Clean Test of the Electroweak Theory by Measuring Weak Boson Masses

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Role of the weak boson masses in the studies of electroweak higher order effects is surveyed. It is shown that precise measurements of these masses give us quite useful information for performing a clean test of the electroweak theory, and for a heavy fermion search. Effects of supersymmetric particles in these studies are also discussed.

§1. Introduction

The standard SU(2)×U(1) electroweak theory, i.e., the Glashow-Weinberg-Salam theory,[1),2)] has been very successful, and widely accepted as a theory describing consistently the low energy weak phenomena. Furthermore, the discovery of the weak bosons[3)] has shown that it is also valid in the region up to $\sim 10^2$ GeV.

This success is, however, restricted to the analyses at the lowest order of the perturbation. Therefore, more precise tests beyond the tree approximation are indispensable as a next step. As a matter of fact, many authors have made efforts for this purpose, and consequently it is known that the higher order corrections to various cross-sections and decay-widths which are normalized by G_F, the Fermi coupling constant, are generally very small.[4)] We can thereby conclude that the success of the theory is not affected by the inclusion of the higher order effects. However, it is quite passive confirmation of the theory. How can we test the theory much more clearly? This is the main theme I would like to talk about here.

In relation to this problem, it is found by the studies of several authors that the weak boson masses, M_W and M_Z, take quite important roles. By the use of the theoretical relation between M_W and M_Z (the M_W-M_Z relation), we are able to make interesting investigations.[5),6)] I will survey those studies as follows: First I briefly summarize the calculations of the electroweak higher order effects (§2). Then, the M_W-M_Z relation is derived, numerically examined and its application to a heavy fermion search is described

(§3). Recently supersymmetric theories[7] are drawing attention of particle physicists in relation to the anomalous events at CERN $p\bar{p}$ collider,[8] so I show in §4 the effects of supersymmetric particles in the M_W-M_Z relation.[9] A conclusion is given in the final section.

§2. Electroweak Higher Order Effects

Higher order effects in the electroweak theory have been investigated for more than ten years. However, the purpose at an early stage was a rather theoretical one, i.e., the confirmation of the UV-divergence cancellation by concrete computations. It is after the phenomenological success of the theory (especially after the discovery of $W^{\pm}$ and Z bosons[3]) that particle physicists have become really interested in the experimental verification of these effects.

Let us summarize renormalization calculations. Necessary steps are as follows:

i) Fix a set of independent parameters through which we work.

ii) Introduce renormalization constants, and divide thereby the bare Lagrangian into the tree one (from which the Feynman rules are produced)[2] and the counterterms.

iii) Choose a subtraction scheme to fix the counterterms.

iv) Make actual calculations with a suitable regularization of the UV-divergence.

v) Determine the values of the renormalized parameters by taking appropriate input data, and substitute them into the results.

In the following I briefly explain the above steps.

i) The basic Lagrangian of the electroweak theory includes five kinds of independent parameters except for the Kobayashi-Maskawa mixing parameters.[10] They are g, g′(the SU(2) and U(1) coupling constants), μ, λ (the Higgs potential parameters) and g_f (the fermion-Higgs Yukawa coupling constant). We may, of course, work with any other combinations of them. Very convenient ones are e (the electric charge), M_W, M_Z, m_f (fermion mass) and m_ϕ (Higgs mass), which I adopt here in relation with the renormalization scheme.

ii) and iii) At present, several schemes are known.[4] In principle, all are equally good, but I think the on-mass-shell renormalization[2),11)] is easiest to understand for people who are familiar with the renormalization in QED. This is because the former scheme is the most natural extension of the latter. I describe this on-mass-shell scheme briefly. The renormalization constants are introduced as

$$W^\pm \text{ boson: } M_{W0}^2 = M_W^2 + \delta M_W^2 \,, \quad W_{0\mu}^\pm = Z_W^{1/2} W_\mu^\pm \,, \qquad (2\text{-}1)$$

Z boson and A (photon):

$$M_{Z0}^2 = M_Z^2 + \delta M_Z^2 \,, \quad \begin{pmatrix} Z_{0\mu} \\ A_{0\mu} \end{pmatrix} = \begin{pmatrix} Z_{ZZ}^{1/2}, & Z_{ZA}^{1/2} \\ Z_{AZ}^{1/2}, & Z_{AA}^{1/2} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \qquad (2\text{-}2)$$

$$\text{Electric charge: } e_0 = Ye \,. \qquad (2\text{-}3)$$

The other constants for the fermions, the Higgs boson, the Nambu-Goldstone bosons and the Fadeev-Popov ghosts are introduced in a similar way. These renormalization constants are determined by the on-mass-shell conditions. For example, the conditions for δM_W^2, Z_W, δM_Z^2 and Z_{ij} (i,j=Z,A) are

$$\mathrm{Re}\ \Pi^W(M_W^2) = \mathrm{Re}\ \Pi^{W\prime}(M_W^2) = 0\ , \tag{2-4 a}$$

