a mainly diffractive character, in close analogy with the diffraction of a plane wave in classical optics, and its dependence on the momentum transfer $t$ is the Fourier transform of the spatial distribution of the hadronic matter inside the interacting particle. In the simple case of an absorbing disc of radius $R$ and uniform greyness $\xi \leq 1$, to the inelastic cross-section $\sigma_{\text {inel }}=\pi R^{2} \xi$ corresponds an elastic hadronic diffraction pattern which is a steep function of $t R^{2}$. Its integral ol is bigger for larger $\xi$ up to a maximum value $\sigma_{e l}=\sigma_{\text {inel }}$ in the limit of a completely black disc $(\xi=1)$.

In the very forward direction $\left(-t<10^{-3} \mathrm{GeV}^{2}\right)$ the elastic differential cross-section is dominated by an almost real Coulomb amplitude, which is well understood theoretically and can easily be calculated. In the $|t|$ region between $10^{-3}$ and $10^{-2} \mathrm{GeV}^{2}$ the Coulomb and the nuclear amplitude are of the same order of magnitude and may give rise to a non-negligible interference effect, if the hadronic amplitude is not purely imaginary. The measurement of the interference term allows the determination of the phase of the nuclear amplitude $F$ in the forward direction, which is usually expressed as the ratio of the real to the imaginary part of the amplitude $\rho(s)=[\operatorname{Re} F(s, t) / \operatorname{Im} F(s, t)]_{t=0}$.

In this very forward t-region the elastic differential cross-section is determined by both the nuclear and Coulomb amplitude: $\mathrm{d} \sigma / \mathrm{dt}=\left|\mathrm{f}_{\mathrm{N}} \pm \mathrm{f}_{\mathrm{C}} \exp [\mathrm{i} \phi(\mathrm{t})]\right|^{2}$, where the upper sign is used for pp and the lower sign for $\bar{p} p$ scattering. The nuclear amplitude $f_{N}$ can be parametrized in terms of the total cross-section $\sigma_{\text {tot }}$ and of the slope $B$ of the forward elastic peak, by means of the optical theorem, as $f_{N}=(1 / 4 \sqrt{ } \pi)(i+\rho) \sigma_{\text {tot }} \exp (B t / 2)$, disregarding spin effects and assuming that the real and imaginary parts have the same exponential $t$-dependence in this region. The Coulomb amplitude $f_{C}$ represents the well-known Rutherford scattering and is expressed by $f_{C}=-2 \sqrt{ } \pi \alpha G^{2}(t) /|t|$, where $\alpha$ is the fine structure constant and $G(t)$ is the proton electromagnetic form factor. The small phase factor $\phi(t)$ arises from the simultaneous presence of both hadronic and electromagnetic exchanges in the same
diagram [3] and has opposite $s i g n$ in the $p p$ and in the $\bar{p} p$ channels. The relative importance of the interference term is maximum when the nuclear and the Coulomb amplitudes are comparable (at $-t \approx 8 \pi \alpha / \sigma_{\text {tot }}$ ). Its contribution to the cross-section in a first approximation is $\approx-( \pm)(\rho+\phi) \alpha \sigma_{\text {tot }} /|t|$ and can easily be distinguished from the steeper $1 / t^{2}$ dependence of the Coulomb term and from the relatively flat nuclear contribution. If the factor ( $p+\phi$ ) is positive the interference is destructive for $p p$ and constructive for $\bar{p} p$ scattering.

A sizeable destructive interference, which becomes more and more pronounced as energy increases, was indeed observed in the ISR pp data [4]. This was the experimental evidence that. $p$, which was observed to go through zero at Fermilab energies [5] , increases towards higher and higher positive values over the whole ISR energy range. Data on the $\bar{p} p$ elastic differential cross-section in this t-region have also become available recently at the ISR [6] . In this case a constructive interference is observed, which indicates a rising positive value of $\rho$ in this energy range for $\bar{p} p$ scattering too. A summary of ISR data on $\rho$ for both $p p$ and $\bar{p} p$ scattering is shown in Figure 1 , together with lower energy data.

