Supersymmetry and Compositeness
— Dynamical Weak Gauge Bosons —

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ABSTRACT

We show at the leading order of 1/N expansion that composite gauge fields in a supersymmetric $U(4n+2)/U(4n)\times SU(2)$ non-linear sigma model become dynamical in some range of parameters. Since the hidden local symmetry in this sigma model is identifiable with a weak SU(2) of the standard electroweak model, we suggest that the observed weak bosons $W^{\pm}$ and Z^0 are indeed the dynamical gauge fields in the non-linear sigma model. Some phenomenological difficulies in identifing such composite fields with $W^{\pm}$ and Z^0 are discussed. We also give a brief review on a supersymmetric preon model which generates our non-linear sigma model as a low-energy effective theory.

I. Introduction

In spite of the excellent success of the standard electroweak gauge theory [1], it is still unclear to us what the origin of the spontaneous breakdown of the $SU(2)\times U(1)$ symmetry is. In the standard scheme, an elementary Higgs scalar φ is assumed to have a vacuum-expectation value $\langle\varphi\rangle \neq 0$ which induces the breaking of the electroweak symmetry. The Fermi scale $G_F^{-1/2}$ is thus related basically to the undetermined, free parameter, $\langle\varphi\rangle$.

Motivated by dynamical understanding of the $SU(2)\times U(1)$ breaking, a technicolor theory has been proposed, in which the electroweak symmetry is supposed to be broken by a condensate of techni-fermions [2]. The needed Higgs scalar φ is, here, not a fundamental particle but a composite state of a pair of techni-fermions which are bound by the technicolor confining forces. This is a natural and very beautiful theory for generating the Fermi scale in a dynamical way. However, no compelling mechanism has been found for giving masses to quarks and leptons without phenomenological difficulties (e.g. a flavour-changing neutral current problem [3]), unfortunately.*)

One alternative option for the dynamical generation of Fermi scale $G_F^{-1/2}$ is a composite model of quarks, leptons and weak bosons, in which the weak interactions are no longer fundamental gauge interactions but rather residual ones originating in the compositeness [5]. The weak bosons are assumed to be spin-one bound states of more elementary particles (say preons) and hence their masses, i.e. Fermi scale, are in principle calculable by the underlying preon dynamics, (similarly to ρ-meson masses in QCD). Some departures from the standard scenario are expected. However, the masses of the weak bosons $W^{\pm}$ and Z^0 discovered at the CERN p-$\bar{\text{p}}$ collider [6] are in good agreement with the prediction of the standard model. Therefore, there might exist a dynamical reason for why such composite objects look like elementary gauge fields and why the standard description of electroweak interactions is an excellent success.

Kugo, Uehara and the present author [7] have recently found that the supersymmetric U(4n+2)/U(4n)×SU(2) non-linear sigma model possesses a hidden local symmetry which is identifiable with the standard electroweak SU(2).**) In this talk, I will show that the hidden local symmetry becomes indeed physical at quantum level and hence the dynamical gauge fields in our non-linear sigma model may be identified with the observed weak bosons $W^{\pm}$ and Z^0.

In sec. II I will review briefly a supersymmetric preon model which generates, as a low-energy effective field theory, the U(4n+2)/U(4n)×SU(2) non-linear sigma model that will concern us. A particular importance of supersymmetry in a composite model will be stressed also. In sec. III I will give a detailed discussion on the dynamical generation of physical poles of the hidden gauge fields in our effective Lagrangian. Problems for identifing such composite states with $W^{\pm}$ and Z^0 are also noted. The last section will be devoted to conclusions.

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*) An alternative scenario, called "composite Higgs model", has been proposed to solve this neutral-current problem by raising the technicolor scale [4].

**) It has been recently pointed out that the chiral Lagrangian on $SU(2)_L\times SU(2)_R/SU(2)_V$ in QCD also possesses a hidden local SU(2) [8]. The ρ mesons may be considered as composite gauge fields of the hidden symmetry.