$$\begin{aligned} &\mathrm{Re}\ \Pi^Z(M_Z^2) = \mathrm{Re}\ \Pi^{Z\prime}(M_Z^2) = \mathrm{Re}\ \Pi^{ZA}(M_Z^2) = 0\ , \\ &\Pi^A(0) = \Pi^{A\prime}(0) = \Pi^{ZA}(0) = 0\ , \end{aligned} \tag{2-4 b}$$

where Π^W, Π^Z, Π^A and Π^{ZA} are the transverse parts (the coefficients of $g_{\alpha\beta}$) of the $W^\pm$, Z, A and Z-A proper self-energies respectively. (Note that in the six conditions in Eq.(2-4 b), only five are linearly independent due to the remaining U(1) gauge symmetry.) Then, after similar applications for the other fields, we obtain the physical masses M_W, M_Z etc. and the properly normalized fields $W^\pm$, Z, A etc..

iv) Much efforts have been paid for evaluating radiative corrections for various processes[4]: $\nu_\mu e \to \nu_\mu e$, $\nu_\mu e \to \mu\nu_e$, $\nu_\mu q \to \nu_\mu q$, $eq \to eq$, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to Z\phi$, $\cdots$. As for the regularization of the UV-divergence, the dimensional method is often adopted. All those calculations are lengthy and tedious, and it is impossible to mention details of them. I only show the relevant Feynman diagrams for the one-loop correction to the muon decay-width Γ in Fig.2.1

as an example.

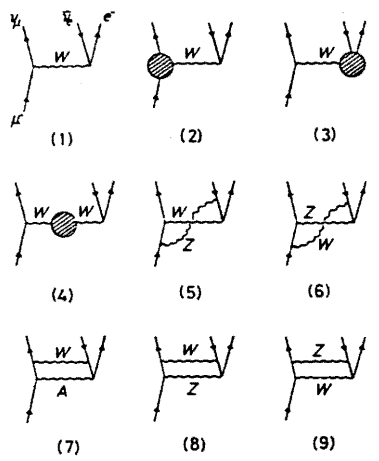

Fig.2.1.

Relevant diagrams for the one-loop correction to the muon decay. Blobs stand for all possible one-loop diagrams. (The scalar exchange diagrams are neglected as usual.)

v) In principle, any set of input data will do as long as we have very precise experimental information on them. Actually, however, we are interested in studying how the success of the theory at tree level is affected by the higher order contributions. Hence we should use the same input data as those in the tree analyses. There, the fine structure constant α^{exp}(= 1/137.036) and the muon decay-width Γ^{exp} which is commonly expressed in terms of G_F^{exp}(= (1.16632 ± 0.00002) $\times 10^{-5}$ GeV^{-2}) are always taken since their experimental uncertainties are remarkably small. In addition, various particle masses (except for M_W and M_Z)[#1] and the Weinberg angle $\sin^2\theta_W^{exp}$ ($\simeq$ 0.21-0.22) are usually used. (The superscript "exp" means the "experimental value".)

For the electric charge and the various masses, we can directly substitute the input data thanks to the on-mass-shell renormalization. (Concerning the top-quark mass m_t and the Higgs mass m_ϕ, we have to assume some appropriate values. The m_t and m_ϕ dependence of results will be discussed later, §§ 3.1 and 3.2.) On the other hand, a little preparation is necessary in order to determine the values of the remaining parameters M_W and M_Z by G_F^{exp} and $\sin^2\theta_W^{exp}$. Suppose that θ_W^{exp} is obtained, e.g., in $\nu_\mu e \to \nu_\mu e$ process, and let us express the corresponding amplitude (under a suitable approximation) as

$$\mathcal{A}(\nu e \to \nu e) = \bar{e}\gamma_\alpha\{A^{NC}(q^2)+B^{NC}(q^2)\gamma_5\}e\cdot\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu \quad . \qquad (2\text{-}5)$$

Similarly, the μ decay amplitude as

$$\mathcal{A}(\mu \to e\nu\bar{\nu}) = A^{CC}(q^2)\bar{e}\gamma_\alpha(1-\gamma_5)\nu^c\cdot\bar{\nu}\gamma^\alpha(1-\gamma_5)\mu \quad . \qquad (2\text{-}6)$$

Here, of course, A^{NC}, B^{NC} and A^{CC} are the functions of e, M_W, M_Z, m_f and m_ϕ. Then, two constraints from G_F^{exp} and θ_W^{exp} are written as

$$\frac{1}{4}\left[1+ \frac{A^{NC}(q^2)}{B^{NC}(q^2)} \right]\Bigg|_{q^2=\langle q^2\rangle_{exp}} \left(=1-\frac{M_W^2}{M_Z^2} \text{ at tree level } \right) = \sin^2\theta_W^{exp},$$

(2-7 a)#2

$$A^{CC}(0) \left(= - \frac{\pi\alpha M_Z^2}{2M_W^2(M_Z^2-M_W^2)} \text{ at tree level } \right) = - \frac{G_F^{exp}}{\sqrt{2}} \quad . \qquad (2\text{-}7 \text{ b})$$ #2,#3