The real part of the elastic amplitude is related to the imaginary part via dispersion relations. On the other hand, the imaginary part at $t=0$ is related to the total cross-section by the optical theorem. As a consequence, it is possible to write the parameter $\rho$ at a given energy as an integral of the total cross-section over energy. Such an integral relation can be approximated by a local expression which relates $p$ to the derivative of $\sigma_{\text {tot }}$ with respect to energy [7] [8] :

$$
\rho(\mathrm{s}) \quad \approx\left(\pi / 2 \sigma_{\text {tot }}\right) \mathrm{d} \sigma_{\text {tot }} / \mathrm{d} \ln \mathrm{~s}
$$

This equation is not really adequate to compute the values of $\rho$ as a function of energy in a quantitative way but has the merit of describing the qualitative connection between $\rho$ and $\sigma_{\text {tot }}$ at asymptotic energies in a transparent way. In particular, it allows to understand easily the result, rigorously proved in Reference [9] , that a rising total cross-section implies a positive value of
p. For instance, in the case of an asymptotic behaviour which saturates the Froissart bound ( $\sigma_{\text {tot }} \sim 1 n^{2} s$ ), $\rho$ goes to zero from positive values as $\pi / \ln s$.

One of the most salient feature of the elastic differential cross-section do/dt observed for $p p$ scattering at the $I S R$ is the presence of a pronounced narrow peak around the forward direction, which decreases almost exponentially in $t$ by more than six orders of magnitude, down to $-\mathrm{t}=1.4 \mathrm{GeV}^{2}$. In $a$ geometrical picture such a smooth diffraction pattern over so wide a t-interval indicates that the transverse distribution of matter inside the proton is nearly Gaussian. As energy increases, the slope of the pp forward peak becomes steeper indicating, in the optical analogy, an expansion of the interaction radius.

Recent data on $\bar{p} p$ elastic scattering at the $I S R$ are compared in Figure 2 with pp data taken at the same energies by the same experiment [10]. It is remarkable how the shape of the $\bar{p} p$ distribution, which is notably steeper at lower energies, becomes similar to that of pp distribution in the ISR energy range. The energy dependence of the slope of the elastic differential cross-section for $\bar{p} p$ and $p$ scattering, shown in Figure 3, indicates a shrinkage of the diffraction peak at a rate of at least $\ln s$ [11] [12] [13]. It appears evident from there that, as energy increases, the antiproton and the proton tend to behave exactly in the same way. Such a behaviour is indeed expected at asymptotic energies, since the Cornille-Martin theorem states that the elastic differential cross-sections of particle and antiparticle in the region of the diffraction peak tend to be the same for $s \rightarrow \infty$.

From Figure 3 one can also notice that the value of the slope $B$ is different at different values of $t$. As a matter of fact, the forward elastic peak deviates from a pure exponential in $t$, and rather than a constant slope $B$ one should introduce a local slope $B(t)=(d / d t) \ln (d \sigma / d t)$. A slope which continuously decreases with increasing $|t|$ is observed up to Fermilab energies, where the elastic diffraction peak is reasonably well described by an exponential with the addition of a small quadratic term [14] , of the kind $\exp \left(B t+C t^{2}\right)$. On the other hand, ISR data exhibit a rather sudden break
localized around $-t \simeq 0.13 \mathrm{GeV}^{2}$. A similar feature is observed in the $\bar{p} p$ elastic differential cross-section measured at the SPS Collider at $\forall \mathrm{s}=546 \mathrm{GeV}$. As illustrated in Figure 4, in the region $0.03<-t<0.15 \mathrm{GeV}^{2}$ the data show no hint of curvature and are well fitted by a single exponential of slope $B=15.2 \pm 0.2 \mathrm{GeV}^{-2}[15]$ with no need for a quadratic term. A significant quadratic dependence can only be obtained from fits over a t-region that extends up to at least $0.3 \mathrm{GeV}^{2}$. However, such a parametrization appears to be inadequate to represent the data in the t-range from 0.03 to $0.5 \mathrm{GeV}^{2}$. On the other hand, a single exponential fits the data in the region $0.2<-t<0.7 \mathrm{GeV}^{2}:$ it is remarkable that in such a wide $t$-interval the slope appears to remain constant [15] [16] [17] [18] , with a value of $13.4 \pm 0.3 \mathrm{GeV}^{-2}$. The phenomenon of a fast slope variation by about two units in a small t-interval between 0.1 and $0.2 \mathrm{GeV}^{2}$ is represented in Figure 5 (from Reference [19], where the local slope of the diffraction peak is shown in various $t$-intervals, at $\quad \checkmark s=53$ and 546 GeV . Independently of this double-slope structure, the antishrinkage observed in $\bar{p} p$ elastic scattering at lower energy turns now into a shrinkage similar to that of the pp diffraction peak. In a geometrical picture the slope of the diffraction peak is proportional to the square of the mean interaction radius: the shrinkage of the diffraction peak therefore reflects an increase of the dimension of the nucleon.