II . A supersymmetric composite model

An advantage of supersymmetry is that it always provides us with natural mechanisms to guarantee light composite fermions. If a global symmetry G in a preon theory remains unbroken, certain massless composite fermions are required to satisfy the 't Hooft consistency condition [9]. On the other hand if G is broken, for instance by a preon-preon condensate, there will necessarily appear massless fermion bound-states as superpartners of Nambu-Goldstone bosons (quasi N-G fermions) [10] in the presence of supersymmetry. We discuss some interesting aspect of the latter case by using an example.

Our example [11] is based on an $SU(2)_H$ supersymmetric confining theory with (4n+2) doublet preons χ^i_α ($i = 1 \sim 4n+2$ and $\alpha = 1, 2$). The classical global symmetry is

$$G_{cl.} = SU(4n+2) \times U(1) \times U(1)_R \tag{1}$$

where the last $U(1)_R$ refers to R symmetry. However, both two U(1)'s, U(1) and $U(1)_R$, have strong-$SU(2)_H$ anomalies and hence only a linear combination of these two U(1)'s is anomaly free and conserved. The global symmetry is reduced to at quantum level

$$G = SU(4n+2) \times U(1)' \equiv U(4n+2) \ . \tag{2}$$

The G-invariance is most likely broken, since we find no solution of the 't Hooft consistency condition about $[G]^3$ anomalies, except for n = 1 [12]. Let us suppose the simplest condensate

$$\varepsilon^{\alpha\beta} < \chi^1_\alpha \chi^2_\beta > = v \tag{3}$$

which causes the breaking

$$U(4n+2) \longrightarrow U(4n) \times SU(2) \ . \tag{4}$$

Corresponding to this breakdown there arise 8n+1 massless N-G chiral-multiplets which are bound states of preons,

$$\begin{aligned} \phi^a_i &= \varepsilon^{\alpha\beta} \chi^a_\alpha \chi^i_\beta \qquad \begin{pmatrix} i = 1 \sim 2 \\ a = 3 \sim 4n+2 \end{pmatrix} \\ \phi &= \varepsilon^{\alpha\beta} \chi^1_\alpha \chi^2_\beta - v \end{aligned} \tag{5}$$

It is easy to see that fermion components of the first chiral-multiplets ϕ^a_i are identifiable with n families of left-handed quarks and leptons, by assigning the suitable $SU(3)_c \times U(1)_{em}$ charges to preons [11]. The ϕ corresponds to the physical Higgs field in the supersymmetric standard model.

The quasi N-G fermions (i.e. the left-handed quarks and leptons) are supersymmetric partners of N-G bosons arising from the U(4n+2)→U(4n)×SU(2) breaking. Besides this the quasi N-G fermions in (5) have another important role, because the massless fermions in (5) are complex with respect to the unbroken group. The 8n+1 massless fermions are precisely the states needed for the 't Hooft consistency condition. No other fermions are required to satisfy the chiral anomaly matching. Therefore, as long as the H = U(4n)× SU(2) in kept unbroken, the quasi N-G fermions in (5) remain massless even after the supersymmetry breaking is switched on [10,11,13].

Introducing n families of right-handed quarks and leptons f_a which are elementary fields here, the mass term of quarks and leptons is engendered by Yukawa couplings of f_a to preons as

$$\mathcal{L} = G^{(i)}_{ab}\, \varepsilon^{\alpha\beta} \chi^{a}_{\alpha} \chi^{i}_{\beta} f_b \quad + h.c. \tag{6}$$

The masses of quarks and leptons, $\phi^a_i \hat{m}_{ab} f_b$, are proportional to the Yukawa coupling $G^{(i)}_{ab}$ (similarly to those in the standard model) and hence there arise no flavour-changing neutral current problem. This is a nice point of this model, although the way to produce quark and lepton masses may not be necessarily the whole story of elementary particle physics.