These simultaneous equations lead to "indirect" determinations of M_W and M_Z as functions of α^{exp}, G_F^{exp}, $\sin^2\theta_W^{exp}$, m_f^{exp} and m_ϕ^{exp} (assumed value). For example, we get the well-known results from the tree expressions of A^{NC}, B^{NC} and A^{CC},

$$M_W^{(0)} \left(= \sqrt{\frac{\pi\alpha^{exp}}{\sqrt{2}G_F^{exp}\sin^2\theta_W^{exp}}} = \frac{37.2813}{|\sin\theta_W^{exp}|}\right) \sim 77 \text{ GeV} ,$$

$$M_Z^{(0)} (=M_W^{(0)}/\cos\theta_W^{exp}) \sim 88 \text{ GeV} , \qquad (2\text{-}8)$$

where "(0)" means the lowest order approximation. Similarly, at one-loop level,[4]

$$M_W^{(1)} \sim 79.2 \text{ GeV} , \quad M_Z^{(1)} \sim 90.5 \text{ GeV} . \qquad (2\text{-}9)$$

Radiative correction to a cross-section is estimated with thus determined parameters as

$$\Delta = \frac{\sigma^{(1)}(\alpha, M_{W,Z}^{(1)}, m_f, m_\phi) - \sigma^{(0)}(\alpha, M_{W,Z}^{(0)}, m_f, m_\phi)}{\sigma^{(0)}(\alpha, M_{W,Z}^{(0)}, m_f, m_\phi)} . \qquad (2\text{-}10)$$

(Here and in the following, we sometimes neglect the superscript "exp" for simplicity.)

As was mentioned in §1, it is known that resultant numerical results for various processes are very small, and it seems quite difficult to check Δ experimentally. For example, the corrections to $\sigma^{(0)}(\nu e \to \nu e)$ are[2]

$$
\begin{array}{ll}
[\, E_\nu^{Lab} \,] & [\, \Delta \,] \\
1 \text{ (GeV)} & 0.86 \ (\%) \\
10^2 & 0.89 \\
10^4 & 0.99
\end{array}
\qquad (2\text{-}11)
$$

Must we abandan any clean test of the higher order effects? No! The discovery of the weak bosons[3] has given us a new possibility. Equations (2-8) and (2-9) are "theoretical predictions" for M_W and M_Z, and should be compared with M_W^{exp} and M_Z^{exp}. If $M_{W,Z}^{(1)}$ are more favored than $M_{W,Z}^{(0)}$ definitely, it will be the first confirmation of higher order effects. It seems to become possible in the near future since the differences (the one-loop effects) $\Delta M_{W,Z} \equiv M_{W,Z}^{(1)} - M_{W,Z}^{(0)}$ are unexpectedly large. (As will be explained in §3.1, the main origin of this large effects is the logarithmic term of the form $\alpha \, \ell n(m_f/M_{W,Z})$.)

However we need a further device to realize this possibility since we cannot draw a definite conclusion from the mere comparison of $M_{W,Z}^{(0),(1)}$ with $M_{W,Z}^{exp}$. The reason is as follows: The results (2-9) have been derived from the fixed $M_{W,Z}^{(0)}$ (Eq.(2-8)), but the actual $M_{W,Z}^{(0)}$ have non-negligible uncertainties ($M_W^{(0)} = 77.9 \pm 1.7$ GeV and $M_Z^{(0)} = 88.8 \pm 1.4$ GeV) because the present data on θ_W include a rather large error, at least ~ 5 % ($\sin^2\theta_W^{exp} = 0.229 \pm 0.010$).[12] Therefore, the one-loop effects become totally unclear. Furthermore it will not be easy to make the precision of θ_W^{exp} much higher due to difficulties in neutrino experiments.

It is a theme of the next section to mention the necessary device.

§3. Weak Boson Mass Relation

Needless to say, the defect appeared in the end of the last section comes from the use of the data on θ_W. Therefore, let us eliminate θ_W from the analyses. That is, let us make analyses with Eq.(2-7b) or

$$\Gamma(\alpha, M_W, M_Z, m_f, m_\phi) = \Gamma^{exp}$$

only. Consequently, we will obtain the interrelation between M_W and M_Z instead of the separate predictions for them.[5] We call it the M_W-M_Z relation. First (§3.1), I explain this relation assuming that all particles are relatively light ($\lesssim$ 100 GeV), and subsequently study the effects of heavy particles (§3.2).