## 3. Total Cross-Section

The total cross-section is one of the basic parameters of hadron scattering processes. Although this quantity is directly the sum of the many cross-sections of all accessible final states, it is simply related via the optical theorem to the imaginary part of the amplitude of forward elastic scattering. To such a simple process, axiomatic approaches are possible exploiting the general principles of scattering theory, such as unitarity, analyticity, and crossing
symmetry. Therefore a number of theorems and bounds can be derived, which constrain the asymptotic behaviour of the total cross-section in the limit $s \rightarrow \infty$. Well-known examples are the Froissart bound, which limits the asymptotic energy dependence of $\sigma_{\text {tot }}$ to a $\ln ^{2} s$ growth at maximum, and the Pomeranchuk theorem, which states that the ratio of the total cross-sections of particle and antiparticle should tend to unity. Experimental measurements of $\sigma_{\text {tot }}$ at the highest available energies therefore play a fundamental role in probing the setting in of an eventual asymptotic regime.

Total cross-sections cannot be measured at colliding-beam machines by traditional transmission techniques. Three different methods have been used instead, by measuring simultaneously two of the three following quantities: total interaction rate, forward elastic rate, machine luminosity. The most direct method [20] determines $\sigma_{\text {tot }}$ by the ratio between the total interaction rate, measured in a detector with full solid-angle coverage, and the luminosity. A second method [21] exploits the optical theorem, extrapolating to $t=0$ the measured rate of small-angle elastic scattering. In both cases an absolute calibration of the machine luminosity is required, usually performed either by the van der Meer method [22] [23]: or by direct measurements of beam profiles [24] [25] , or even by the measurement of a known electromagnetic process such as Coulomb scattering [26] . When a high-precision luminosity calibration is not available, the simultaneous measurement of low-t elastic scattering and of the total rate allows the determination of $\sigma_{\text {tot }}$ without the need for an independent luminosity measurement [27] [28] [29]. The use of the optical theorem in the second and third methods implicitly assumes that spin effects at small $t$ can be disregarded. The plausibility of this assumption was firmly supported by the precise measurements performed at the ISR, where all three methods were exploited simultaneously [27], obtaining consistent results to an acuracy of better than $1 \%$ (see a discussion in Reference [30]).

Until the coming into operation of the ISR, total cross-sections were believed to approach a finite limit with increasing energy. Actually some hints
of a growth were present in cosmic-ray data at very high energy [31] in addition to the well-known rising trend of $\sigma_{\text {tot }}\left(K^{+} p\right)$ that was considered as a transient feature. Moreover, the possibility of indefinitely rising hadron cross-sections had also been considered theoretically [32] [33] . Nonetheless, the generally-accepted prejudice was that total cross-sections should tend to constant values, and the discovery of the pp rising cross-section at the ISR [21] [20] undeniably came as a surprise. Afterwards, this trend was found to be a general feature of all hadronic total cross-sections.

The value of $p$ is sensitive to the variation of $\sigma_{\text {tot }}$ with energy, via dispersion relations. The simultaneous study of $\rho$ and $\sigma_{\text {tot }}$ therefore provides a better understanding of the energy dependence of $\sigma_{\text {tot }}$ and allows a sensible extrapolation in a domain which extends well beyond the accessible energy range. This kind of analysis, pioneered in Reference [4] on $\rho$ and $\sigma_{\text {tot }}$ data then available up to ISR energies, was indeed able to predict that the pp and $\overline{\mathrm{p}} \mathrm{p}$ total cross-sections should keep rising at least to $\sqrt{ } \approx 300 \mathrm{GeV}$ at a rate very close to $\mathrm{ln}^{2} \mathrm{~s}$.

Recently, the $\overline{\mathrm{p}} \mathrm{p}$ total cross-section has been measured at the SPS Collider [29] at an energy as high as $\sqrt{s}=546 \mathrm{GeV}$. The observed value of $61.9 \pm 1.5 \mathrm{mb}$ is $\sim 50 \%$ higher than at the ISR and indicates that a $\sigma_{\text {tot }}$ rise at a rate compatible with $\ln ^{2} s$ persists also in this new energy domain. This Collider measurement is shown in Figure 6 together with a compilation of lower energy pp and $\overline{\mathrm{p}} \mathrm{p}$ total cross-section data which includes the recent $\overline{\mathrm{p}} \mathrm{p}$ results from the ISR [6] [34] . The dispersion relation fit of Reference [4] mentioned above is also shown in Figure 6 for comparison (the simultaneous fit to the real part is the curve shown in Figure 1). One can notice that the recent $\overline{\mathrm{p}} \mathrm{p}$ data at the highest energies agree well with the prediction of this analysis and their inclusion in fits of this kind confirms the $\ln ^{2} s$ behaviour of $\sigma_{\text {tot }}$ [13] [35]. It is striking that data at present energies indicate a rate of growth of $\sigma_{\text {tot }}$ which is just the fastest function allowed at asymptotic energies by the Froissart bound. A suggestive hypothesis is that the observed qualitative
saturation of the Froissart bound could be the manifestation of an asymptotic regime, appearing already at present energies, which leads hadron total cross-sections to increase indefinitely at this rate.