The low-energy effective Lagrangian of the N-G chiralmultiplets ϕ^a_i and ϕ is obtained by the method of supersymmetric U(4n+2)/U(4n)×SU(2) non-linear realization [7,14] as ;

$$\mathcal{L} = \int d^4\theta \; v^2 \; F\left(\det\left[\bar{\xi}^{\dot{i}}_{\alpha} \; \xi^{\alpha}_{\dot{j}} \right] \right) \quad , \tag{7}$$

$$\xi^{\alpha}_{i} = \begin{pmatrix} e^{k\Phi} \delta^{\alpha}_{i} \\ \Phi^{a}_{i} \end{pmatrix} \quad . \tag{8}$$

Here, i = 1,2, $\alpha = 1 \sim 4n+2$ and $a = 3 \sim 4n+2$ with the normalization constant v and κ determined by

$$v^2 \frac{\partial^2 F}{\partial \bar{\Phi}^{i}_{a} \partial \Phi^{b}_{j}} \Bigg|_{\Phi^a_i = \Phi = 0} = \delta^{a}_{b} \, \delta^{j}_{i} \quad , \tag{9}$$

$$v^2 \frac{\partial F}{\partial \bar{\Phi}\, \partial \Phi} \Bigg|_{\Phi^a_i = \Phi = 0} = 1 \quad . \tag{10}$$

The parameter v is a dimension-one constant which corresponds to the energy scale of the preon-preon condensate (3). F(x) is an arbitrary function and it's arbitrariness is basically due to the presence of one extra quasi N-G bosons [11,14]. However, it should be noted that the choice $F(x) = \sqrt{x}$ which we will assume later, has a special geometrical meaning. In this case, our non-compact manifold $GL(4n+2)/GL(4n)\times SL(2)$ has a asymptotic flat metric gij, i.e. $gij \to 1$ for ϕ_i^a and $\phi \to \infty$.

As pointed out in Ref.[6], the residual interactions (7) among quarks and leptons have a hidden local SU(2) invariance in addition to the global U(4n+2). This hidden SU(2) symmetry is made manifest by introducing redundant non-abelian gauge supermultiplets $V^i_j(x,\theta)$ $(i,j = 1\sim 2)$ and rewriting the Lagrangian (7) as

$$\mathcal{L} = \int d^4\theta \, v^2 G[\, \xi^\alpha_i (e^V)^i_j \bar{\xi}^j_\alpha \,] \quad , \tag{11}$$

$$\mathrm{Tr}\, V = 0 \quad . \tag{12}$$

$G(x)$ satisfies the relation $G(2\sqrt{x}) = F(x)$. The gauge multiplets V^i_j are just redundant variables since there is no kinetic term of V^i_j. By integrating over V^i_j we easily see that the Lagrangian (11) is equivalent to the original one (7) [7].

If $G(x) = c+x$, the effective interactions of (11) have precisely the same form as those in the standard Weinberg-Salam model including even Higgs sectors. Therefore, our main question is whether the kinetic term of V^i_j is dynamically generated. If the answer is "yes", there arises an interesting possibility that the observed weak bosons $W^\pm$ and Z^0 may not be elementary but dynamical gauge fields of our hidden symmetry. In the next section we will discuss a possible dynamics to produce physical poles of the hidden gauge fields V^i_j.

II. A dynamical generation of hidden gauge fields

The 1/N expansion is known powerful for analysing the dynamical aspects of non-linear sigma models. In fact it has been proved at the leading order of 1/N expansion that redundant gauge fields of CP^N model become physical in two and three dimensional space-time [15]. We discuss, in this section, quantum effects in the non-linear sigma model on $U(4n+2)/U(4n)\times SU(2)$, using the 1/4n expansion, and show a generation of physical poles of the hidden gauge fields V^i_j.

We addopt an ultraviolet cutoff at the U(4n+2) chiral symmetry breaking scale, $\Lambda_{\chi B}$ to remove all divergencies, since the non-linear Lagrangian (11) is unrenormalizable in the four dimentional space-time. It is, however, quite natural to use the ultraviolet cutoff at the momentum $\sim\Lambda_{\chi B}$ in evaluating the quantum effects of our non-linear sigma model, because our effective theory is valid only at distances larger than $1/\Lambda_{\chi B}$.