3.1. The M_W-M_Z relation and light particle effects

In order to derive more convenient form of the M_W-M_Z relation (the form in which M_W is calculated as a function of α, M_Z, m_f and m_ϕ), we separate A^{CC} into two parts: the tree part and the one-loop correction,

$$A^{CC} = A_0(\alpha, M_W, M_Z) + A_1(\alpha, M_W, M_Z, m_f, m_\phi). \quad (3\text{-}1)$$

The tree relation is obtained by using A_0 only as

$$M_W^{(0)} = M_Z \left[\frac{1}{2} \left(1 + \sqrt{1 - \frac{2\sqrt{2}\pi\alpha}{M_Z^2 G_F}} \right) \right]^{\frac{1}{2}} . \qquad (3\text{-}2)$$

Similarly at one-loop approximation

$$M_W^{(1)} = \left[M_W + \frac{M_W^3 (M_Z^2 - M_W^2)^2}{\pi\alpha M_Z^2 (2M_W^2 - M_Z^2)} A_1 (\alpha, M_W, M_Z, m_f, m_\phi) \right] \Bigg|_{M_W = M_W^{(0)}} . \qquad (3\text{-}3)$$

The explicit form of A_1 is found in Ref.2).

In the table 3.1 is given the numerical M_W-M_Z relation. The rather large one-loop effects, $|M_W^{(1)} - M_W^{(0)}| \sim 1$ GeV, observed in the table are mainly due to the coexistence of terms proportional to $\alpha \ell n\, m_f$ and $\alpha \ell n\, M_{W,Z}$ in the calculations. These terms are combined together in the final result, A_1, and produce the large logarithmic term $\alpha \ell n (m_f / M_{W,Z})$ (especially large for f = e, μ, u, d and s).

M_Z	$M_W^{(0)}$	$M_W^{(1)}$
90.0(GeV)	79.49	78.49
92.0	81.92	80.99
94.0	84.31	83.43

Table 3.1

The numerical M_W-M_Z relation. As for the quark and the Higgs masses, we have chosen $m_u = m_d = m_s = 0.1$(GeV), $m_c = 1.5$ (GeV), $m_b = 4.7$(GeV), $m_t = 30$(GeV) and $m_\phi = 10$(GeV).

Picking up only the log terms, we obtain an approximate formula:

$$M_W^{(1)} = \left[M_W + \frac{\alpha}{3\pi} \left\{ \frac{M_W(M_Z^2-M_W^2)}{2M_W^2-M_Z^2} \sum_f Q_f^2 \; \ell n\left(\frac{m_f}{M_W}\right) \right\} \right]_{M_W=M_W^{(0)}} . \qquad (3\text{-}4)$$

($\sum_f$ is the sum in all flavors and colors, and Q_f is the corresponding electric charge in $|e|$ unit.)

This large one-loop correction is expected to make a clean test of the theory possible. However, we need further studies since it also indicates that the two (and higher) loop effects may be non-negligible. Fortunately, we can easily estimate the size of $\sum_n [\, \alpha \, \ell n(m/M) \,]^n$ contributions by the well-known techniques: the operator analysis combined with the renormalization group equations.[13] I show here only the result, which is obtained by replacing $\frac{\alpha}{3\pi} \sum_f Q_f^2 \ell n(\frac{m}{M})$ in Eq.(3-4) by $\frac{1}{2}\{1-\frac{\alpha(M)}{\alpha}\}$ [14] :

$$M_W^{(1)} = \left[M_W + \frac{M_W(M_Z^2-M_W^2)}{2(2M_W^2-M_Z^2)} \left\{ 1 - \frac{\alpha(M_W)}{\alpha} \right\} \right]_{M_W=M_W^{(0)}} , \qquad (3\text{-}5)$$

where $\alpha(\mu)$ is the running coupling constant

$$\alpha(\mu) = \alpha\left\{ 1 + \frac{2\alpha}{3\pi} \sum_f Q_f^2 \ell n\left(\frac{m_f}{\mu}\right) \right\}^{-1} .$$

I again present the numerical results[14] summing the effects of Eq. (3-5) and the remaining $O(\alpha)$ contributions in Table 3.2. We see that the $[\alpha \; \ell n]^n$ $(n \geq 2)$ effects are less than 0.1 GeV. Furthermore the $\alpha^2 \ell n$ effects have been estimated by Sirlin[15], and been found much smaller.

M_Z	$M_W^{(0)}$	$M_W^{(1)}[\alpha]$	$M_W^{(1)}[\alpha+\alpha\ell n]$	$M_W[\alpha+\Sigma\alpha^n\ell n^n]$
90.0(GeV)	79.49	79.54	78.49	78.41
92.0	81.92	81.97	80.99	80.92
94.0	84.31	84.35	83.43	83.36

Table 3.2

$M_W^{(1)}[\alpha]$, $M_W^{(1)}[\alpha+\alpha\ell n]$ and $M_W[\alpha+\Sigma\alpha^n\ell n^n]$ represent the values with the $O(\alpha)$ (non-leading) effects only, with the full one-loop effects, and with the full one-loop correction plus the all leading logarithmic effects respectively.