At the energy of the Collider the $p p$ and $\bar{p} p$ total cross-sections are expected to have practically the same value. The total cross-section difference $\Delta \sigma=\sigma_{\text {tot }}(\bar{p} p)-\sigma_{\text {tot }}(p p)$ is shown in Figure 7. Its energy dependence exibits a Regge behaviour of the kind $s^{-\alpha}$, with $\alpha \approx 1 / 2$, which makes it tend to zero rather rapidly. Actually, if total cross-sections rise indefinitely, their difference does not necessarily have to vanish and might even increase logarithmically with energy, still preserving the limit $\sigma_{\text {tot }}(\overline{\mathrm{p}} \mathrm{p}) / \sigma_{\text {tot }}(\mathrm{pp}) \rightarrow 1$ as required by the Pomeranchuk theorem at infinite energy. Non-vanishing asymptotic contributions (odderons) to the part that is odd under crossing of the forward elastic amplitude, which determines the behaviour of $\Delta \sigma$, have indeed been considered in the literature [36]. The operation of the ISR both with proton and antiproton beams has allowed the study of the convergence of pp and $\bar{p} p$ total cross-sections in an energy interval where the difference is expected to become very small. While $\sigma_{\text {tot }}(\bar{p} p)$ starts to rise at these energies, the difference $\Delta \sigma$ keeps decreasing following the same inverse power law observed at lower energy. The ISR data from Reference [34] (black points in Figure 7) could actually suggest a small, systematic deviation above the Regge fit. However, this measurement could include a small but significant electromagnetic inelastic contribution to $\Delta \sigma$-- up to 0.4 mb according to the authors -- from which the data of Reference [6] (open circles) are free. As a conclusion, the data are well compatible with the behaviour expected in the framework of a standard Regge exchange picture, with no need for odderons, though a sufficiently small contribution of this kind cannot of course be excluded.

Data on pp and $\bar{p} p$ total elastic cross-section $\sigma_{e l}$ measured at the $I S R$ and at the SPS Collider, and the corresponding ratio $\sigma_{e l} / \sigma_{\text {tot }}$, are shown in Figures 8 a and 8 b , respectively, together with lower energy data. Similarly to $\sigma_{\text {tot }} \sigma_{e l}$ also starts rising at $I S R$ energies, reaching a value of
$13.3 \pm 0.6 \mathrm{mb}$ at the $\operatorname{SPS} \overline{\mathrm{p}} \mathrm{p}$ Collider [29]. More than $\sigma_{e l}$ itself, the ratio $\sigma_{e 1} / \sigma_{\text {tot }}$ has a crucial role in the investigation of the high-energy regime of hadron scattering, since this parameter is directly sensitive to the hadron opacity. Great interest was aroused a few years ago, when this ratio was found to reach a constant value of about 0.175 over the whole energy range of the ISR [37]. This value is below the saturation limit of 0.5 , which is typical of a fully absorbing black disc. This was interpreted as the attainment of a "geometrical scaling", regime in which the proton blackness stabilizes at a rather "grey" value and the observed $\ln ^{2} s$ rise of $\sigma_{\text {tot }}$ only reflects a steady expansion of the proton radius like ln s. However, the validity of this attractive simple picture has been denied by the recent result from the SPS pp Collider. The measured value $\sigma_{e l} / \sigma_{\text {tot }}=0.215 \pm 0.005$ at $\sqrt{ }=546 \mathrm{GeV}$ [29] definitely indicates an increase of the opacity with increasing energy.

This violation of "geometrical scaling" appears quite evident also in an impact parameter analysis [38] of $\bar{p} p$ elastic scattering data at the SPS Collider. Here, in addition to a clear expansion of the interaction radius, the central opacity is also found to be definitely larger than at ISR energies (see a more detailed discussion in Reference [2] ).