We choose the function F(x) in (7) to be the asymptotic flat form $F(x) = \sqrt{x}$, that is $G(x) = x$ in equ.(11). To see massive poles of the gauge field V^i_j, we introduce the following explicit breaking term of U(4n+2) invariance;

$$v\varepsilon^{ij}\left\{G^{(1)}_{ab}\,\phi^a_i\,\xi^1_j\,f_b + G^{(2)}_{ab}\,\phi^a_i\,\xi^2_j\,f'_b\right\} \qquad (13)$$

with f_b and f'_b being the elementary right-handed quarks and leptons. The equ.(13) gives the mass term of quarks and leptons in the broken phase of the hidden SU(2), $\langle\xi^1_1\rangle = \langle\xi^2_2\rangle = 1$. We assume, for simplicity, the common mass m for quarks and leptons;

$$G^{(1)}_{ab}\,v = G^{(2)}_{ab}\,v = \delta_{ab}\,m \quad . \qquad (14)$$

Now our effective Lagrangian is

$$\mathcal{L} = \int d^4\theta \left\{\xi^\alpha_i (e^V)^i_j\, \bar{\xi}^j_\alpha + \bar{f}^b f_b + \bar{f}'^b f'_b\right\}$$
$$+ \int d^2\theta \left\{G^{(1)}_{ab}\,\xi^a_i\,\xi^1_j\,f_b + G^{(2)}_{ab}\,\xi^a_i\,\xi^2_j\,f'_b\right\} + h.c. \qquad (15)$$

Here, the scale factor v has been absorbed in ξ^α_i by redefining $\xi^\alpha_i \equiv v\cdot\xi^\alpha_i$ (old) and as a consequence the Lagrangian (15) has no scale. Notice that we should not add any constraint with a Lagrange multiplier unlike in the case of non-supersymmetry. In our case the constraints are obtained from the equation of motion for D-components of the vector multiplets $V^i_j(x,\theta)$ as

$$\varphi^1_2 = \varphi^2_1 = 0 \quad ,$$
$$\varphi^1_1 = \varphi^2_2 \quad . \qquad (16)$$

with φ^i_j being the scalar components of ξ^i_j $(i,j = 1\sim 2)$. However, it is clear that the magnitude of vacuum-expectation value $\langle\xi^1_1\rangle = \langle\xi^2_2\rangle = v/2$ is not determined by the constraints (16). This situation remains unchanged even at the quantum level, because there is no tadpole diagram for the D-components of V^i_j due to the SU(2) symmetry. Therefore, we consider that the scale v is already fixed by the underlying preon dynamics and hence taken of the order of the composite scale.

Let us first integrate over the $2\times 4n$ ϕ^a_i fields. Using 1/4n expansion of the effective action S_{eff}, we obtain the inversed propagator $D^{-1}_{\mu\nu}(p)$ for the hidden SU(2) gauge bosons W^i_μ ($i = 1\sim 3$) in the leading order approximation;

$$D^{-1}_{\mu\nu}(p^2) = \frac{v^2}{4} g_{\mu\nu} - 4n\,(p^2 g_{\mu\nu} - p_\mu p_\nu)\,\Gamma(p^2),$$

$$\Gamma(p^2) = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} dy \int \frac{d^4 q}{(2\pi)^4}\,\left(q^2 - p^2 y^2 + \frac{p^2}{4} - m^2\right)^{-2}. \tag{17}$$

The ultraviolet cutoff at $q^2 \sim \Lambda^2_{\chi B}$ in the logarithmically divergent integral (17) yields

$$\Gamma(p^2) = \frac{1}{32\pi^2}\left\{ \log\frac{\Lambda^2_{\chi B} + m^2}{m^2} - 1 \right.$$
$$+ 2\sqrt{\frac{4m^2 + 4\Lambda^2_{\chi B} - p^2}{p^2}}\,\arctan\sqrt{\frac{p^2}{4m^2 + 4\Lambda^2_{\chi B} - p^2}}$$
$$\left. - 2\sqrt{\frac{4m^2 - p^2}{p^2}}\,\arctan\sqrt{\frac{p^2}{4m^2 - p^2}} \right\} \tag{18}$$

We find, in a range of parameters (mass of quarks and leptons m, the number of generation n and $\Lambda_{\chi B}$), that the propagator $D_{\mu\nu}(p^2)$ has a pole on a physical sheet of complex momentum plane [16]. Parameter regions where the physical pole appears are shown in Fig.(1) for $m = 100$ GeV with $v = 250$ GeV.