The remaining ambiguities are the quark masses and the Higgs mass. Among them, the light quark mass problem can be avoided by using the data of the Drell-ratio with the dispersion technique.[#4] Moreover, it has been studied recently by another approaches.[17)] And it is known that the consequent ambiguity in $M_W^{(1)}$ is at most $\sim$ $\pm$ 0.05 GeV. Concerning m_t and m_ϕ, the change of the results is small if $m_t \lesssim 100$ GeV: $M_W^{(1)}$ decreases at most $\sim$0.1 GeV for m_ϕ=10 $\rightarrow$ 100 (GeV), and increases $\sim$0.2 GeV for m_t=30 $\rightarrow$ 100 (GeV). Therefore, we have now the results with quite satisfactory precision.

Let us proceed to a more concrete analysis. How precisely must M_W and M_Z be measured for an unambiguous test of the higher order effects? This has been assessed by Grządkowski et al.[18)] Suppose we get data on M_Z as $M_Z=M_Z^{exp}\pm\Delta M_Z^{exp}$. Then the calculated M_W has the following ambiguity,

$$M_W^{(n)} = M_W^{(n)}[M_Z^{exp}] \pm \frac{\partial M_W^{(n)}}{\partial M_Z}\Delta M_Z^{exp} \quad (n=0,1) . \qquad (3-6)$$

Since we can expect $M_{W,Z}^{exp} >> \Delta M_Z^{exp}$, we may set

$$\frac{\partial M_W^{(1)}}{\partial M_Z} \sim \frac{\partial M_W^{(0)}}{\partial M_Z} = [\frac{M_W}{M_Z} + \frac{\pi\alpha M_Z}{\sqrt{2}G_F M_W(2M_W^2-M_Z^2)}]_{M_W=M_W^{(0)}} .$$

Next, let us assume that $W^{\pm}$ boson mass is determined as $M_W=M_W^{exp} \pm \Delta M_W^{exp}$. Then we may conclude that $M_W^{(1)}$ is experimentally more favored than $M_W^{(0)}$ if, e.g., the following criterion is satisfied,

$$|M_W^{exp} - M_W^{(0)}[M_Z^{exp}]| \geq 2\sigma, \quad |M_W^{exp} - M_W^{(1)}[M_Z^{exp}]| \leq \sigma, \qquad (3-7)$$

where

$$\sigma \equiv \sqrt{(\frac{\partial M_W^{(0)}}{\partial M_Z})^2(\Delta M_Z^{exp})^2 + (\Delta M_W^{exp})^2} .$$

A much more convenient, sufficient condition is

$$|M_W^{exp} - M_W^{(1)}[M_Z^{exp}]| \leq \sigma , \quad \frac{1}{3}|M_W^{(1)}[M_Z^{exp}] - M_W^{(0)}[M_Z^{exp}]| \geq \sigma. \qquad (3-8)$$

The second equation shows clearly the precision needed to test the one-loop effects. For example, for M_Z=93 GeV, ΔM_W^{exp} has to be less than 0.31 GeV (0.23 GeV) for ΔM_Z^{exp}=0.1 GeV (0.2 GeV). ($\partial M_W^{(0)}/\partial M_Z$

$\simeq 1.19$)

These conditions are enough mild for our purpose since determinations of M_Z to within the error of ± 0.1 GeV and M_W to within ± 0.25 GeV can be expected in the near future.[19] (See also Ref.20) and references cited therein.) Therefore, we will be able to make a clean test of the electroweak theory as a renormalizable field theory. (A similar analysis shows that $\sin^2\theta_W$ should be determined within ~ 0.005 in absolute value for the same purpose.)[18]

3.2. Heavy particle effects

In the preceding discussions, we have assumed that heavy particles do not exist. How are those discussions modified if, e.g., the top-quark is not found in the region $\lesssim 100$ GeV? Let us investigate here such a situation.

So far, there have been various approaches to get information on heavy particles or upper (or lower) bound on their masses.[21] In relation to the present subject, Veltman's analysis is famous,[22] in which heavy fermion effects are examined by the ρ-parameter. However we have now only an upper bound on m_t ($\lesssim 310$ GeV)[23] due to the non-negligible uncertainty[12] in ρ^{exp}. In contrast with this, we can expect that much more useful analysis will become possible by the M_W-M_Z relation[6] if accurate values for $M_{W,Z}$ are obtained.

Let us first study heavy fermion effects. Various fermions appear in $A_1(\alpha, M_W, M_Z, m_f, m_\phi)$ through the $W^\pm$ boson self-energy. In addition, they also contribute to the $\nu\ell W^\pm$ vertex ($\ell=e,\mu$) since the counterterms for this vertex include the renormalization constants δM_Z^2, δM_W^2 and Z_W. (The charge renormalization constant Y also includes m_f effect, but it is only logarithmic.) In the on-mass-shell renormalization scheme[2),11)] (see Eqs.(2-4a,b)),

$$\delta M_Z^2 = -\mathrm{Re}\ \Pi^Z_{(U)}(M_Z^2),\ \delta M_W^2 = -\mathrm{Re}\ \Pi^W_{(U)}(M_W^2),\ Z_W = \mathrm{Re}\ \Pi^{W'}_{(U)}(M_W^2). \quad (3\text{-}9)$$