It is worth stressing that the energy dependence of the parameter $\sigma_{e l} / \sigma_{\text {tot }}$ is crucial in order to discriminate between the various theoretical models that claim to describe the asymptotic behaviour of hadron scattering. These can be classified coarsely in three groups [39]: in addition to the "geometrical scaling" approach mentioned above, in which the ratio $\sigma_{e l} / \sigma_{\text {tot }}$ is expected to be energy independent, other models have been developed in which this parameter is predicted either to decrease or to increase with energy. In the Reggeon Field Theory with "critical Pomeron" [40] the opacity, and thus $\sigma_{e l} / \sigma_{\text {tot }}$, decreases with increasing energy, while the expansion of the radius prevents $\sigma_{\text {tot }}$ from decreasing. This approach seems now to be excluded by the Collider measurements.

A more interesting approach stems from the original idea that was at the basis of the Chou-Yang model [41]. In this geometrical picture, elastic scattering is considered as the result of the attenuation of the incoming wave through the extended distribution of matter inside the hadron. In this kind of models ("factorizing eikonal") [33] [42] [43] the asymptotic regime is still far away in energy, although $\sigma_{\text {tot }}$ already increases with a dependence close to $\ln ^{2} s$ : the opacity is also increasing slowly with energy, so that hadrons tend gradually to become completely black discs. The ratio $\sigma_{e l} / \sigma_{\text {tot }}$ is therefore predicted to reach the value $1 / 2$ at infinite energy (the "supercritical string" model [44] also exhibits similar features). The recent findings at the SPS $\bar{p} p$ Collider clearly favour the latter class of models.

Another parameter often used to probe the asymptotic behaviour of hadron scattering, which however is not independent of $\sigma_{e l} / \sigma_{\text {tot }}$, is the ratio $\sigma_{\text {tot }} / B$. Similarly to the former, the latter is also sensitive to the hadron opacity. The trend of this quantity is illustrated in Figure 8 c up to Collider energy and appears to be similar to that of $\sigma_{e 1} / \sigma_{\text {tot }}$. After reaching a constant plateau in the ISR energy range at about 8.4 it rises again up to $\sigma_{\text {tot }} / B=10.5 \pm 0.3$ at $\sqrt{ }=546 \mathrm{GeV}$.

As a conclusion, the measurements at the nowadays highest available energies consistently indicate that the hadron opacity, which was believed to have definitely reached a constant value already at ISR energies, is actually increasing: hadrons become not only larger, but also darker as energy increases. Despite the observed qualitative saturation of the Froissart bound, there is still a long way to go in the approach to the elusive asymptopia.

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Figure 1. Summary of high-energy data on the real part $p$ for $p p$ and $\bar{p} p$ forward elastic scattering (from Reference [6]) ; the curve represents the dispersion relation fit of Reference [4].


Figure 2. Comparison of $p p$ and $\bar{p} p$ diffraction peaks at three ISR energies, (from Reference [10])


Figure 3. A compilation of slopes of pp and $\overline{\mathrm{p}} \mathrm{p}$ diffraction peaks (black points and open points, respectively) as a function of energy; (a) in the forward direction (early, low statistics measurements at the SPS Collider [16] [17] have not been included); (b) around $-t \approx 0.2 \mathrm{GeV}^{2}$ (from References [10] [11]).


Figure 4. Elastic differential cross-section for $\overline{\mathrm{p}} \mathrm{p}$ at $\sqrt{\mathrm{s}}=546 \mathrm{GeV}$; the lines are single-exponential fits below and above $-t \simeq 0.15 \mathrm{GeV}^{2}$ (from Reference [15] ).


Figure 5. Local slope of the elastic diffraction peak as a function of $t$ (from Reference [19] ): (a) for $\bar{p} p$ at $\checkmark s=546 \mathrm{GeV}$; (b) for pp at $\sqrt{ } / \mathrm{s}=53 \mathrm{GeV}$. The horizontal bars indicate the t-interval in which the exponential fit was performed.


Figure 6. Total cross-section data for $p p$ and $\bar{p} p$ scattering (early, low statistics measurements at the SPS Collider [17] [28] are not shown). The curve represents the dispersion relation fit of Reference [4].


Figure 7. Total cross-section difference $\Delta \sigma=\sigma_{t o t}(\bar{p} p)-\sigma_{t o t}(p p) \quad$ as a function of energy; the line is a Regge-like fit (from Reference [34]).


Figure 8. Energy dependence of (a) total elastic cross-section $\sigma_{e l}$; (b) ratio $\sigma_{e l} / \sigma_{\text {tot }}$; (c) ratio $\sigma_{\text {tot }} / B$, for $p p$ and $\bar{p} p$ scattering.