The gauge coupling constant α_2, equivalently to the mass of the SU(2) gauge bosons W^i_μ ($m_W^2 = \frac{1}{4} g^2 v^2$), is also given by

$$\alpha_2 = \frac{1}{4\pi}\,\frac{1}{4n\,\Gamma(m_W^2)}. \tag{19}$$

Introducing fundamental U(1) gauge interactions in the Lagrangian (15) we find the Weinberg mixing in the standard form, since our effective Higgs ξ^i_j transform as the SU(2) doublets. Thus, we have the Weinberg formula

$$\rho = m_W / m_Z \cos\theta_W = 1, \tag{20}$$

$$\sin^2\theta_W = \alpha_{em}/\alpha_2 \tag{21}$$

For $n = 5\sim 10$ and $\Lambda_{XB} = 1\sim 10$ TeV, we get

$$\sin^2\theta_W = 0.04 \sim 0.08. \tag{22}$$

It is quite difficult to obtain the observed value, $\sin^2\theta_W \simeq 0.22$, unless the number of generations n is very large or $\Lambda_{XB} \gg 1$ TeV. However, we should not take the result (21) at face value, since we have completely neglected the short-distance contributions ($q^2 > \Lambda_{XB}$) in our calculation.

It should be finally noted that the mass $m_W = 100 \sim 200$ GeV derived from the result (21) is rather small relatively to the preon-condensate scale $\langle\chi\chi\rangle \sim 250$ GeV (compared with QCD; that is $m_\rho \simeq 700$ MeV with $\langle\bar{q}q\rangle \sim 100$ MeV). The main reson for this is the presence of the large number of quarks and leptons ($4n = 20 \sim 40$).

IV. Concluding remarks

In our analysis, we have found that the composite gauge fields of hidden SU(2) symmetry in the supersymmetric U(4n+2)/U(4n)×SU(2) model become dynamical in some range of parameters. Because of the similarity of the effective Lagrangian (11) to those of the Weinberg Salam model, the composite gauge bosons may be identified with the weak bosons $W^\pm$ and Z^0 discovered at the CERN $p\bar{p}$ collider. In our scheme, the non-linear sigma model is a low-energy approximation of the underlying preon theory and hence the appearance of $W^\pm$ and Z^0 bosons is regarded as a dynamical consequence of the preon theory.

However, it turns out that it is quite difficult to obtain $\sin^2\theta w \simeq 0.22$ within the framework of the non-linear sigma model. Perhaps, we should not be, however, too critical here, since we have neglected, for example, the short-distance effects which may be dominated by preon-loop diagrams. In fact the short-range forces in our preon theory will be more important than in the case of QCD, since the $SU(2)_H$-confining hypercolor interactions are not asymptotic free for $n \geq 3$.

Furthermore, if the chiral symmetry breaking scale Λ_{XB} is grater than the confining scale Λ_{conf} as suggested in QCD by Manohar and Georgi [17], we have the effective field theory in the intermediate region between Λ_{XB} and Λ_{conf}, in which constituent preons are also interacting with the Nambu-Goldstone modes. Contributions to the kinetic term of V^i_j from the preon-loop diagrams may have the same sign as those from the ξ-loop diagrams (although the calculation is practically beyond the scope of this talk because of the non-perterbative effects of the hypercolor interactions). If it is the case, the effective gauge coupling α_2 of the composite weak bosons becomes smaller.

In any case, some unconventional dynamics is required to explain the small gauge coupling constant $\alpha_2 \sim 1/25$ if the weak bosons $W^\pm$ and Z^0 are indeed the composite gauge fields as we suggested. This may be the most crucial difficulty for identifing our dynamical gauge fields with $W^{\pm}$ and Z^0. However, we should keep in our mind an intriguing possibility that the short-distant part of the hypercolor forces may dominantly contribute to forming composite states because of the asymptotic non-freedom of the confining forces. The preons inside the quarks and leptons are, therefore, very tightly bound, that will be, perhaps, a physical reson for the small coupling constant of composite weak bosons. Although the dynamical situation of our preon theory is not obvious to us, we hope that the observation of the hidden gauge fields generated dynamically will give a new clue on the Fermi scale problem.

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Figure captions

Fig.(1): Regions where V-poles appear on a physical sheet of the complex momentum plane.