($\Pi^{Z,W}_{(U)}$ express "unrenormalized" quantities)

For example, the finite part of $\Pi^W_{(U)}$ (except for the coupling constants and numerical factor) is

$$\Pi^W_{(U)}(q^2)$$
$$\sim \int_0^1 dx\{m_1^2(1-x)+m_2^2x-2q^2x(1-x)\}\ell n\{m_1^2(1-x)+m_2^2x-q^2x(1-x)\}\ , \quad (3\text{-}10)$$

(m_1, m_2 : masses of fermions in the loop)

so we recognize that A_1 has m_f^2 dependence through $\delta M_{W,Z}^2$. The $\delta M_{W,Z}^2$ dependent part of A_1 is

$$[\ A_1(\alpha,M_W,M_Z,m_f,m_\phi)\ ]_{\delta M_{W,Z}^2} = \frac{\pi\alpha M_Z^2}{2(M_Z^2-M_W^2)^2}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right). \quad (3\text{-}11)$$

By combining this equation with Eq.(3-3), the m_f^2 dependent part of the M_W-M_Z relation reads

$$M_W^{(1)}$$

$$= [\ M_W + \frac{\alpha}{32\pi} C_{color} \frac{M_W M_Z^2}{(M_Z^2-M_W^2)(2M_W^2-M_Z^2)} \{\ m_I^2 + m_i^2 + \frac{4m_I^2 m_i^2}{m_I^2-m_i^2} \ell n(\frac{m_i}{m_I})\ \}\ ]_{M_W=M_W^{(0)}}, \quad (3\text{-}12)^{\#5}$$

for a doublet (ψ_I, ψ_i)$_L$ whose masses are m_I and m_i. (C_{color} is the color factor (=3 for quarks and =1 for leptons).) Furthermore

$$\longrightarrow \quad [\ M_W + \frac{\alpha}{32\pi} C_{color} \frac{M_W M_Z^2}{(M_Z^2-M_W^2)(2M_W^2-M_Z^2)} m_f^2\ ]_{M_W=M_W^{(0)}} \quad (3\text{-}13)$$

in the case m_I(or m_i)>>m_i(or m_I) and $M_{W,Z}$. ($m_f \equiv \max[m_I,m_i]$)

Before showing numerical results, let us examine large m_ϕ effect. Due to the same reason as the case of heavy fermions, $\delta M_{W,Z}^2$ include also m_ϕ^2 dependent terms, which are as follows in the limit of m_ϕ>> $M_{W,Z}$,

$$[\ \delta M_Z^2\ ]_{m_\phi^2} \longrightarrow - \frac{\alpha M_Z^4}{32\pi M_W^2(M_Z^2-M_W^2)} m_\phi^2 \quad ,$$
$$[\ \delta M_W^2\ ]_{m_\phi^2} \longrightarrow - \frac{\alpha M_Z^2}{32\pi (M_Z^2-M_W^2)} m_\phi^2 \quad . \qquad (3\text{-}14)$$

As is easily seen, the m_ϕ^2 terms cancel out in A_1.

There appear another seemingly non-negligible terms in the μ decay-width. They are in the contributions of the Nambu-Goldstone boson, χ, exchange,[#6] which are usually neglected because of the suppression factor $\sim m_e m_\mu / M^2_{W,Z}$. For example, the $\chi\chi\phi$ vertex is proportional to m_ϕ^2, so the diagram with the $\chi\phi$-loop-corrected χ propagator seems to produce a m_ϕ^4 dependent term! However, we can again confirm by some explicit calculations[24] that all such terms completely cancel out, and there remains, at best, $\ell n\ m_\phi$ dependence in the final result.[25]

Therefore, we conclude that the effect of the Higgs-scalar in the M_W-M_Z relation is small even if m_ϕ is very large, and the dominant contribution of heavy particles comes from the m_f^2 terms. We show the $M_W^{(1)}$-m_f curve[24] for the case $m_f=m_t>>m_b$ in Fig.3.1. We see

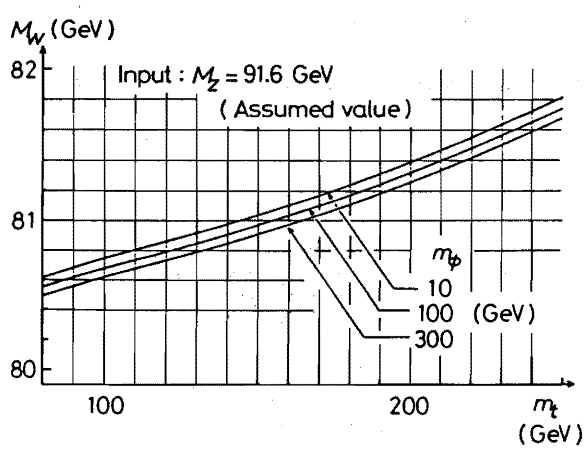

Fig.3.1 .

The $M_W^{(1)}$-m_t curve for an assumed value M_Z=91.6 GeV.
(This value satisfies M_W=81(GeV)=$M_W^{(1)}$[M_Z=91.6 and m_t=150].)

that the m_ϕ dependence of the result is in fact weak, and that precise determinations of $M_{W,Z}$ give us a valuable information on m_t. We can, of course, make a similar analysis for the fourth generation fermions if $m_t \simeq$ 30-50 GeV as the preliminary data[26] show.

Finally I wish to comment on the usual separate predictions for M_W and M_Z like Eq.(2-9) in relation with heavy fermion search. As an example, I take the frequently referred ones[27]:

$$M_W^{(1)} = 83.0 \begin{smallmatrix} +\ 3.0 \\ -\ 2.8 \end{smallmatrix} \text{ GeV} , \quad M_Z^{(1)} = 93.8 \begin{smallmatrix} +\ 2.5 \\ -\ 2.4 \end{smallmatrix} \text{ GeV} . \qquad (\text{ for } m_t=18 \text{ GeV }) \qquad (3-15)$$

If we get experimental data, e.g.,

$$M_W^{exp} = 84.0 \text{ GeV} , \quad M_Z^{exp} = 92.8 \text{ GeV} ,$$

how do you think? At first sight, the agreement of $M_{W,Z}^{(1)}$ and $M_{W,Z}^{exp}$ seems quite good. Actually, however, the difference $M_Z^{exp}-M_W^{exp}=$ 8.8 GeV is too small to be consistently fitted by the theory if m_t =18 GeV. As a matter of fact, we need $m_t \simeq 325$ GeV in order to make realize the relation $M_W^{exp}(=84.0)=M_W^{(1)}[M_Z^{exp}=92.8]$ in the framework of three generations and m_ϕ=10 GeV. (The value of m_t becomes much larger for larger m_ϕ.)

The reason why such a confusion occurs is apparent: The ambiguities in Eq.(3-15) come from only one origin, i.e., $\Delta \sin^2\theta_W^{exp}$, and are correlated with each other, so we are not allowed to consider a situation M_W=83.0+1.0 and M_Z=93.8-1.0. Such a trivial point is, of course, known by the authors.[27] Nevertheless I would like to stress that the presentation like Eq.(3-15) is quite misleading especially for non-experts, and the use of the M_W-M_Z relation is effective for avoiding such a trouble.

§4. Supersymmetry?

The electroweak theory has been so far very successful, which everyone recognizes. From the theoretical point of view, however, this theory includes several unsatisfactory points. In particular, the elementary scalar fields (a help of which we need in order to realize the desirable spontaneous symmetry breaking) cause the so-called "gauge hierarchy" problem when one proceeds to grand unification.[28] As is well-known, one possible way to avoid this is to make the theory supersymmetric.[7]

In addition, several "anomalous" events have recently been observed in the CERN $p\bar{p}$ collider,[8] a consistent explanation of which may be difficult within the present theory. Although we should wait for more accurate information before drawing some definite conclusions on them, they seem to allow an interpretation in terms of supersymmetric (SUSY) theories.[29]

Therefore it is significant to study SUSY effects in the M_W-M_Z relation.[30] I restrict here the discussion to the minimal supersymmetric extension of the electroweak theory with soft SUSY breaking terms. Furthermore, I only consider the case in which scalar-quark ($\tilde{q}$) masses (strictly speaking, the mass difference between scalar-

quarks in a same SU(2) doublet) are very large. This is because:
i) We can expect sizable effects by an analogy with the case of the ordinary quarks if $\tilde{q}$ is heavy,
ii) Otherwise, no large effects are expected and we cannot get any phenomenologically interesting results since the remaining small one-loop effects will depend on various undetermined parameters.

According to the preceding analyses, it is sufficient to examine the $\tilde{q}$-effects in $\delta M^2_{W,Z}$. I consider the effects of $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$ and $\tilde{d}_R$ (the superpartners of u_L, u_R, d_L and d_R). In contrast to the preceding cases, a little complication occurs due to the $\tilde{q}_L$-$\tilde{q}_R$ mixings (q=u,d) which are induced by the soft breaking terms. I use a notation where θ_q and $\tilde{m}_{q_i}$ denote the $\tilde{q}_L$-$\tilde{q}_R$ mixing angle and the eigenvalue of $\tilde{q}_L$-$\tilde{q}_R$ mass matrix (i=1,2).

By picking up $\tilde{m}^2_q$ dependent terms, the result in the limit $\tilde{m}_q \gg M_{W,Z}$ is[9])

$$M_W^{(1)} = \left[M_W + \frac{M_W^3}{2(2M_W^2-M_Z^2)} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right)_{\tilde{m}_q^2} \right]_{M_W=M_W^{(0)}} , \qquad (4\text{-}1\ a)$$

where

$$\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right)_{\tilde{m}_q^2} \longrightarrow \frac{3\alpha M_Z^2}{16\pi M_W^2 (M_Z^2 - M_W^2)}$$

$$\times \{ \cos^4\theta_u \tilde{m}_{u_1}^2 + \sin^4\theta_u \tilde{m}_{u_2}^2 + \cos^4\theta_d \tilde{m}_{d_1}^2 + \sin^4\theta_d \tilde{m}_{d_2}^2$$

$$+ \cos^2\theta_u \cos^2\theta_d L(\tilde{m}_{u_1}, \tilde{m}_{d_1}) + \cos^2\theta_u \sin^2\theta_d L(\tilde{m}_{u_1}, \tilde{m}_{d_2})$$

$$+ \sin^2\theta_u \cos^2\theta_d L(\tilde{m}_{u_2}, \tilde{m}_{d_1}) + \sin^2\theta_u \sin^2\theta_d L(\tilde{m}_{u_2}, \tilde{m}_{d_2})$$

$$- \cos^2\theta_u \sin^2\theta_u L(\tilde{m}_{u_1}, \tilde{m}_{u_2}) - \cos^2\theta_d \sin^2\theta_d L(\tilde{m}_{d_1}, \tilde{m}_{d_2}) \} . \quad (4\text{-}1\ b)$$

$$\left(L(x,y) \equiv \frac{4x^2y^2}{x^2-y^2} \ln \frac{y}{x} \right)$$

This formula becomes a much simpler form in the limit $\theta_q \to 0$:

$$\to \frac{3\alpha M_Z^2}{16\pi M_W^2(M_Z^2-M_W^2)} \{ \tilde{m}_{u_1}^2 + \tilde{m}_{d_1}^2 + \frac{4\tilde{m}_{u_1}^2 \tilde{m}_{d_1}^2}{\tilde{m}_{u_1}^2 - \tilde{m}_{d_1}^2} \ln(\frac{\tilde{m}_{d_1}}{\tilde{m}_{u_1}}) \} , \quad (4\text{-}2)$$

which is exactly the same as the ordinary heavy quark contribution. Therefore, we may be able to obtain some useful information on $\tilde{m}_q$ from accurate values $M_{W,Z}^{exp}$, but it will be less definite.

In addition to these studies, much efforts should also be paid, at present, to confirm whether the observed anomalous events really indicate the supersymmetry or not, and to make phenomenological analyses on them if it is true. Does the Nature demand a new physics?

§5. Conclusion

In this talk, I have shown how valuable information we can get from precise measurements of the weak boson masses for testing the electroweak theory.

If the top-quark mass lies, in fact, between 30 and 50 GeV as the preliminary UA1 report says, we are able to predict the value of M_W very accurately as a function of M_Z (except for quite small ambiguities from $m_{u,d,s}$ and m_ϕ), and consequently it will serve as a clean test of the electroweak higher order effects.

Conversely if the existing or the near-future accelerators fail to observe the top-quark, the M_W-M_Z relation will be useful for predicting its mass.

A similar analysis is also possible even though the present theory is obliged to be extended to a supersymmetric version (although some ambiguities are inevitable). Moreover, even if we are forced to consider a scheme in which the weak bosons are composite particles, the measurements of $M_{W,Z}$ are important and will give us strong conditions on a model building.

Anyway we hope that a clean test becomes possible for the electroweak theory, which is undoubtedly one of the most successful theories in the history of the particle physics.

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Footnotes

#1 Needless to say, we could not use the values $M_{W,Z}^{exp}$ before their discovery.

#2 In the one-loop-corrected A^{NC}, B^{NC} and A^{CC}, the purely electromagnetic (E.M.) effects are not contained due to the following reason. In the case of the muon decay, for instance, the following equation is used

$$\Gamma^{exp} = \frac{G_F^2 m_\mu^5}{192\pi^3} \left(1 - \frac{8m_e^2}{m_\mu^2} \right) \left\{ 1 + \frac{\alpha}{2\pi} \left(\frac{25}{4} - \pi^2 \right) \right\},$$

when expressing the data in terms of G_F. That is, the $O(\alpha)$ purely E.M. effects (including the effect of the real photon emission) have already been taken into account at this step.

#3 Strictly speaking, the constraint by G_F^{exp} should be expressed as

$$\Gamma(\alpha, M_W, M_Z, m_f, m_\phi) = \Gamma^{exp}$$

partly because the contribution of the Nambu-Goldstone boson exchange cannot be written in the form like Eq.(2-7b), but the resultant difference is negligible.

#4 As a matter of fact, the values used in Tables 3.1 and 3.2, $m_u = m_d = m_s = 0.1$ GeV, have been derived by this method.[16)]

#5 The $m_{I,i}$ dependence of this equation is same as that of the ρ-parameter. It is not accidental since $\delta M_{W,Z}^2$ contribution to ρ is

$$\rho = 1 + (\delta M_Z^2 / M_Z^2 - \delta M_W^2 / M_W^2) .$$

#6 In this case, we must use $\Gamma = \Gamma^{exp}$. (See footnote #3.)

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